Endogenous Mortality and the Quantity and Quality of Children

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Abstract

This paper examines the effect of endogenous mortality and parental care on the well-known quantity-quality model of population growth. We associate child quality to nutrition and other parental investments targeted to lower mortality. We show that, due to the quantity-quality trade-off, parents in poor countries have large incentives in favor of more births as changes to the average quality of children represents an expense that has to be applied to a large number of births. As income increases, the incentive for child quality raises so parents asymptotically prefer investments in child quality at the expense of a fixed number of births. In between, we observe an inverse U-shape relation between income and population growth characteristic of all demographic transitions. We test the trade-off with the use of twins as an instrument for unexpected increases in family size. To obtain a sufficiently large and representative data, we combine demographic and health surveys in 37 less developed countries.

Keywords: Mortality; population growth; economic development

JEL classification: I12; O11; O33.

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1 Introduction

The well-known quantity-quality model of fertility offers one explanation for the pervasive negative relation between income and fertility in developed countries but in its current form it cannot provide a unified view of modern demographic changes for several reasons.[1] First, although child mortality would appear to depend on any sensible measure of child quality, variations in mortality have not been systematically studied as part of the quantity-quality model leading to some ambiguity on the interpretation of the quantity of children either as the number of births or the number of survivors. Also, a unified model of the demographic transition must explain the positive relation between fertility and income at the early stages of development as Galor [22] advocates.[2] In the quantity and quality model, a positive relation between income and fertility would require quantity and quality to be complements instead of substitutes posing problems for the existence of a solution in the model. The thesis of this paper is that once child quality is related to factors influencing child mortality, the modern demographic transition and its aftermath can be seen as a unified movement from a low-quality/high-quantity equilibrium to a high-quality/low-quantity equilibrium.

In this paper we systematically examine the effect of endogenous child mortality on the quantity-quality model of population growth. We develop a model of lifetime fertility and parental care that follows Becker [2], and Becker and Lewis [3]. We assume that parents have preferences for the quantity and quality of children and are subject to a budget constraint in which quantity and

[1] Additional reasons for a negative relation between fertility and income in altruistic models of fertility are changes in children costs associated with the mother's time involved in child nurturing (Galor and Weil [25], and Jones [31]) and the intergenerational trade between parents and children (Ehrlich and Liu [17], and Boldrin and Jones [10]) Exogenous changes in mortality are also able to account for the decline in population growth if the uncertainty faced by parents in regards to child mortality is sufficiently important (Sah [45], Kalemli-Ozcan [32], Eswaran [18], and Doepke [16]). There are additional models that study population growth regardless of its cause (Galor and Weil [24] and Galor and Moav [23]).

[2] The negative relation between fertility and income also applies to the cross-section of countries (Birdsall [7]). It also seems to apply to modern within country data (Becker [2] and Turchi [50]), although there is also evidence of a lack of correlation between income and fertility (Freedman and Thornton [21]). Clark and Hamilton [13] present evidence of positive correlations between income and the number of survivors from wills of male testators in pre-industrial England circa 1600. Lee [34] also describes evidence of positive effects of income on population growth in pre-modern economies. Hunter-gatherers and traditional societies also exhibit a positive relation between resources or power and reproductive success, Hill and Kaplan ([30], 420)

quality interact. We also assume that investments in child quality reduce the risk of death for children. As in quantity-quality models, if the income elasticity with respect to child quality is sufficiently large, economic development leads to an increase in child quality and a reduction in child quantity that ultimately produces a decline in mortality, a decline in fertility, and a decline in population growth. Under non-homotetic preferences and endogenous mortality, the decline in population growth follows an initial population increase consistent with the inverse U-shape relation between income and population growth during the demographic transition (see Figures 1 and 2 below for the representation of the British demographic transition).

We show that under endogenous mortality, the marginal cost of child quality is a non-trivial function of quality whereas in Becker and Lewis [3] marginal costs are independent of their own arguments. This has two implications: First, we can rely on the shape and concavity of the survival function to ensure interior solutions at low income levels while isocost curves in Becker and Lewis [3] are quasi-concave so they have to rely exclusively on indifference curves for optimality. Second, the marginal cost of child quality is itself increasing in quality so parents are limited in the amount of resources they can use to achieve low mortality. Since high mortality requires a high number of births for reproduction, at low income levels parents might opt for more births since they are "less costly." As income increases, the relative cost of quality declines and the trade-off favors low mortality and low fertility.

Quantity-quality interactions under endogenous mortality also serve to study the effect that exogenous mortality changes have on population growth. Following Fernandez-Villaverde [19], Boldrin and Jones [10], Doepke [16] and Galor [22] (among others) argue that a decline in mortality could not have triggered the decline in population growth during the later stages of the demographic transition because lower mortality reduces the cost of survivors and increases the expected benefits from survivors in altruistic models of fertility (i.e., the Barro and Becker [1] model). Under endogenous mortality, an exogenous decline in mortality could also reduce the cost of child quality if the exogenous and the endogenous components in mortality are complements. If the cost of child quality also declines, an increase in child quality can potentially increase once again the cost of survivors and reduce population growth. Although it was not considered as part of a quantity-quality trade-off, the previous effect can be related to Cigno [11].

Considerations of endogenous mortality seem to be particularly relevant for traditional societies, current less developed countries, and the early stages of modern demographic changes. As Fogel [20] shows, changes in nutrition in early life are an important driving force for the secular decline in mortality. In the case of France, Weir [52] has shown that parental decisions regarding child quantity and quality had an effect on the health of children measured by their final height. Using marital fertility rates as a proxy for the relative price of child quantity, Weir [52] shows that the early fertility decline in France had a beneficial influence in stature (see also Schneider [46]). Similarly, the close birth spacing associated with high fertility is often seen as a cause of the elevated mortality risk within families in poor countries (see for example Cleland and Sathar [14] and Curtis et al. [15]). Olsen and Wolpin [38] and the surveys of Wolpin [53], Schultz [47] and Behrman and Deolalikar [6] include other examples of behavioral factors in child mortality often studied as inputs in a health production function but no consideration is provided to a direct quantity-quality interaction.

The scope of quantity-quality interactions extends well beyond behavioral aspects in human families. Work in biology has extensively studied optimal reproductive strategies that involve a form of quantity-quality trade-off.[3] The optimal balance between size and the number of offspring when there is a size-number trade-off was first studied by Smith and Fretwell [49]. They assumed that natural selection maximizes the expected number of survivors when the survival function is subject to diminishing returns to offspring size (quality). Their model (and also the bio-economic

[3]Biology sometimes classify species according to their reproductive strategies. In environments in which offsprings have a high probability of death, it is optimal for parents to produce a large quantity of offspring ($r-$selection) while in environments in which offsprings have plenty of chances to survive, it is optimal to aim for high quality ($k-$selection). Although both strategies are viable, a high number of births is only desirable in $r-$type situations, Pianka [39].

model of Robson [43]) produce an optimal strategy characterized by an unit elasticity between births and resources (or income). Hence, parents are set to divide resources equally among births and whenever more resources become available, they should be devoted toward more offspring. For this reason, pure biological concerns struggle with the modern demographic transition (Hill and Kaplan [30]).

The rest of the paper proceeds as follows: Section two describes the empirical motivation and the basis for the interpretation of quality. Section three describes the model, the main results, and some quantitative analyses. Section four considers some generalizations, section five an empirical test and section six concludes.

2 Interpreting Human Quality

Child quality can lead to multiple interpretations and the lack of adequate measures often serves to argue that quality is a "complex and somewhat esoteric" concept (Robinson [42], 65). We interpret child quality as a productive factor in child survival and a valuable attribute in utility. According to Becker [2], child quality represents a bundle of attributes that make children more valuable to parents. For its role in mortality, we regard child quality as synonymous to embodied forms of capital such as health, human, somatic, and physiological capital.

Although no specific example of quality was provided in Becker [2], following Becker and Tomes [4], the dominant view of child quality in economic models of population growth is that of education or human capital accumulation. In different forms, a negative relation between education and fertility can be found in quantity-quality models such as Becker and Tomes [4], Barro and Becker [1] and Becker, Murphy and Tamura [5]. This form of interactions is often the one most commonly tested in empirical analysis in economics (i.e., Rosenzweig and Wolpin [44], and Black et al. [8]) although not the only one available.

Anthropologist have long recognized the value of quantity-quality models in understanding human reproduction and have provided several attempts for testing the trade-off using the number of children or grandchildren as measures of fitness, or with proxies for fitness like child nutrition and growth (in addition to new estimates, Hagen et al. [28] provides a recent overview). Evidence shows that parents value child attributes differently and "preferentially invest, and reduce aggression towards, biological children, healthy children, children with higher intrinsic reproductive value, and the sex with the higher reproductive potential," Hagen et al. [28]. Evidence however has been mixed with regards to a trade-off between quantity and quality mostly in part by the lack of adequate controls for cofounding factors, Hill and Kaplan [30].

The low prevalence of twinning and lactation amenorrhea provide indirect evidence for a quantity-quality trade-off in human mortality. Hill and Kaplan ([30], 416) provide the following interpretation: "women's bodies cannot evolve the capacity to convert food onto milk at a rate high enough to support two infants simultaneously, that women cannot evolve the ability to carry two fetuses to term given optimal body size at birth, or that caring for one infant interferes with the successful care of a second." However, since women are usually able to raise infants simultaneously with older children, the suppression of ovulation by lactation and the lack of twinning do not unambiguously indicate a trade-off, Hill and Kaplan [30].[4]

3 Model and Assumptions

Assume that parents have preferences over child quality c and child quantity n given by the number of survivors. Preferences are represented by $U : \mathbb{R}^2_+ \to \mathbb{R}_+$. The number of surviving children n is the product of the number of births b and the child survival rate $P(c;\theta)$ that depends on child quality c and θ, an exogenous parameter that reflects environmental aspects or public health. Parents receive an endowment of income y, and allocate quality equally among all births.[5]

[4] Twins and multiple births have several times higher mortality rates and the effects of income or socioeconomic characterisitics are amplified for twins compared to singleton births (Guo and Grummer-Strawn [27]).

[5] As in Becker and Lewis [3], we take the quantity-quality interaction in the budget constraint as given but such budget constraint, $cb = y$, can be seen as the result of a symmetric allocation in which parents first assign quality between different births in order to maximize the expected number of survivors.

Additional goods and different time periods can be included but at this stage they just represent unnecessary complications.

Assumption 1 establishes some regularity conditions on preferences and the survivor function:

Assumption 1. $U(c,n)$ is strictly increasing, strictly quasi-concave in (c,n) and $P(c;\theta)$ is strictly increasing and strictly concave in c, and increasing in θ.

Optimal allocations: Optimal allocations are given by continuous functions $\{n(y,\theta), c(y,\theta)\}$ that satisfy the budget constraint and the first order condition:

$$\frac{U_n(c,n)}{U_c(c,n)} = \frac{\Pi(c,n;\theta)}{\Theta(c;\theta)}, \tag{1}$$

with $\Pi(c;\theta) \equiv cP(c;\theta)$ and $\Theta(c,n;\theta) \equiv n\{P(c;\theta) - cP_c(c;\theta)\}$ as the marginal costs of child quantity and quality.[6] As in standard allocation problems, $\Pi(c;\theta)/\Theta(c,n;\theta)$ represents the marginal rate of transformation between child quantity and quality; which must equal the marginal rate of substitution, $U_n(c,n)/U_c(c,n)$.

The second order condition is:

$$\Delta = -\left(\frac{\Pi}{U_n}\right)^2 |\mathbf{H}| + U_n \Pi P_c \left[\frac{cP_{cc}}{P_c} + 2\left(\frac{P - cP_c}{cP_c}\right)\right] < 0,$$

with $|\mathbf{H}|$ as the determinant of the bordered Hessian of U, positive under strict quasi-concavity, and with the expression inside square brackets negative if P is sufficiently concave or if $c\Theta_c/\Theta \geq 2$.

Second order conditions, in contrast to Becker and Lewis [3], are not forced by the patterns of substitution between n and c so long $P(c;\theta)$ is sufficiently concave. Also, marginal costs are not independent of their own arguments so the marginal cost of child quality varies with quality itself whereas in Becker and Lewis [3] marginal costs of quality only depend of quantity and viceverza.

[6] The outcomes of Becker and Lewis [3] follow if we substitute the budget constraint by $cn = y$. In this case, marginal costs are $\Pi = c$ and $\Theta = n$. When $P = 1$ and $P_c = 0$, both settings are identical.

As in Becker and Lewis [3] and Blomquist [9], the first order condition (1) can be seen as the outcome of a utility maximization subject to a linearized budget constraint in order to derive compensated changes. Instead, we study comparative statics based on observable changes in income and public health. For that we make a standard assumption.

Assumption 2. c and n are non-inferior goods under a linear budget constraint in which $\Pi(c;\theta) = \Pi$ and $\Theta(c,n;\theta) = \Theta$.

Comparative statics: The following relation can be established through comparative statics:

$$b_\theta(y,\theta) = -\frac{bP_\theta}{(-\Delta)}\left[U_{cn}\Pi - U_{nn}\Theta\right] + \frac{U_n}{(-\Delta)}\left[P_\theta - cP_{c\theta}\right].$$

The first term is related to the standard income effect on the number of survivors under a linear budget constraint (or under constant prices) and the second term depends on the direct and indirect changes in the survival probability. Under Assumption 2, the first term is negative suggesting that an improvement in public health reduces the number of births. If c and θ are complements in mortality, public health further reinforces a fertility decline because higher θ increases quality investments that can only take place if the number of births declines.

The effect of income changes on fertility is:

$$b_y(y,\theta) = \frac{1}{(-\Delta)b}\left[U_{cn}\Theta - U_{cc}\Pi\right] - \frac{P_c}{(-\Delta)}\left[U_{cn}\Pi - U_{nn}\Theta\right] - \frac{U_nP_c}{(-\Delta)b}\left[\frac{cP_{cc}}{P_c} + \left(\frac{P - cP_c}{cP_c}\right)\right], \quad (2)$$

where the first two terms are again related to the standard income effects in quality and quantity, while the last element is due to the quantity and quality interaction under endogenous mortality. Under Assumption 2, since quality investments increase the chance of survival, a relatively large income effect on quality is able to produce a negative sign in (2).

Population changes: We can relate the previous changes in fertility to changes in population growth through the following expressions: $n_\theta(y,\theta) = P_\theta b(y,\theta) + Pb_\theta(y,\theta)$, and $n_y(y,\theta) =$

$P_c + \Theta b_y(y,\theta)$. Since $\Theta > 0$, negative changes in fertility will translate into negative changes in population growth if the birth response to income is sufficiently large. The effect of changes in public health also involve a purely mechanical effect in P_θ and a behavioral change in $b_\theta(y,\theta)$.

We next consider allocations when income tends to infinity and restrict the utility function to obtain a bounded fertility rate.

Asymptotic allocations: Assume that income tends towards infinity. We are interested in finding an allocation in which fertility rates are constant and quality increases with income. For that we consider two additional assumptions:

Assumption 3. $\lim_{y\uparrow\infty} P(y/b;\theta) = 1$ and $\lim_{y\uparrow\infty}\{P_c(y/b;\theta)(y/b)\} = 0$ for all θ and b.

Assumption 4. $U(c,n) = \varphi(c)\phi(n)$, with $\phi_n(n)n/\phi(n)$ non-trivial in n and

$$\lim_{c\uparrow\infty} \frac{\varphi_c(c)c}{\varphi(c)} = \sigma < \infty.$$

Assumption 3 states that mortality is eradicated as income increases and that the marginal effects of quality in mortality are bounded. Assumption 4 imposes a particular functional form on the utility function so that in the limit, the first order condition (1) becomes $\sigma = \phi_n(b)b/\phi(b)$, with a bounded b provided that the elasticity of $\phi(n)$ is bounded.[7] If the conditions on Assumption 4 are reversed and we assume that ϕ has a constant elasticity, the quantity-quality trade-off will settle at a constant quality and an infinite amount of births.

Logarithmic utility: Consider the following utility functions $\varphi(c) = (c+\mathbf{c})^\sigma$ and $\phi(n) = (n-\mathbf{n})^\varepsilon$ with $\sigma > \varepsilon$. Both functions have non-trivial elasticities but in the limit (as $c\uparrow\infty$), the elasticity of $\varphi(c)$ is σ. The elasticity in ϕ is non-trivial in n as Assumption 4 requires since parents want to have at least $\mathbf{n}$ survivors. A baseline quality $\mathbf{c}$ in $\varphi(c)$ serves to reduce the marginal value of

[7] A constant asymptotic fertility requires an homogeneity restriction similar to the one found in balanced growth models, see Nikaido ([37], Chap. 11).

quality at low income levels but it has no effect at high income levels.[8] Using a log transformation, we can consider: $U(c,n) = \sigma \ln(c + \mathbf{c}) + \varepsilon \ln(n - \mathbf{n})$ with the optimal number of births given by the solution to: $\varepsilon [P - P_c c] \{c + \mathbf{c}\} - \sigma c [P - \mathbf{n}/b] = 0$.[9]

When mortality is exogenous and $\mathbf{c} = 0$, the optimal policy involves a target fertility rate given by $bP = \sigma\mathbf{n}/(\sigma - \varepsilon)$. Under exogenous mortality and $\mathbf{c} > 0$, population growth and income are negatively related at all income levels (i.e., there is no inverse U-shape relation between income and population growth) and under endogenous mortality, when $y \uparrow \infty$, fertility also satisfies a target $b = n = \sigma\mathbf{n}/(\sigma - \varepsilon)$ so the quantity-quality trade-off will settle at a constant fertility rate determined exclusively by preference parameters.

Public health: The relation between public health and fertility and population growth is:

$$b_\theta(y,\theta) = \frac{-(\sigma - \varepsilon)cP_\theta - \varepsilon c(c + \mathbf{c})P_{c\theta}}{(-\Delta)}, \text{ and}$$

$$n_\theta(y,\theta) = \left[b - \frac{(\sigma - \varepsilon)cP}{(-\Delta)}\right] P_\theta - \frac{\varepsilon c(c + \mathbf{c})PP_{c\theta}}{(-\Delta)}.$$

Thus, improvements in public health reduce the number of births if c and θ are complements in mortality (i.e., $P_{c\theta} > 0$) and they also reduce population growth if besides complementarity, the income effect on quality is large (i.e., $(\sigma - \varepsilon)\, cP > -b\Delta$).

As in Fernandez-Villaverde [19] and Barro and Becker [1], the model still generates the first order effect that suggests that low mortality should increase population growth since it reduces the cost of survivors. However, as Cigno [11] notes, the fact that c and θ are complements in mortality implies that quality also increases as θ changes so population growth and fertility must decline to accommodate the change in public health.[10]

[8] Greenwood et al. [26] also rely on a baseline quality but they offer an alternative interpretation from household production models.

[9] The second order condition can be written as

$$\Delta = \frac{\varepsilon c^2 P_c}{b}\left[\frac{(c + \mathbf{c})\, P_{cc}}{P_c} + \frac{\mathbf{c}}{c}\left(\frac{P - cP_c}{cP_c}\right) + \frac{\sigma}{\varepsilon} - \frac{\sigma}{\varepsilon}\frac{\mathbf{n}}{cP_c b}\right],$$

so when $y \uparrow \infty$, Δ tends to zero from below.

[10] There is no direct evidence that suggests that c and θ are complements or substitutes in P. From a purely

Income: The income-fertility relation satisfies:

$$b_y(y,\theta) = \frac{\varepsilon c P_c}{(-\Delta) b}\left[-\frac{(c+\mathbf{c})\,P_{cc}}{P_c} - \frac{\sigma}{\varepsilon} - \frac{\mathbf{c}}{c}\left(\frac{P - cP_c}{cP_c}\right)\right], \text{ and}$$

$$n_y(y,\theta) = \frac{\varepsilon c P_c P}{(-\Delta) b}\left[-\frac{(c+\mathbf{c})\,P_{cc}}{P_c} - \frac{\sigma}{\varepsilon}\left(1 - \frac{\mathbf{n}}{n(y,\theta)}\right) - \frac{\mathbf{c}}{c}\left(\frac{P - cP_c}{cP_c}\right)\right].$$

The first term in the square brackets is positive since $P_{cc} < 0$ by Assumption 1. The second and third terms are related to the quantity-quality trade-off with σ/ε and $\mathbf{c}$ favoring a negative relation. The effects of quality in population growth however are required to be stronger since the differentials in income elasticities have a positive effect in population growth (instead of a negative one as in the case of births). If baseline quality $\mathbf{c}$ succeeds in lowering the demand for quality at low income levels, population growth and income will display a positive association at low levels of income and a negative relation at high income levels. Asymptotically, income will have no effect in fertility since the number of births and population growth will be set according to a fixed target.

Assumption 5. $P(c) = 1 - (c - \bar{c})^{1-\alpha}$, with $\bar{c} > 0$, and $\alpha > 1$.

Assumption 5 considers that public health and child quality vary according to changes in subsistence. Without loss of generality we ignore the dependence of P on θ. We can state the following result for the relation between fertility and income:

Proposition 1 *Let Assumptions 1-5 hold. Assume that households have a logarithmic utility function and that $1 + \alpha > \sigma/\varepsilon > \alpha$. Then, the sign in the relation between fertility and income is:*

$$\mathsf{sgn}\left\{b_y(y,\theta)\right\} = \mathsf{sgn}\left\{\alpha\left(\frac{c+\mathbf{c}}{c-\bar{c}}\right) - \frac{\sigma}{\varepsilon} - \frac{\mathbf{c}}{(\alpha-1)c^2}\left[(c-\bar{c})^{\alpha} - \alpha c + \bar{c}\right]\right\},$$

mechanical point of view, evidence that relates infectious diseases to nutrition and mortality seems to suggest that c and θ tend to be substitutes (i.e., $P_{c\theta} < 0$) instead of complements because as public health improves, the nutritional demands from a given diet decline (Scrimshaw et al. [48]).

with

$$\lim_{c\downarrow\bar{c}}\left[\mathsf{sgn}\left\{b_y(y,\theta)\right\}\right] > 0 \geq \lim_{c\uparrow\infty}\left[\mathsf{sgn}\left\{b_y(y,\theta)\right\}\right],$$

so there exists a unique quality level c^* such that fertility is an increasing function of income if $\bar{c} < c(y,\theta) \leq c^*$ and a decreasing function of income if $c(y,\theta) > c^*$.

Proof. The proof follows from the standard application of the Intermediate Value Theorem because $b_y(y,\theta)$ is a continuous function in $(\bar{c},\infty)$. The first expression inside square brackets tends to infinity as $c \downarrow \bar{c}$ while the second and third terms tend to a constant $-\sigma/\varepsilon + \mathbf{c}/\bar{c}$. Thus, fertility is increasing in income near $\bar{c}$. When consumption tends to infinity, the first term tends to α so if the second term is such that $\sigma/\varepsilon > \alpha$ the sign of $b_y(y,\theta)$ would be negative. If $\alpha < 2$, the third term vanishes as c increases since the denominator is of order 2. ■

If we assume that $\mathbf{c} = 0$, it is possible to show that $c^* = \bar{c}/(\sigma/\varepsilon - \alpha) > \bar{c}$. Therefore, fertility and income will be positively related if child quality is below c^* or if income is below y^* defined implicitly as $c^* = b(y^*;\theta)/y^*$. As income increases, the relative cost of quality decreases and parents find it optimal to assign more resources to a fewer number of births. Since the date of the switch depends on $\bar{c}$ and α, public health can be seen as affecting the transition from a regime with a positive income-fertility relation to one with a negative relation.

We can extend the previous finding of an inverse U-shape relation between income and fertility to population growth:

Proposition 2 *Let Assumptions 1-5 hold. Assume in addition that $\alpha \geq 2$. Then, the sign in the relation between population growth and income is:*

$$\mathsf{sgn}\left\{n_y(y,\theta)\right\} = \mathsf{sgn}\left\{\alpha\left(\frac{c+\mathbf{c}}{c-\bar{c}}\right) - \frac{\sigma}{\varepsilon}\left(1 - \frac{\mathbf{n}}{n(y,\theta)}\right) - \frac{\mathbf{c}}{(\alpha-1)c^2}\left[(c-\bar{c})^{\alpha} - \alpha c + \bar{c}\right]\right\}, \tag{3}$$

with

$$\lim_{c\downarrow\bar{c}}\left[\mathsf{sgn}\left\{n_y(y,\theta)\right\}\right] > 0 \geq \lim_{c\uparrow\infty}\left[\mathsf{sgn}\left\{n_y(y,\theta)\right\}\right],$$

so there exists a unique quality level $c^{**} > c^*$ such that population growth is an increasing function of income if $\bar{c} < c(y,\theta) \leq c^{**}$ and a decreasing function of income if $c(y,\theta) > c^{**}$.

Proof. The proof is as before, but we should note that the negative effects that the quantity-quality trade-off impose are no longer positive since $n(y,\theta) \geq \mathbf{n}$. In addition, for a single reversal in the effect of income on population growth we need $\alpha \geq 2$. If $\alpha < 2$, the third term vanishes as before when c increases so there is an additional rise in fertility at very high levels of income due to the fact that the first term converges to α. ■

The reversal in the relation between income and fertility is due solely to changes in the cost of quality while the reversal in the relation between income and population growth depends on baseline quality $\mathbf{c}$. A positive level of subsistence consumption $\bar{c}$ in mortality increases the cost of quality at low income levels while a positive $\mathbf{c}$ reduces the marginal benefit from quality investments. Both factors lower quality demanded for poor households. A similar effect can be attributed to $\mathbf{n}$ since low population growth creates a high value for quantity when mortality is high (or income is low). As income increases, the marginal cost of quality declines and the marginal benefit rises so parents move to the other side of the quantity-quality trade-off. As in the case of births, public health changes will affect the pace and timing of the demographic transition according to the relation between endogenous and exogenous mortality changes.[11]

The divergence in the costs of quantity and quality follows from our assumptions in the production of survivors. An increase in b or in c raise population growth but the marginal product of quality is decreasing in c by the concavity in the survivor function (while the marginal product of b is constant). Since $\Theta = n(P - cP_c) > 0$ and $\Theta_c = -nP_{cc}c > 0$, quality additions are not only expensive for parents with many children but they also have increasing marginal costs due to the concavity of the survivor function. As quality declines to $\bar{c}$, the curvature of the survivor function increases so the cost of changes in quality also increases steeply. In summary, additions to the

[11] The relation between preference parameters and tuning points $\{c^*, c^{**}\}$ is in general ambiguous but if $\bar{c} = 0$ and $\alpha = 2$, an increase in the baseline value of quality $\mathbf{c}$ will reduce c^* and c^{**} while an increase in $\mathbf{n}$ increases c^{**}. The next section considers numerical examples that illustrate the directions of change.

quality of children at very low income levels are "too costly" since they have diminishing marginal productivity and must be spread over a high number of births.

Numerical Examples: This sub-section constructs numerical examples to illustrate the features of the model. Although we try to ground the parameters of the model on data, there are no estimates for some preference parameters so we consider asymptotic allocations. We assume that asymptotic population growth is zero and assume that the lower bound in the number of survivors is $\mathbf{n} = \frac{1}{2}$ (as in a one-child policy). From the asymptotic population growth, $\sigma\mathbf{n}/(\sigma - \varepsilon) = 1$, so we obtain $\sigma/\varepsilon = 2$.[12] We set $\alpha = 1.2$ based on aggregate estimates of the elasticity of income to infant mortality that range between -0.2 and -0.4 (see Pritchett and Summers [41] for one example).

Table 1. Baseline simulations.

	Child survival rates		Peak values of: Income		Peak values of: Fertility	Peak values of: Population growth
	$P(\underline{y})$	$P(\bar{y})$	y^*	y^{**}	$b(y^*)$	$n(y^{**})$
England and Wales (1540-2000)	0.49	1.00	450	450	2.88	1.70
Baseline model	0.48	0.84	309	333	3.48	1.92

Note: Peak values include the income levels at which fertility and population growth peak (y^* and y^{**} respectively). Mortality values in England are from 1541 and 2000. In England and Wales, NRR and GRR peaked in 1821 (Wrigley et al. [54], Appendix 9). The income at the turning point is assumed as one third of the average of the real GDP (in 1985 dollars) between 1800 and 1850 from Lucas ([35], Table 5.3).

We assume that 450 dollars is the lower bound on income per-capita available for poor countries but we scale spending in children by one-third, i.e., $\underline{y} = 150$. According to Lucas ([35], Table 5.2), Africa, the region with the lowest income per-capita, had an income of 455 dollars in 1750. We increase spending until it reaches 10,000 dollars; variations in behavior beyond this level are very small to be considered interesting. We take subsistence consumption to be $\bar{c} = 40$ since $P(\underline{y}) = 0.5$. Baseline quality is more difficult to infer since $\mathbf{c}$ has no effect at high income levels. The baseline case considers $\mathbf{c} = 100$. Greenwood et al. ([26], Table 1) present estimates of $\mathbf{c}$ to study the baby

[12] In the quantity-quality model of Greenwood et al. [26], $\varepsilon = 1.2$. The absolute values of ε and σ have no effect on our problem. The only factor that is relevant for the model is the ratio σ/ε.

boom with values of $\mathbf{c} = 49$. This alternative value lowers the peak of population growth and the fertility rate but still preserves the inverse U-shape relation between income and population growth.

Table 1 includes the results of the baseline simulation and Figure 2 displays the paths of fertility and population growth under the alternative scenarios. The table also includes information from England and Wales that indicates that the peak in fertility and population growth was obtained in 1821 with a NRR of 1.7 and GRR of 2.9.[13]

The model produces an inverse U-shape relation between population growth and income as in the demographic transition. The turning point for population growth, in terms of income is $1,000 dollars. As in the data, when income increases asymptotically mortality, fertility and population growth decline although the model suggests mortality values that are "too high." At total incomes of $450 (in 1985 dollars), mortality is near 50 percent as in most pre-modern economies, but when income is $30,000, mortality is 15 percent while current developed countries have survival rates of almost 100 percent. The values produced by the model are also higher than in the data.

We consider marginal variations in the parameters $\bar{c}$, α, and $\mathbf{c}$ in Figures 1 and 2. The figures also depict the time series (per decade) between population growth, fertility and income for England and Wales for nearly 500 years and a 4th-order polynomial trend (different orders have no effect on the pre-modern data and only serve to capture the baby boom). The results from the simulations indicate that a lower subsistence consumption $\bar{c}$ increases population growth and the income level at which fertility and population growth peak. An increase in α, or in the effect of quality in mortality, also increases population growth but it reduces the income required to reach the turning point. If we assume that public health made a given quality more effective in achieving low mortality, we should expect recent demographic transitions to occur at lower income levels and with higher population growth rates (as they actually did).[14]

[13] Galor [22] argues that income changes produce counter-factual results since incomes in 1870 in England were twice the income of Germany but their demographic transitions occurred in the same decade. However, population

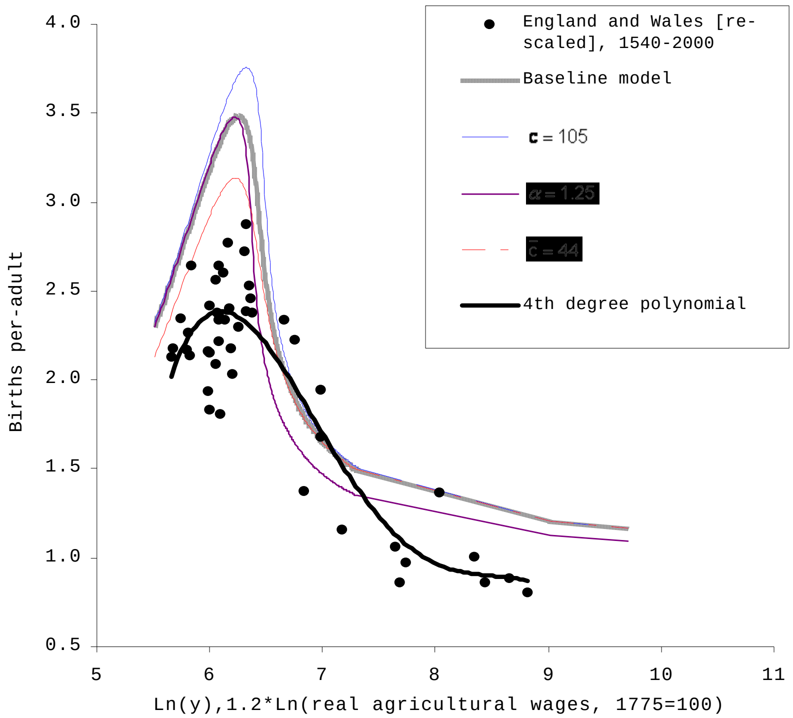

Figure 1: – Fertility rates as a function of child spending. Baseline parameters as in Table 1 and reparametrization as described in the figure. Time series for fertility (GRR) and log of real agricultural wages (1775=100) in England and Wales, 1540-2000, from Wrigley et al. [54], Keyfitz and Flieger [33], Clark [12], and Mitchell [36]. Log-wages multiplied by 1.2 to make the figures comparable. The 4th-order polynomial trend is applied to the data for England and Wales.

Two generalizations (the budget constraint and additional goods): The trade-off between quantity and quality is not the only, and perhaps not the main, decision in regards to fertility since intrahousehold allocations vary with gender composition of the children and additional economic and social influences. In this section we only sketch two possible generalizations: changes in the budget constraint and in the number of goods in the model assuming that preferences for quantity and quality remain unaltered.[15] We also discuss the role of quality in non-human families and the

growth peaked in England and Wales near 1825, at least 50 years before Germany.

[14] If we assume that $\mathbf{c} = 50$, as Greenwood et al. [26], we obtain a small value of population growth and low incomes associated with the turning points. If the value of quality increases through changes in σ/ε, we also reduce the income needed to achieve a demographic transition and we obtain lower peak values of population growth and fertility.

[15] Straightforward extensions also include the possibility of risk in income. It is well-known that if the first order condition is convex in income, parents will have a precautionary demand for births. Convexity however, depends on $\mathsf{sgn}\{P_{ccc}\}$ being not too negative and no prior information serves to sign the expression.

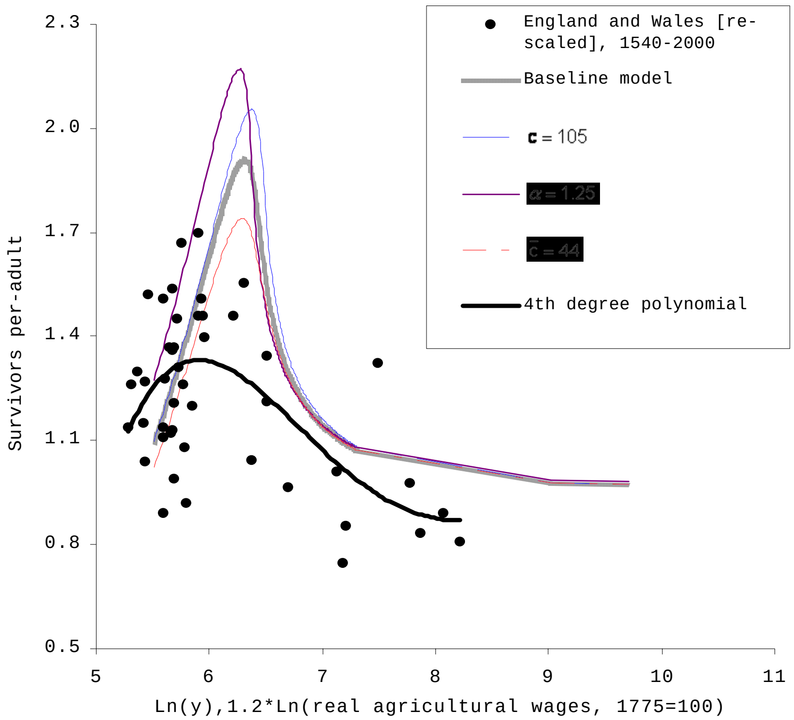

Figure 2: − Population growth as a function of child spending. Baseline parameters as in Table 1 and reparametrization as described in the figure. Time series for population growth (NRR) and log of real agricultural wages (1775=100) in England and Wales, 1540-2000, from Wrigley et al. [54], Keyfitz and Flieger [33], Clark [12], and Mitchell [36]. Log-wages multiplied by 1.2 to make the figures comparable. The 4th-order polynomial trend is applied to the data for England and Wales.

behavioral conditions consistent with a demographic transition.

Consider cost components related to an interaction between births and child quality, births, quality, survivors and an interaction between survivors and child quality. Costs for each component correspond to $\boldsymbol{\pi}_i$ with $i = cb, b, c, n, nc$. From the first order conditions, augmented marginal costs to quality and quantity are given by:

$$\hat{\Theta}(c, n; \theta) = \boldsymbol{\pi}_{cb}\left[\frac{n\,(P - cP_c)}{P^2}\right] - \boldsymbol{\pi}_b \frac{nP_c}{P^2} + \boldsymbol{\pi}_c + \boldsymbol{\pi}_{cn} n, \text{ and}$$

$$\hat{\Pi}(c; \theta) = \frac{\boldsymbol{\pi}_{cb} c}{P} + \frac{\boldsymbol{\pi}_b}{P} + \boldsymbol{\pi}_n + \boldsymbol{\pi}_{cn} c.$$

Hence, it is possible to show that

$$\hat{\Theta}_c(n, c; \theta) = -\boldsymbol{\pi}_{cb} nc \left(\frac{P_{cc}}{P^2}\right) + 2n\boldsymbol{\pi}_b \left(\frac{P_c}{P^2}\right) \left[\frac{P_c}{P} - \frac{1}{2}\frac{P_{cc}}{P_c}\right], \text{ and}$$

$$\hat{\Pi}_c(c; \theta) = \frac{1}{P}\left[\boldsymbol{\pi}_{cb} - \frac{(\boldsymbol{\pi}_{cb} c + \boldsymbol{\pi}_b) P_c}{P} + P\boldsymbol{\pi}_{cn}\right].$$

1Since $-P_{cc}$ and P_c are positive, a quality level that increases mortality sufficiently at low income levels creates an increase in the marginal cost of child quality and a large reduction in the marginal cost of child quantity. That is, we can find a $\bar{c}$ as before, such that: $\lim_{c \downarrow \bar{c}} \hat{\Theta}_c(n, c; \theta) > 0 \geq \lim_{c \downarrow \bar{c}} \hat{\Pi}_c(c; \theta)$. Thus, even for extended cost structures, at low income levels parents might prefer to invest in more births than in higher child quality since marginal changes in child quality are more costly due to the concavity in the survivor function and to the large number of births needed in environments with high mortality.

As income increases asymptotically, the bias toward child quantity disappears.

Corollary 1 *Under the conditions above, as income increases, the slope of the marginal cost of child quality tends to zero while the slope in the marginal cost of child quantity tends to a positive constant determined by the quantity-quality trade-off. That is,* $\lim_{c \uparrow \infty} \hat{\Pi}_c(c; \theta) = \boldsymbol{\pi}_{cb} + \boldsymbol{\pi}_{cn} > 0$ *and* $\lim_{c \uparrow \infty} \hat{\Theta}_c(c, n; \theta) = 0$.

An interaction between child quality and child quantity, even if quantity is considered in terms of the number of survivors rather than in terms of births, preserve the asymptotic incentives in favor of child quality. However, the endogeneity of mortality will make the predictions of the quantity-quality interaction indistinguishable from costly births. If all interaction terms are omitted and only $\boldsymbol{\pi}_b$ remains in the previous expressions, the limit as $c \downarrow \bar{c}$ will also indicate a trade-off and a preference for quantity at low income levels since marginal costs will still be a function of survival rates. The asymptotic allocation however will invoke a continuous increase in fertility due to the normality of survivors.

Additional goods and periods can be introduced under weak separability and although separability imposes strong restrictions in the patterns of substitution, if spending in children is a normal good, an increase in household income will increase spending in children leaving our previous findings unaffected.

Corollary 2 *Let* $V(a) = \max\{W(v(\mathbf{x}), U(c, bP(c;\theta)), V(a')) : a' = (1+r)a + w - (bc)p - \mathbf{x}\}$ *define a dynastic problem in which* $\{\mathbf{x}, w, a\}$ *are* N *additional goods (including parental consumption), per-capita labor income and household assets. If the aggregator function* $W : \mathbb{R}_+^{N+2} \to \mathbb{R}_+$ *is linear (or homothetic in general), and* $v(\mathbf{x})$ *is strictly concave, spending in children will be an increasing function of household assets and income and a decreasing function of the relative price* p*.*

The application of our findings to dynamic and more general models serves to overcome some of the limitations in the analysis of modern economic and demographic changes. We obtain an inverse U-shape relation between income and population growth regardless of the structure of production of final goods and the production of quality. In that sense, economic development and modernization will induce a demographic transition trough its effects in mortality and the cost of children.[16] The analysis of Malthusian outcomes will also apply within the increasing range of the inverse U-shape curve since along that segment, a Malthusian equilibrium will be stable.

Biology and the non-human family: The negative relation between income and population growth is puzzling from evolutionary forces since the sole valuation of survivors fails to deliver a negative relation between income (resources) and reproductive success. In a model in which only quantity is valued, quality becomes independent of income and hence any increase in resources leads to a constant quality and an increase in offspring.

Corollary 3 (Smith and Fretwell [49]) *In a reproductive strategy that maximizes (any concave function of) the expected number of survivors (or fitness), subject to a quantity-quality constraint,*

[16] For instance, the model described above complements Hansen and Prescott [29] in which the link between income and population growth was exogenously assumed rather than derived from economic considerations.

the elasticity of the number of births with respect to income (or resources) must be equal to one, i.e., $b_y(y, \theta) = b(y, \theta)/y$.

Since the objective function becomes $bP(y/b; \theta)$, optimality conditions now require births to increase one-to-one with income or resources (equation 1 in Robson [43] arrives to the same condition under an alternative interpretation). By aggregation conditions of the budget constraint, the previous result is equivalent to a decision in which quality remains unaltered as income increases. If changes in mortality are assumed to be only a consequence of θ, and if $P_c - cP_{c\theta} < 0$, a simultaneous increase in income and in θ produces ambiguous effects in fertility. Unfortunately, as soon as mortality becomes insignificant, the income effect will still prevail leading to an increase in fertility.[17]

4 Identification of the Trade-Off [incomplete]

The most direct implication of a quantity-quality trade-off under endogenous mortality is that modern population growth cannot be regarded as simple independent movements of mortality and fertility but that interactions between both factors are important. And although the association is highly feasible (as the previous figures show) and often invoked (as pointed out in the introduction), our numerical exercise is not a direct test of the implications of the trade-off.

We next extend the analysis through the empirical estimation of the trade-off. We write the standard model, in which c represents quality, as:

$$c = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1(n + T) + \boldsymbol{\beta}_2 X + U,$$

with n as the number of births or children in the family ($n = \sum_{j \geq 0} b_j$ or $\sum_{j \geq 0} b_j P_j$ with P_j as the

[17] An alternative specification in biology considers competition between offspring for parental care with $P(c, b; \theta)$. The possibility of offspring competition generalizes the first order conditions and the marginal costs, and although quality and quantity vary with income even under survival maximization, the possibility of an inverse U-shape due to biological concerns alone is highly unlikely.

survival probability), T as the twin outcome and X as a vector of observable control variables. Since fertility decisions are not randomly drawn form the population of potential births, Rosenzweig and Wolpin [44] suggested using twins as unplanned exogenous variations in family size in order to test the quantity-quality trade-off in education. Due to their small sample size, their implementation is not free from statistical problems but it is often amended to evaluate the effect of quantity on child quality (see for example Black et al. [8]). Whether an increase in the number of births leads to a reduction in subsequent births (or an increase in spacing), as predicted by the quantity-quality model, has not been consistently studied but seems rather intuitive.

Having a birth implies either that a pregnancy was attempted or unintended (unwanted plus mistimed) so the possibility of twinning in itself is not entirely random though it is conditional on having a birth if the fertility control is perfect. That is, $E[T, U|X, n] = 0$. It is possible that the twin birth is not entirely random (i.e., due to the age of the mother) or that effects beyond what the trade-off predicts exist (a twin birth might not be perceived as entirely random if twinning is believed to occur with higher frequencies in families with more twins), but we try to avoid biased results due to the interaction between taste and fertility.

Data: In the last two decades, a new and comprehensive source of demographic and health data for less developed countries has become available: the Demographic and Health Surveys (DHS). The DHS are nationally representative household surveys for less developed countries with sample sizes of about 5,000 households. The DHS program is funded by the United States Agency for International Development (USAID) and coordinated by Macro International.[18] The surveys provide data in the areas of population, health, and nutrition including aspects of household's socioeconomic background, fertility, family planning and maternal and child health measures. Height, weight and other anthropometric measures of children and women aged 15-49 are also included.

[18] The surveys follow a stratified-cluster sample design selecting random sample of women aged 15-49 within each cluster. By the cluster-sampling design within correlations are exhacerbated so, to control for unobserved community-level factors, we use cluster-level fixed effects.

Although the socioeconomic information collected by the DHS+ is rather limited, there almost no other large nationally representative survey to collect health histories for young children. Since all surveys use standardized questionnaires, aggregation of multiple countries is now feasible serving to overcome the main limitation of studies that use twins as an instrument for changes in family size. With twinning occurring in less than 3 percent of the population, a large sample is needed to obtain reliable results.

Estimation: We first assume that births are actually random and use only families with no contraceptive use.

[to be completed]

5 Concluding Remarks

Parents in modern economies opt for investments in education and other quality attributes that make fewer children more valuable and successful. In pre-modern economies, in which children faced high mortality risks, this strategy is unfeasible because if parents had opted for child quality investments in education, the reduction in fertility needed to increase child quality would have lead to an unsustainable equilibrium due to extinction.

In the paper we consider endogenous mortality as an alternative and complementary view of child quality. We assume that parents in pre-modern economies faced a trade-off between the number of births and the quality of each birth. Besides the nonlinear effects on the budget constraint borrowed from Becker and Lewis [3], we assume that parental investments in child quality had a positive effect on the child survival rate.

The paper offers a unified view of the demographic transition in which economic development brings about a decline in child mortality, an initial increase in fertility and population growth followed by a reduction in the growth rate of population at the later stages of the demographic transition. There are two building block for the model: First, the quantity-quality interaction and second, the endogenous link between quality and child mortality. The workings of the model can be summarized as follows. Since pre-modern economies faced high child mortality, a large number of births was needed just to ensure population growth near replacement. If parents face a quantity-quality trade-off, changing the average quality of each child born represents a large expense since the change had to be applied to a large number of births. At low income levels, the presence of a high number of births creates an incentive in favor of the quantity of children. The incentive is stronger under endogenous child mortality if the marginal productivity of the survival function declines as quality increases. As income increases, the marginal cost of child quality declines since the number of births needed to achieve a given number of survivors declines as mortality declines. If the value of child quality also increases as income increase due to non-homotetic preferences, the increase in income leads parents to asymptotically prefer investments in child quality at the expense of a fixed number of births.

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