Using the heterogeneity model and the Lee-Carter method to estimate mortality surface at old ages

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Death rates cannot be measured reliably at old ages, yet their values are necessary in studies such as population forecasts. In estimating the true values of death rates at old ages, general smoothing techniques cannot apply to extremely old ages at which there are too few survivors, and they also involve arbitrary choice between smoothness and fitting. To avoid the arbitrariness, we call for the Lee-Carter method that is designed to distinguish persistent trends and disturbances in mortality change. To extend estimates to any age, we utilize the heterogeneity-mortality model. Composing cohorts born in long periods, we obtain the robust estimate of heterogeneity variance, and subsequently the targets are extended from mortality curves to surfaces. Identifying the baseline mortality surface, we eliminate disturbances using the Lee-Carter method and derive estimates of the true values of cohort mortality. We provide examples using data from 17 low-mortality countries.

How mortality changes across age is perhaps the oldest issue in demography. Ideally, this issue should be approached on the basis of observation. Mortality studies have been progressing closely along this line, especially in recent decades when data collection improved remarkably. However, no matter how much efforts were paid on collecting data, they could reliably measure mortality only up to some age, older than which there are severe disturbances that include random fluctuation and binomial noise

[1] The views expressed are those of the authors and do not necessarily reflect the position of the United Nations

due to too few survivors. In this paper, we propose a method to estimate mortality at old ages at which direct measure cannot be reliable. Why do we need to know mortality at so old ages? Among other reasons, population forecasts provides perhaps the most obvious one: Survivors at old ages are increasing fast in the course of mortality decline. Therefore forecasting the number of nonagenarians or centenarians is becoming increasingly necessary. Such forecasts, obviously, cannot be done without knowing mortality at ages older than 90 or 100 years.

In estimating the true values of mortality, which we refer to observed death rates in the absence of disturbance, smoothing techniques based on spline functions (see Direckx, 1993) cannot apply to extremely old ages at which there are too few survivors; we therefore turn to demographic models. Using the values of death rate at younger ages measured under slight disturbance, parameters of the models can be calculated, and then the values of death rate at older ages can be estimated. This strategy was adopted to extend the maximum age for both model life tables (Caole and Guo, 1990) and observed death rates (Lee and Carter, 1992).

The model

In describing mortality changes at ages older than 85 years, the Coale-Kisker (1990) and Kannisto (Thatcher, Kannisto and Vaupel, 1998) models are perhaps the easiest to use. The Coale-Kisker model assumes a quadratic function for the logged death rate, while the Kannisto model recommends a logistic curve. If the modeling errors were identically and independently distributed across age, parameters could be well estimated using observed data. Unfortunately, the variance of the binomial noises rise with age; and random fluctuations are always correlated across age. Since direct estimate is difficult, the Coale-Kisker model set 0.8 for male and 1 for female death rate at age 110, and the Kannisto model assumes 1 as the ultimate level, both arbitrarily.

Similar to the findings of leveling-off mortality among other species (Vaupel, et al, 1998), human mortality plateau has been revealed recently (Robine et al, 2005). Specifically, human death rate is observed to level off at about 0.65 for males and 0.75 for females at ages around 115 years. This finding disfavors the Coale-Kisker model in which death rate declines after reaching some maximum level, and suggests substantially reduce the assumed leveling-off value in the Kannisto model.

Given that mortality should gradually level off, there are still countless paths that could lead to the plateau. For example, the arctangent curve (Lynch and Brown, 2001) may also be a candidate besides logistic. Thus, what path should the death rate follow is still a question. And this question is perhaps more important than that of the leveling-off value, because there are more survivors on the slope than on the plateau. Yet this question cannot be properly answered by inductive methods, which recommend particular curves by comparing fittings of limit data. A specific curve may fit a certain set of data best among a certain group of curves. But no matter how large the set and how representative the group may be, this specific curve cannot be guaranteed the best either in fitting other data or when more curves are considered.

The mortality-heterogeneity model (Vaupel, Manton and Stallard, 1979, henceforth VMS) answers this question on a deductive basis. Admitting that individuals are different with respect to mortality even if they share the same status such as age and gender, the fact that weaker people tends to die earlier provides the deductive basis of the VMS model. On this basis, the rate of mortality increase with age should decline at advanced ages, because individuals are more robust there. This basis, on the other hand, prefers cohort data to that of period. The VMS model was criticized for arbitrarily assuming the baseline mortality and distribution of heterogeneity (e.g., Trussell and Rodríguze, 1992). The situation is improved in the extension of the VMS model (Li and Vaupel, 2005), in which the baseline mortality is speculated to rise exponentially according to the Gompertz (1825) law, and the heterogeneity distribution is specified as gamma according to the observed mortality plateau. Standing on qualitative observations of the Gompertz law and mortality plateau, the VMS model deduces that a cohort's death rate should follow a logistic curve. Different also quantitatively from the Kannisto model that approaches 1; the logistic curve should level off at about 0.65 for males and 0.75 for females, the values of observed human mortality plateau. Although the level of mortality plateau may vary among populations, taking the only observed values would be better than making other assumptions arbitrarily.

Choosing the VMS model, severe disturbance will still cause problem in estimating death rates for cohorts born in single year. We solve this problem through two steps. First, we compose cohorts born in long periods to obtain the robust estimate of heterogeneity variance, which could be argued to be more constant than the death rate itself that apparently declines over time. Using the robust estimate of heterogeneity variance, we identify the baseline mortality surface on which the effects of disturbance do not cumulate over age. In the second step, we apply the Lee-Carter method (Lee and Carter, 1992, henceforth the LC) as a filter to smooth the baseline mortality surface. Using the robust estimate of heterogeneity variance and the smooth baseline mortality surface, the true values of cohort mortality can be estimated by the VMS model.

The relevant features of the VMS model

For a cohort at age x, denote its force of mortality by $\mu(x)$, and its baseline force of mortality by $\mu_o(x)$, which represents the force of mortality of individuals whose values of z is 1. Then, $\mu(x)$ can be expressed by $\mu_o(x)$ as

$$\mu(x) = \frac{\mu_o(x)}{1+\sigma^2 \int_{85}^{x} \mu_o(y)dy}, \quad x \geq 85, \qquad (1)$$

where σ^2 is the variance of heterogeneity at age 85, the starting age of the VMS in this paper. We choose such a starting age is because younger than which the Gompertz law may work well. In measuring mortality, life expectancy removes effects from age structure, and hence is better than crude death rate. Similarly, owing to eliminating mortality heterogeneity among individuals, $\mu_o(x)$ measures mortality at a more fundamental level than $\mu(x)$, and hence should suffer less disturbance than $\mu(x)$. To illustrate this, suppose there is no disturbance at age x; but at a younger age y, $\mu_o(y)$ is raised up by a disturbance. Then, according to (1), $\mu(x)$ would be smaller than that of no disturbance on $\mu_o(y)$, because the integral in the denominator of (1) continues the disturbance to older ages. This can be explained as that some individuals would normally die at age x died earlier at age y. In other words, $\mu(x)$ could be disturbed even if there is no disturbance on $\mu_o(x)$. Of course, using $z\mu_o(x)$ to express individual mortality is a simplification, which implies that when there is a disturbance at age x for individuals with z=1, there will also be disturbance for other individuals with strength proportional to their z. In other words, disturbances are perfectly correlated across individuals at the same age.

The VMS model can also to describe $\mu_o(x)$ by $\mu(x)$ as

$$\mu_o(x) = \mu(x)\exp[\sigma^2 \int_{85}^{x} \mu(y)dy] = \mu(x)s(x)^{-\sigma^2}. \qquad (2)$$

In (2), s(x) is the cohort's survival probability from age 85 to x. Since disturbance is smoothed in s(x) over age, (2) also indicates that $\mu_o(x)$ is less disturbed than is $\mu(x)$, consistent with (1). Furthermore, if the value of σ is estimated, $\mu_o(x)$ can be identified by (2) using the observed values of $\mu(x)$.

Estimating σ for cohorts born in long period

In reality what can be measured is the death rate that can be regarded as the over-age average of the force of mortality in an age group. Therefore we will not distinguish death rate and the force of mortality, and accordingly will use discrete expression. In the extension of the VMS model, $\mu_o(x)$ is speculated to follow the Gompertz law,

$$\mu_o(x) = \mu(85)\exp[r(x-85)]. \quad (3)$$

Inserting (3) into (1), $\mu(x)$ is a logistic curve,

$$\mu(x) = \frac{\mu(85)\exp[r(x-85)]}{1+(\sigma^2/r)\mu(85)\{\exp[r(x-85)]-1\}}; \quad (4)$$

and the ultimate level of mortality is

$$\mu(\infty) = \frac{r}{\sigma^2} \ . \quad (5)$$

In principle, parameters r and σ could be estimated from minimizing errors of using (4) to describe the observed values of $\mu(x)$. In practice, however, even for cohorts born in long period such estimates may not yield $\mu(\infty)$ that is close to the observed mortality plateau because of disturbance. To avoid strong disturbances, parameter estimating should exclude the values of $\mu(x)$ at ages older than some threshold, namely w. On the other hand, the observed values of mortality plateau, which we denote as $\tilde{\mu}(\infty)$, should be utilized. We therefore introduce the below constraint according to (5)

$$r = \tilde{\mu}(\infty)\sigma^2 \ . \quad (6)$$

Inserting (6) into (4), σ can be estimated from the below nonlinear least squares,

$$\min \sum_{x=85}^{w} \{\mu(x) - \frac{\mu(85)\exp[\tilde{\mu}(\infty)\sigma^2 (x-85)]}{1+[\mu(85)/\tilde{\mu}(\infty)][\exp[\tilde{\mu}(\infty)\sigma^2 (x-85)]-1]}\}^2 . \qquad (7)$$

We apply this estimate to cohorts born in long periods, because increasing cohort size reduces binomial noises, and because over-time averaging eliminates random fluctuations. The performance of this estimate can be evaluated by the average-relative error

$$E(VMS) = \frac{\sqrt{\sum_{x=85}^{w} \{1 - \frac{1}{\mu(x)} \frac{\mu(85)\exp[\tilde{\mu}(\infty)\sigma^2 (x-85)]}{1+[\mu(85)/\tilde{\mu}(\infty)]\{\exp[\tilde{\mu}(\infty)\sigma^2 (x-85)]-1\}}\}^2}}{w-85} . \quad (8)$$

Estimating mortality for cohorts born in single year

We now denote by $\mu(x,c)$ the force of mortality at age x for cohort born in single year c, and by $\mu_o(x,c)$ the baseline force of mortality of individuals in this cohort. Because the σ^2 describes the variance of heterogeneity with respect to mortality, it should not change significantly over cohort born in single year when the society is in stable state. This is different with situation that death rate should decline stably when the society is in stable state. We therefore assume that the value of σ^2 does not change over cohort. Under this assumption, the value of σ^2 can be robustly estimated from the largest cohort whose members are born in all the single years.

Using the robust estimate σ^2, the $\mu_o(x,c)$ is identified by (2). We now turn to eliminate disturbances, because their effects do not cumulate over age in $\mu_o(x,c)$. Using general smoothing techniques, however, arbitrary choice between smoothness and fitting (see Direckx,1993) is inevitable. Because now the concern is not general but mortality, one may naturally recall the LC, which first eliminates binomial noise by averaging and then distinguishes historical mortality trends from random fluctuations. Although the LC was originally proposed for period data, it applies also to that of cohort as can be seen next.

In the cohort version of the LC, the over-cohort average of $\log[\mu_o(x,c)]$, a(x), is calculated first, in which the binomial noises are largely eliminated. The difference between $\log[\mu_o(x,c)]$ and a(x) contains therefore mainly persistent trends and random fluctuations, and is modeled as

$$\log[\mu_o(x,c)] = a(x) + b(x)k(c) + \varepsilon(x,c)\,. \qquad (9)$$

Equation (9) transfers the task of modeling an age-specific vector $\log[\mu_o(x,c)]$ into modeling a scalar k(c), when the average-relative error

$$E(LC) = \frac{\sqrt{\sum_{c=1}^{C}\sum_{x=85}^{w}\{1-\frac{\exp[a(x)+b(x)k(c)]}{\mu_o(x,c)}\}^2}}{C(w-85)} \qquad (10)$$

is small and therefore $\varepsilon(x,c)$ is negligible. Subsequently the task of identifying a large number of persistent trends in $\mu_o(x,c)$ is transferred into identifying only one persistent trend in k(c).

For forecasts including uncertainty, time-series models are the natural tools to deal with k(c). And in the LC a random walk with drift (RWD) is usually the model. In order to smooth out random fluctuations in k(c), we also need time-series models, because the k(c)s are correlated and in time-series models the modeling errors are independent.

To remove random fluctuations from k(c), however, the situation is different from forecasting. On the one hand, mistaking persistent trend as random fluctuation introduces additional modeling error, which cannot be attributed into forecasting uncertainty. Therefore, better fitting is more important than in the case of forecasting. On the other hand, behaving reasonable in long term is not the concern of selecting model, therefore models more general than RWD could be chosen. For these reasons, we use a random walk with cohort-varying drift (RWVD) model, which is written as

$$k(c) = k(c-1) + d_0 + d_1 c + e(c). \qquad (11)$$

When the cohort-varying coefficient d_1 is estimated as or close to 0, (11) reduces to RWD. Because e(c) is deemed as random fluctuation, which will be eliminated, we do not measure it as modeling error in the way of (8) or (10). Since e(c)s can be assumed as independently and identically distributed (i.i.d.) variables without resulting in k(c)s to be independent, the coefficients in (11) can be estimated using OLS (see Kendall and Ord, 1990) as

$$\begin{pmatrix} d_0 \\ d_1 \end{pmatrix} = \begin{pmatrix} C+1 & \sum_{c=1}^{C} c \\ \sum_{c=1}^{C} c & \sum_{c=1}^{C} c^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{c=1}^{C} [k(c) - k(c-1)] \\ \sum_{c=1}^{C} [k(c) - k(c-1)] \cdot c \end{pmatrix}. \qquad (12)$$

The persistent trend in k(c), which we denote as K(c), is then identified as the difference between k(c) and e(c). Using the estimated of parameters in (12) and an initial value K(0), K(c) is obtained according to (11) as:

$$K(c) = K(c-1) + d_0 + d_1 \cdot c = K(0) + d_0 \cdot c + d_1 \cdot \frac{c(c+1)}{2}. \qquad (13)$$

The value of K(0) may differ from k(0) that includes disturbance e(0), and can be determined by minimizing the removed disturbances as

$$K(0)=\frac{1}{C+1}\sum_{c=1}^{C}[k(c)-d_0\cdot c-d_1\cdot\frac{c(c+1)}{2}]. \qquad (14)$$

Using K(c), the persistent trends in the baseline mortality surface are obtained as

$$\log(\mu_f(x,c))=a(x)+b(x)K(c). \qquad (15)$$

We call (9)—(15) the LC filter that specifies the persistent trends $\log(\mu_f(x,c))$ in the baseline mortality surface $\log(\mu_o(x,c))$. We might take $\mu_f(x,c)$ as non-parametric smooth values of $\mu_o(x,c)$ to estimate the true values of $\mu(x,c)$. But by doing so we can only obtain the true values of $\mu(x,c)$ for x≤w. In order to estimate $\mu(x,c)$ at any age, we further derive the smooth estimates of r(c), which is the r in (3), using $\mu_f(x,c)$:

$$r(c)=\frac{\sum_{x=85}^{w}(x-85)\log\frac{\mu_f(x,c)}{\mu_f(85,c)}}{\sum_{x=85}^{w}(x-85)^2}. \qquad (16)$$

The performance of (3) can also be evaluated by the average-relative error

$$E(G)=\frac{\sqrt{\sum_{c=1}^{C}\sum_{x=85}^{w}\{1-\frac{\mu_f(85,c)\exp[r(c)(x-85)]}{\mu_f(x,c)}\}^2}}{C(w-85)}. \qquad (17)$$

Consequently, the true values of $\mu_o(x,c)$ and $\mu(x,c)$ are estimated according to (3) and (1) as

$$\mu_o(x,c)=\mu_f(85,c)\exp[r(c)(x-85)], \qquad (18)$$

and

$$\mu(x,c) = \frac{\mu_f(85,c)\exp[r(c)(x-85)]}{1+[\sigma^2/r(c)]\mu_f(85,c)\{\exp[r(c)(x-85)]-1\}}. \qquad (19)$$

Examples and discussion

We use Human Mortality Database (www.mortality.org) to illustrate our method. The data include male and female populations and death rates, specified in single year of age from 0 to 109 years, and in single year of time. To avoid the impact of World War II, we choose the starting year as 1950 or the earliest available later than 1950. We also limit our application to countries whose mortality declined stably and data are believed reliable. These countries are: Austria, Belgium, Canada, Denmark, England and Wales, Finland, France, the former West Germany, Italy, Japan, the Netherlands, Norway, Spain, Sweden, Switzerland, and the US.

Following Thatcher, Kannisto and Vaupel (1998), we set the maximum age of estimating the model, w, as 98. This is because centenarians are scarce and at age 99 misreporting errors are usually high. We then construct the largest cohort, which is composed by individuals who survived to age 98 at and before the latest year and who reached age 85 at and after the earliest year, at which the data are available. Taking Austria with data available between 1950 and 1999 as example, the largest cohort includes individuals who reached age 85 from1950 through 1986. The numbers of population in the largest cohorts at age 85 are listed in the second column for each country in Tables 1 and 2.

We apply the constraint of leveling off, $\tilde{\mu}(\infty)$, as 0.65 for males and 0.75 for females for each country. The values of the average-relative error, the E(VMS) in (8), are listed in the third column in Tables 1 and 2. Among all countries and for both sexes, the maximum E(VMS) is 1.4%, which is expectantly observed from Finland males that numbered the second smallest. And the value of Eσ is larger for males than females for each country expect for Japan and Spain, whose males' Eσ is marginally smaller than that of females'. These indicate that larger Eσ correspond to smaller populations, and that the logistic curve fits observed data well for both sexes in all the 17 countries. The observed and estimate values of $\mu(x)$ for Finland males are shown in the first panel of figure 1.

The estimate values of σ^2 are larger for males than for females for each country, as can be seen in the seventh column in tables 1 and 2. For males, the values of σ^2 are estimated in the range of 0.1645 to 0.2185, with mean 0.1958 and standard deviation 0.0176. For females, the range, mean and standard deviation of σ^2 are 0.1342 to 0.1717, 0.1532 and 0.0106, respectively.

Having the estimate value of σ, the $\mu_o(x,c)$ is identified by (2), which is shown by the triangles in the second panel of figure 1, taking the first cohort of Finland males as example. Applying the LC to the identified surface of $\mu_o(x,c)$, the average-relative errors E(LC) are listed in the fifth column in tables 1 and 2. For all the 17 countries and for both sexes, the maximum Elc is 0.7%, implying that the LC worked well for cohort data. On this basis, we transfer the changes in a vector $\mu_o(x,c)$ into that of a scalar k(c), which is shown by the triangles in third panel of figure 1. Applying the RWVD model, the persistent trend in this scalar k(c) is identified as the solid curve in third panel of figure 1.

Using the persistent-trend k(c) and (15), $\mu_o(x,c)$ is filtered as $\mu_f(x,c)$ at ages younger than 99. The values of $\mu_o(85,c)$ and $\mu_f(85,c)$ are displayed in the fifth panel of figure 1. Applying (16) to $\mu_f(x,c)$, the smooth values of r(c) are estimated, as is depicted in the fourth panel of figure 1. The maximum average-relative error in estimating the smooth r(c), E(G), is 1.1% among all the 17 countries and for both sexes, as can be seen in the sixth column in tables 1 and 2. Using $\mu_f(85,c)$, the smooth estimates of r(c), and (18), the $\mu_o(x,c)$ is estimated for ages older than 98, and the values of $\mu_o(x,1)$ are shown as the solid curve in the second panel of figure 1. Because the heterogeneity model is deductive, we believe its good performances at ages younger than 98 years indicate accurate estimates at older ages. Using (19), we obtain the estimates of cohort mortality surface $\mu(x,c)$.

Models (3) and (1) can be extended to any age. The leveling-off value, $r(c)/\sigma^2$, however, is no longer constant but changes over cohort closely around the constraint $\tilde{\mu}(\infty)$. Taking the smooth values of r(c) for the first and last cohort in the fourth panel of figure 1 as example, the leveling-off values for cohorts reached age 85 between 1950 and 1989 reduced smoothly from 0.7008 to 0.6210. The leveling-off value will also change over country and sex, but should not differ too much from $\tilde{\mu}(\infty)$ either. Because observed cohort death rate stops at age 109, we compare the observed and estimate values of $\mu(x,c)$ up to age 109, in figure 2 for Finland males, and in figure 3 for the US females that numbered the largest among the 17 countries. The first panels of figures 2 and 3 show the observed and estimated mortality surfaces; the second panels provide projections of the surfaces along the time dimension; and the third panels display the over-cohort average death rates.

Utilizing the LC method that is designed to identify persistent trends from disturbances in mortality change, we estimate the true values of $\mu_o(x,c)$ for x between 85 and 98 years. Using the VMS model, which is deductive and hence could be regarded accurate according to its good performance at younger ages, we extend estimates to ages older than 98 years. Our estimates depend also on the constraints $\tilde{\mu}(\infty)$, which are observed from the large cohorts that were born in long periods in more than 15 low-mortality countries (Robine, et al, 2005). Viewing a cohort born in a specific country and a single year as a random sample from this large cohort, the estimate $\mu(x,c)$ should be asymptotically unbiased around its empirical average $\tilde{\mu}(\infty)$. Our estimates are therefore demographically empirical and asymptotically unbiased. This unbiasedness implies that when the population is larger the difference between the estimate and observation would be smaller, and could be illustrated presently by figures 2 and 3. The differences between the estimates and observations are enormous for Finland and negligible for the US; while the population is small in the former and large in the latter.

References

Coale, Ansley J. and Guang Guo, 1990. Revised Regional Model Life Table at Very Low Levels of Mortality. *Population Index*, 55:613—43.

Coale, Anslay J. and Ellen E. Kisker, 1990. Defects in Data on Old – Age Mortality in the United States: New Procedures for Calculating Mortality Schedules and Life Tables at the Highest Ages. *Asian and Pacific Population Forum*, 4(1):1—31.

Direckx, Paul. 1993. *Curve and Surface Fitting with Splines*. Oxford University Press Inc,. New York.

Gompertz, Benjamin. 1825. On the nature of the function expressive of the law of human mortality. *Philosophical Transactions*, XXVII, 51—585.

Human Mortality Database. (University of California, Berkeley, USA, and Max Planck Institute for Demographic Research, Germany. Available at www.mortality.org or www.humanmortality.de, data downloaded on May 2005.)

Lee, Ronald D. and Lawrence R. Carter. 1992. Modelling and Forecasting the Time Series of U.S. Mortality. *Journal of the American Statistical Association* 87: 659—71.

Li, Nan and James W. Vaupel, 2005. Extending the heterogeneity approach to model human mortality plateau. Manuscript.

Lynch, Scott M. and J. Scott Brown. 2001. Reconsidering mortality compression and deceleration: An alternative model of mortality rates. *Demography*, 38(1):79—95.

Kendall, Maurice and J. Keith Ord. 1990. *Time Series*. Edward Arnold, Suffolk, Great Britain.

Oeppen, Jim. and James W. Vaupel. 2002. Broken limits to life expectancy. *Science* 296:1029—31.

Robine, Jean-Marie, Jutta Gampe, Amandine Cournil and James Vaupel. 2005. IDL, the International Database on Longevity. Presented at the meeting of Society of Actuaries, Orlando, FL, USA.

Thatcher A. Roger, Vaino Kannisto and James W. Vaupel. 1998. *The Force of Mortality at Ages 80 to 120*. Odense: Odense University Press.

Trussell, J. and G. Rodríguze. 1992. Heterogeneity in Demographic Research, pp 111—32. In Adams et al (ed.), *Convergent Issues in Genetics and Demography*, Oxford University Press, New York.

Vaupel, James W., Kenneth G. Manton and Eric Stallard, 1979. The Impact of Heterogeneity in Individual Frailty on the Dynamics of Mortality. *Demography*, 16(3): 439—54.

Vaupel, James W., James R. Carey, Kaare Christensen, Thomas E. Johnson, Anatoli I. Yashin, Niels V. Holm, Ivan A. Iachine, Väinö Kannisto, Aziz A. Khazaeli, Pablo Liedo, Valter D. Longo, Yi Zeng, Kenneth G. Manton, and James W. Curtsinger et al, 1998. Biodemographic Trajectories of Longevity. *Science*, 280: 855—60.

Table 1. Results of models of cohort and individual mortality for males

	Years in which data are available	Population of the largest cohort at age 85 (x1000)	E(VMS)	E(LC)	E(G)	σ^2
Austria	1950—1999	134	0.0083	0.0033	0.0038	0.2125
Belgium	1950—2002	229	0.0055	0.0034	0.0039	0.1996
Canada	1950—1996	317	0.0067	0.0021	0.0027	0.1804
Denmark	1950—2002	129	0.0102	0.0039	0.0048	0.2185
England	1950—1998	864	0.0063	0.0017	0.0022	0.1892
Finland	1950—2002	54	0.0141	0.0066	0.0105	0.2029
France	1950—2002	1,119	0.0038	0.0017	0.0020	0.1834
Germany(W)	1956—1999	946	0.0048	0.0019	0.0022	0.1921
Italy	1950—2001	1,023	0.0042	0.0018	0.0021	0.1893
Japan	1950—1999	946	0.0039	0.0024	0.0027	0.1645
Netherlands	1950—2003	292	0.0106	0.0026	0.0046	0.2141
New Zealand	1950—2003	51	0.0131	0.0057	0.0063	0.1901
Norway	1950—2002	111	0.0056	0.0026	0.0030	0.2180
Spain	1950—2002	566	0.0059	0.0029	0.0037	0.1730
Sweden	1950—2003	249	0.0083	0.0024	0.0030	0.2174
Switzerland	1950—2003	133	0.0073	0.0034	0.0038	0.2054
USA	1959—1999	3,083	0.0063	0.0014	0.0020	0.1686

Table 2. Results of models of cohort and individual mortality for females

	Years in which data are available	Population of the largest cohort at age 85 (x1000)	E(VMS)	E(LC)	E(G)	σ^2
Austria	1950—1999	297	0.0031	0.0024	0.0027	0.1609
Belgium	1950—2002	434	0.0028	0.0022	0.0024	0.1500
Canada	1950—1996	471	0.0038	0.0022	0.0025	0.1432
Denmark	1950—2002	206	0.0046	0.0029	0.0033	0.1674
England	1950—1998	2,121	0.0034	0.0013	0.0016	0.1467
Finland	1950—2002	135	0.0050	0.0049	0.0054	0.1493
France	1950—2002	2,683	0.0021	0.0013	0.0014	0.1520
Germany(W)	1956—1999	1,977	0.0024	0.0019	0.0021	0.1511
Italy	1950—2001	1,837	0.0022	0.0016	0.0017	0.1473
Japan	1950—1999	1,836	0.0040	0.0016	0.0017	0.1342
Netherlands	1950—2003	472	0.0082	0.0030	0.0045	0.1717
New Zealand	1950—2003	93	0.0056	0.0035	0.0032	0.1495
Norway	1950—2002	182	0.0048	0.0025	0.0027	0.1689
Spain	1950—2002	115	0.0061	0.0022	0.0032	0.1460
Sweden	1950—2003	401	0.0070	0.0020	0.0025	0.1621
Switzerland	1950—2003	266	0.0050	0.0027	0.0030	0.1640
USA	1959—1999	5,880	0.0051	0.0018	0.0023	0.1448

Figure 1. Observed and estimate values of variables in the Lee-Carter method and the heterogeneity model, Finland males

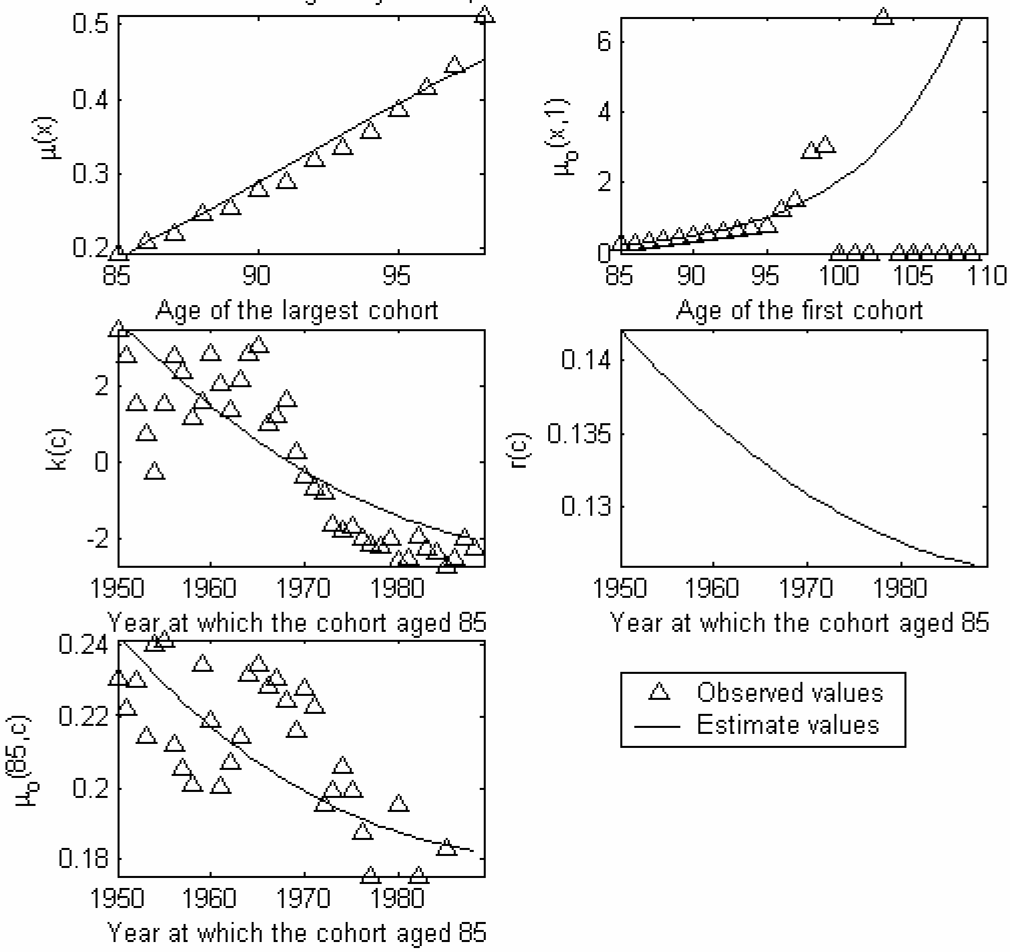

Figure 2. Observed and estimated mortality surfaces, Finland males

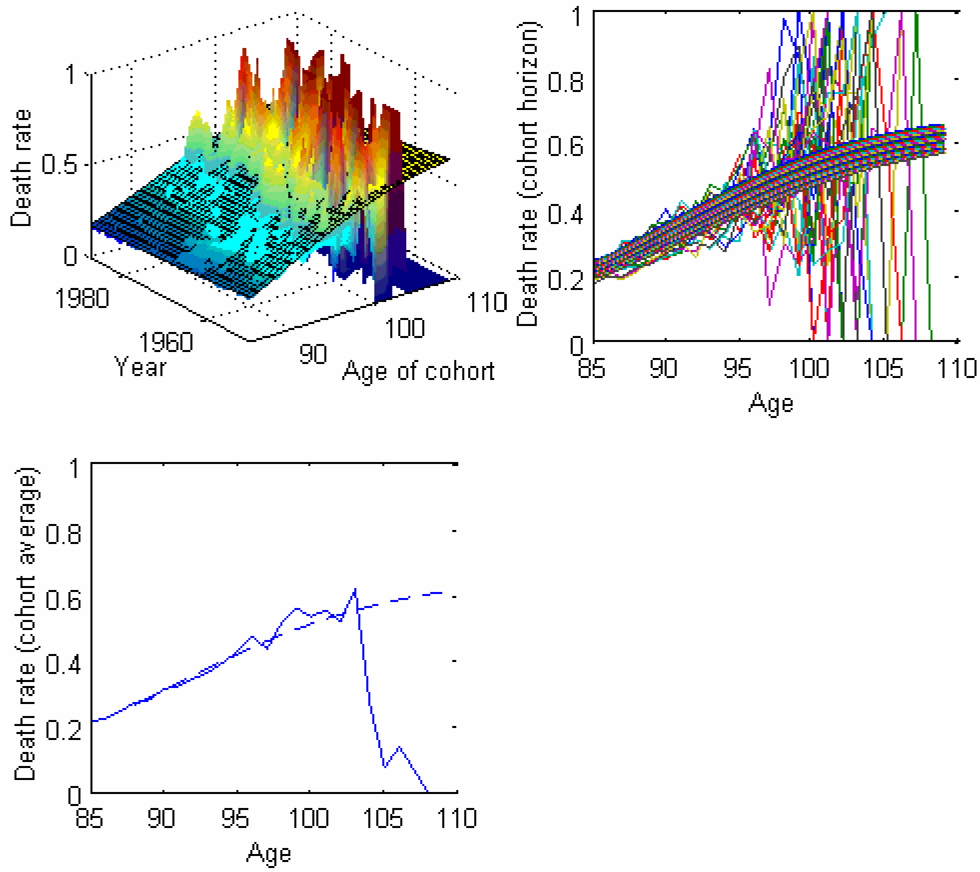

Figure 3. Observed and estimated mortality surfaces, the US females