THE MODAL AGE AT DEATH AND THE SHIFTING MORTALITY HYPOTHESIS.

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Abstract

A mathematical expression for the modal age at death is used to calculate the number of deaths at this age. Models that capture change in mortality over time show an asymptotic approximation towards a constant number of deaths, and survivors, at the modal age. The bell-curve for the number of deaths centered around the modal age at death is also constant, while the modal age moves to higher ages over time. These findings are confirmed through applications to populations with historical mortality data. Results reveal a need to revise the rectangularization hypotheses of approaching a limit for longevity.

INTRODUCTION

Life expectancy, or the mean of the distribution of deaths, is the indicator most frequently used to describe this distribution. Lexis (1878) considered that the distribution of deaths consisted of three parts: a decrease in the high number of deaths with age after birth to account for infant mortality; deaths that follow a bell-curve centered around the late modal age at death (referred to hereafter as modal age at death), accounting for senescent mortality; and premature deaths that occur infrequently at young ages between the high infant mortality and senescent deaths. In a regime with a high level of infant mortality, life expectancy will be within the age range of premature deaths. The early stages of the epidemiological transition (Omran 1971) are characterized by a reduction in infant mortality. These abrupt changes in infant mortality have been captured very accurately with the rapid increase in life expectancy over time.

Currently, mortality is concentrated at older ages in most countries. Life expectancy has slowed down its rapid increase and approached the modal age at death. Therefore, a study of modal age at death provides an opportunity to understand changes in the distribution of deaths, and to explain the change in mortality at older ages (Cheung et al. 2005; Kannisto 2000; Kannisto 2001; Robine 2001).

This article contains a review of the modal age at death, the number of survivors and deaths at this modal age, and the concentration of deaths around this measure. Analytical expressions for the increase in longevity, seen as the change in modal age at death, and variability of age at death are found. As pointed out by Wilmoth and Horiuchi (1999), these expressions can be used in criticism of the rectangularization of the survival curve hypothesis proposed by Fries (1980), and to provide an alternative perspective on aging.

The paper is divided into four parts. The first section introduces the reader to the formal definition of modal age at death, denoted in this paper by the letter M. The second section shows the constancy of survivors and number of deaths under certain mortality models. The third part contains an examination of the concentration of deaths around the modal age. Finally, applications to human populations with historical data are presented.

MODAL AGE

Let the force of mortality at age a and time t be denoted as $\mu(a,t)$. We can write the life table survivorship function at age a under the rates at time t as

$$\ell(a,t) = e^{-\int_0^a \mu(x,t)dx}. \tag{1}$$

If the radix of the life table is one, i.e., $\ell(0,t) = 1$, then $\ell(a,t)$ is the life table probability of surviving from birth to age a.

The most commonly known measure of mortality is life expectancy. In the life table context, life expectancy at age a and time t is normally calculated as the person-years lived above age a divided by the number surviving to age a. An alternative expression for life expectancy is the mean of the distribution of deaths. Let $d(a,t)$ be the density function describing the distribution of deaths (i.e., life spans) in the life table population at age a and time t; then life expectancy can be expressed as

$$e(0,t) = \frac{\int_0^\omega ad(a,t)da}{\int_0^\omega d(a,t)da} = \frac{\int_0^\omega ad(a,t)da}{\ell(0,t)} = \int_0^\omega ad(a,t)da, \tag{2}$$

where ω is the highest age attained and the denominator is equal to 1. For every given age the distribution of deaths is the product of the survival function up to that age multiplied by the force of mortality, $d(a,t) = \ell(a,t)\mu(a,t)$.

Figure 1 shows the distribution of life table deaths for the Netherlands at the beginning, middle, and end of the twentieth century.

[FIGURE 1 HERE]

Also included in Figure 1 are the life expectancy, modal age, and mode number of deaths after age 2 in 1900, 1950, and 2000. Here it is possible to appreciate how life expectancy is found at age 48.5 in 1900, while most of the deaths in this year are concentrated at ages below 5 and around the late modal age of 76.4. At the turn of the century in year 2000, life expectancy reached a value of 78.4 years, almost double that of 1900, and reduced markedly its distance to the modal age, which has moved to 84.5. The modal number of deaths moved from 2.4% of all deaths in 1900 to be 4.2% in 2000.

In industrialized countries where infant mortality has dropped dramatically, the modal age of the distribution of deaths is found at older ages. More generally, for any population the modal age at death after age 5 can be calculated as the age at which the derivative of $d(a,t)$ is equal to zero. To simplify the notation, let the partial derivative of a variable with respect to age be denoted by a dot on top of the variable and an acute accent over the variable to represent the relative partial derivative with respect to age. This notation has proven to be very helpful in reducing the clustering of equations (Canudas-Romo and Schoen 2005; Vaupel and Canudas-Romo 2002; 2003). Assuming continuity over age in functions $d(a,t)$, $\ell(a,t)$ and $\mu(a,t)$, the partial derivative of the distribution of deaths with respect to age is

$$\dot{d}(a,t) = \dot{\ell}(a,t)\mu(a,t) + \ell(a,t)\dot{\mu}(a,t) = \ell(a,t)\mu(a,t)[\acute{\ell}(a,t) + \acute{\mu}(a,t)]$$

by substituting the definition of the survival function in equation (1) we obtain

$$\dot{d}(a,t) = d(a,t)[\acute{\mu}(a,t) - \mu(a,t)]. \tag{3}$$

Equation (3) is equal to zero when $d(a,t)$ or $[\acute{\mu}(a,t) - \mu(a,t)]$ are equal to zero. In the first case there are no deaths, and therefore also no modal age. In the second case, it implies that at the modal age M the force of mortality equals its relative derivative with respect to age,

$$[\acute{\mu}(a,t) = \mu(a,t)]. \tag{4}$$

To further add to this special age in the distribution of deaths, Wilmoth and Horiuchi (1999) showed that the modal age is also the inflection point in the survival curve.

Model populations provide a useful way to examine changes in mortality. In the next section we show the change over time in modal age in three types of mortality models, that have been adopted by demographers as good approximations of force of mortality. The three mortality models are the Gompertz Mortality Change Model, the Logistic Model and the Siler Mortality Change Model.

MORTALITY MODELS

Gompertz Mortality Change Model

Bongaarts and Feeney (2002; 2003) have stimulated a new debate about how to interpret period life expectancy when rates of death vary over time. The parallel shift in adult mortality analyzed by Bongaarts and Feeney can be characterized by a Gompertz mortality change model (Vaupel 1986; Vaupel and Canudas-Romo 2000; 2003). Their formulation is an extension of the Gompertz (1825) model of mortality, which has a changing initial force of mortality component,

$$\mu(a,t) = \mu(0,t)e^{\beta a}, \tag{5}$$

where $\mu(0,t)$ reflects the value of the rate of mortality decrease over time; parameter $\beta > 0$ is the fixed rate of mortality increase over age.

Substituting the Gompertz mortality change model in equation (4) gives us the modal age for this model when the following condition is fulfilled

$$\beta = \mu(0,t)e^{\beta a},$$

or in terms of the modal age $a = M$,

$$M = \frac{\ln(\beta) - \ln[\mu(0,t)]}{\beta}. \tag{6}$$

The survival function for this model is obtained by substituting the force of mortality of equation (5) in equation (1):

$$\ell(a,t) = exp[\mu(0,t)\frac{(1-e^{\beta a})}{\beta}], \tag{7}$$

and at the value of the modal age, (6), the survival function can then be simplified to

$$\ell(M,t) = exp[\frac{\mu(0,t)}{\beta} - 1], \tag{8}$$

with a maximum number of deaths of

$$d(M,t) = \ell(M,t)\mu(M,t) = \beta exp[\frac{\mu(0,t)}{\beta} - 1]. \tag{9}$$

Bongaarts and Feeney (2002; 2003) showed that the value of $\mu(0,t)$ declines over time. When the reduction in mortality is almost negligible, and the value of $\mu(0,t)$ approaches zero, equations (8) and (9) decrease to a constant number of survivors

$$\lim_{\mu(0,t)\to 0} \ell(M,t) = e^{-1} = 0.37,$$

and thus the numbers of deaths is $d(M,t) = \beta e^{-1}$. However, the modal age at death increases to infinity. Therefore, under this model the rectangularization process of the survival curve has stopped completely. Instead, a shift occurs in the modal age towards advanced ages.

A particular case of equation (5) is where the $\mu(0,t)$ is parameterized, and the force of mortality at age 0 and time t is $\mu(0,t) = e^{\alpha-\rho t}$ (Schoen et al. 2004a; Vaupel 1986). The force of mortality at age a and time t is defined as

$$\mu(a,t) = e^{\alpha-\rho t+\beta a}, \tag{10}$$

where the α is a constant that reflects the value of $\mu(0,0)$ and parameter ρ is the rate of mortality decrease over time.

In this model of continuous mortality decline we take $\beta = 0.1$, the conventional value for the pace of mortality increase over age. Reasonable values for contemporary western low mortality populations are $\alpha = -11$ and $\rho = 0.01$. Using these values and equations (6) and (10), we obtain a modal age for time t of $M = 87 + .1t$, i.e., increasing one year of age every ten calendar years. However, the survivors and number of deaths in (8) and (9) change very modestly over time, reaching their limit values of $\ell(M) = e^{-1}$ and $d(M) = \beta e^{-1} = 0.037$, respectively.

Figure 2 shows the survival function and distribution of deaths in the continuous declining mortality model (10) over 400 years.

[FIGURE 2 HERE]

As shown in Figure 2 the modal age increases every 200 years by 20 years of age. However, the number of survivors and deaths at the modal age remain constant. Life expectancy for this model moves at a similar speed as modal age at death, but starts at a lower value of 82 and reaches a life expectancy of 122 after 400 years. (Values underlying Figure 2 are shown in Table 2.)

The shifting model observed by Bongaarts and Feeney (2002) in mortality has many implications for the survival function and the distribution of deaths. These authors have advanced some possible implications for this type of shift in mortality, as the need to find alternative measures to life expectancy. As shown here, for populations where mortality is concentrated at adult ages the modal age at death comes as a good candidate for finding out how long do we live? This model is not unique, however. These results are tested in the next subsections under alternative mortality models.

Logistic Model

The logistic model has been used in place of the Gompertz model to account for the overestimation of mortality at older ages (Thatcher et al. 1998; Thatcher 1999). Bongaarts (2005) studied separately the two components of the logistic model, senescent and background mortality, with this second term not varying over age. Similarly, here we will examine only the senescent component, which changes over age. The force of mortality is here expressed as

$$\mu(a,t) = \frac{e^{\alpha(t)+\beta(t)a}}{1+e^{\alpha(t)+\beta(t)a}}, \tag{11}$$

where the parameters $\alpha(t)$ for the level of mortality and $\beta(t)$ for the rate of increase in mortality change over time. The survival function for this model is obtained from equation (1) as

$$\ell(a,t) = \left[\frac{1+e^{\alpha(t)}}{1+e^{\alpha(t)+\beta(t)a}}\right]^{\frac{1}{\beta(t)}}. \tag{12}$$

The corresponding modal age for the logistic equation (11) can be found by applying the relation in equation (4) as

$$M = \frac{\ln[\beta(t)] - \alpha(t)}{\beta(t)}, \tag{13}$$

and the number of deaths at this modal age is

$$d(M,t) = \ell(M,t)\mu(M,t) = \left[\frac{1+e^{\alpha(t)}}{1+\beta(t)}\right]^{\frac{1}{\beta(t)}} \frac{\beta(t)}{1+\beta(t)}. \tag{14}$$

Bongaarts (2005) examined change over time for $e^{\alpha(t)}$ and $\beta(t)$ parameters in several countries in the second half of the twentieth century. During this time, the first parameter decreased to levels around $\alpha(t) = -11$, while $\beta(t)$ has remained almost constant at 0.1. According to these values, the number of deaths at the modal age is 0.035, just slightly less than the results for the Gompertz mortality change model.

If further decline is observed in the parameter for mortality level over time, then the value of $e^{\alpha(t)}$ will continue to decrease, which is the same as saying that the value of $\alpha(t)$ becomes much bigger with a negative value. Equation (14) then depends only on the measure of the rate of increase in mortality with age. The survivors and distribution of deaths follow similarly shifting patterns over time to those observed in Figure 2, because modal age at death continues to increase with $\alpha(t)$. As shown by Thatcher et al. (1998) and Thatcher (1999), most mortality models fall between the overestimation of the Gompertz model and the logistic curve. Therefore, this constant value of deaths at modal age is likely to appear in those models as well.

Table 1 presents the modal age and value at death using equations (13) and (14) and the logistic parameters presented by Bongaarts (2005).

[Table 1 HERE]

Similar to the comparison of life expectancies between sexes, here the modal values can be contrasted. As observed in Table 1, females have higher modal age and higher modal value at death than their male counterparts. The largest difference between males' and females' modal ages is found in Finland, while the smallest occurs in Japan. The largest difference between the sexes in modal values is seen in Finland, while the smallest is in England and Wales.

The Siler Mortality Change Model

The mortality models presented above assume that infant mortality has already declined and the distribution of deaths is only composed of deaths at senescent age. However, the first stages of the epidemiological transition were characterized by a decline in infant mortality, which was followed later by declines at advanced ages. Therefore, to have a complete understanding of change over time in modal age at death at advanced ages and its modal value, it is necessary to include infant mortality and the premature component of the distribution of deaths.

The Gompertz model with a continuous rate of decline of equation (10), can be extended to include these two additional components. A proposal by Canudas-Romo and Schoen (2005) combines the mortality model used by Siler (1979) and parameters that account for improvement in mortality over time

$$\mu(a,t) = e^{\alpha_1-\beta_1 a-\rho_1 t} + e^{\alpha_2-\rho_2 t} + e^{\alpha_3+\beta_3 a-\rho_2 t}, \tag{15}$$

where three constant terms reflect the value of $\mu(0,0) = e^{\alpha_1} + e^{\alpha_2} + e^{\alpha_3}$; the parameters β_1 and β_2 are fixed rates of mortality decline and increase over age, respectively, and account for infant and senescent mortality; the parameters ρ_1 and ρ_2 are constant rates of mortality decrease over time. Parameters αs and βs come from the Siler model, while the ρs are used in Gompertz models with a continuous rate of decline (Schoen et al. 2004a; Vaupel 1986). In the remaining text we refer to equation (15) as the Siler mortality change model.

In the model we begin with a fairly high infant mortality (203 per thousand), resulting from the values of $e^{\alpha_1} = 0.2$, $e^{\alpha_2} = 0.003$ and $e^{\alpha_3} = 0.0002$. The early decline over age proceeds at a pace of $\beta_1 = 1$ with an overall increase with age at a rate of $\beta_2 = 0.1$. These values for parameters α and β have been adapted from a comparison of the Siler model with the different model life tables elaborated by Coale and Demeny (Gage and Dyke 1986). At time 0, period life expectancy is 38.5, the modal age at death at advanced ages is 62, and the modal value at this age is 0.023 deaths. These values approach those observed in populations with historical data. For example, in Sweden in the year 1800 infant mortality was 227 per thousand, life expectancy 32.19 years and the late modal age 71. For the pace of mortality improvement we have chosen $\rho_1 = 0.015$ and $\rho_2 = 0.01$. These values correspond to a 1.5% decline at younger ages and mortality improvement of one percent at older ages. The decline at younger ages in several European countries occurred at an even faster rate (Woods et al. 1988, 1989). The rate of one percent is below the current average mortality decline in the West.

Figure 3ab show the change in survivors, distribution of deaths, modal age and modal number of deaths over time under the Siler mortality change model.

[FIGURE 3a & 3b HERE]

As observed in Figure 3ab, the modal age at death at advanced ages increases linearly, while the modal value increases asymptotically to its maximum value. Prevented deaths in infancy have very little probability of occurring during the premature period before the modal age. Therefore, the new survivors add to the distribution of deaths of senescent mortality and increase the modal value of deaths. (Values underlying Figures 3ab are shown in Table 2.)

The increase in the late modal age at death over the twentieth century has been observed in several countries (Cheung et al. 2005; Kannisto 2001). For example, in figures for France reported by Robine (2001), there are almost linear trends in this measure for the entire century. However, the number of deaths at this age or, as studied by Robine, the verticalization of the survival curve, began a slow downward trend in the 1950s. This discontinuity during the second half of the century is in fact noted by Robine. Nevertheless, findings from the analysis reported here provide important information about what is happening with the modal number of deaths. To further support the results from the above models, the concentration of deaths around the modal age is analyzed in the next section.

CONCENTRATION OF THE NUMBER OF DEATHS AROUND THE MODAL AGE AT DEATH

Standard deviations around the late modal age at death have been used to study the dispersion of deaths under the bell-curve centered in the modal age at death (Cheung et al. 2005; Kannisto 2001). To prove the constancy of the concentration of deaths around the mode here we study two measures: the standard deviation from the mode, and the number of years needed before the modal age to obtain 90%, 75%, and 50% of deaths.

Let the standard deviation from the modal age at death, SDM, be defined as

$$SDM = \sqrt{\int_0^{\omega} (a - M)^2 d(a,t) da}, \tag{16}$$

where the denominator of this measure is equal to one, $\int_0^\omega d(a,t)da = 1$.

Table 2 presents the modal age and value at death, and the standard deviation from the mode for the Gompertz mortality change model of Figure 2 and equation (10), and the Siler mortality change model of Figure 3 and equation (15).

[TABLE 2 HERE]

As observed in Table 2 under both models the modal age at death increases linearly. The modal value and SDM are constant at values of 0.037 and 14.02 for the Gompertz mortality change model. The modal value for the Siler mortality change model increases over time reaching the Gompertz' value of 0.037 after 400 years. However, for the Siler mortality change model the standard deviation never reaches the value of 14.02 of its Gompertz counterpart, having a minimum value of 14.72.

Let K be the proportion of the number of deaths around the modal age at death that it is desired to find. This is equivalent to looking at the area under the curve of the distribution of deaths that goes from age $M - A$ to $M + A$, and provides the total number of K deaths. The number of deaths in this interval can also be calculated as the difference between those surviving up to age $M - A$ minus those surviving to age $M + A$,

$$K = \int_{M-A}^{M+A} d(a,t)da = \ell(M - A, t) - \ell(M + A, t). \tag{17}$$

Similar quantities that do not, however, depend on the modal age, are the interquartile range based on the values of 75% and 25% of survivors (Wilmoth and Horiuchi 1999) or the compression measures used by Kannisto (2000). In the rest of the analysis we use equation (17), because we intend to study the concentration of deaths around the modal age.

Equation (17) is easily generalized to the case involving a different number of years before and after the modal age. An interesting case is the one observed with a continuous declining mortality model of the type in (10). Over time the number of survivors at the modal age becomes a constant, (8), which is equivalent to the number of deaths from the modal age until the last age attained by a person, $e^{-1} = 0.37$. Given that the value for the remaining number of deaths after the modal age does not change, the desired proportion K is obtained from the lower limit age. The lower limit age in the distributions of deaths may be calculated from equation (17) as $K = \ell(M - A, t)$, and substituting (7) a value for the interval A is obtained:

$$A = M - \frac{ln\left[1 - \beta ln[K]e^{\rho t - \alpha}\right]}{\beta}. \tag{18}$$

This value for the lower limit of the interval is almost a constant. In the initial stages of the model the age $M - A$ moves slightly slower than the modal age but this difference becomes negligible over time. The value needed to have 90% of the deaths is $A = 22.5$ years before the modal age, while 75% requires $A = 12.5$ and 50% only $A = 3.7$. This constant result confirm the distribution of deaths observed in Figure 2.

In the Siler mortality change model shown in Figure 3 and equation (15), at time 0 the number of years before the modal age at death to cover 90%, 75% and 50% of deaths are 86.84, 84.84, and 17.84, respectively. After 400 years these values for the intervals for the different proportions decreases to levels slightly higher than those mentioned earlier for the Gompertz mortality change model.

This analysis confirms that the rectangularization process of mortality compression dramatically decreases once infant mortality has become a minor factor (Kannisto 2000; Wilmoth and Horiuchi 1999). However, our results further the rectangularization debate by suggesting that the current situation might be the beginning of a shifting trend in mortality.

Cheung et al. (2005) concluded that the transformations occurring in the survival function for Hong Kong may be interpreted as resistance to human longevity. However, as shown in our analysis of the modal age at death, the changes in Hong Kong may also be seen as a transition towards a shifting mortality era. In this era the rectangularization process has stopped while the modal age at death keeps moving towards older ages, taking with it all of the deaths concentrated around it. For example, as shown in Table 2 in Cheung et al. (2005), over a 25-year period in Hong Kong, the modal ages for females and males move 6.3 and 7.8 years to older ages, respectively. In this period, the standard deviation shows minor reductions.

To this point we have analyzed the distribution of deaths and modal age at death under mortality model assumptions. In the next section we contrast the results presented above with those for changes in human populations.

FIVE INDUSTRIALIZED COUNTRIES: AN ILLUSTRATION

Figure 4ab show the change over time in the modal age at death as suggested by Kannisto (1996), and the modal value at death in England and Wales, Japan, The Netherlands, Sweden and the United States.

[FIGURE 4a & 4b HERE]

Figure 4a shows the common linear trend in the modal age at death for these five countries. Particularly interesting is Japan, which started with low values for modal age at death but moved rapidly to become the country with the highest value. The Netherlands, Sweden, and the US followed for modal age values at the end of the twentieth century, but their pace of increase is moderate compared to Japan's. England and Wales changed very modestly during this period, continuing to exhibit the lowest values for this group of countries. These results parallel the changes in life expectancy where Japan is the leader.

Figure 4b shows the logistic trend in the number of deaths at the modal age for the five selected countries. The trend over time is less clear than for modal age. However, the increase from low to high values and leveling off in the second half of the century are clear. The last decade of the century reveals unexpected changes, which include a decline for Japan and increase for the rest of the countries. An analysis of these changes is beyond the scope of this article.

Table 3 presents the modal age and modal number of deaths, and the number of years needed before the modal age to obtain 90%, 75%, and 50% of deaths, as well as the standard deviation from the mode, for these five industrialized countries from 1970 to 1995. For the year 1995, or 2000 if available, estimated values for α and β have been calculated under a Gompertz model assumption as in (10) for ages 30 to 90. These parameters provide us with the information we need to calculate estimates of the modal age and value at death, the intervals needed to obtain the different proportions and the standard deviation from the mode for the last year, whether 1995 or 2000.

[TABLE 3 HERE]

All of the countries included in Table 3 show common trends. An increase in the modal age and modal value at death is found, with the second changes less clear for Japan. The three age intervals studied and the SDM decline over time, although an increase is observed in the US in 1995. The estimated results using the Gompertz mortality change model are slightly below and above the values observed in 1995 and 2000. Most of the differences between observed and estimated results in 1995 and 2000 are due to mortality at younger ages that is not accounted for in the estimation. These results suggest that some countries might be nearer than others to achieve the shifting mortality era. As already shown in Figure 4a, the modal age at death in England and Wales has remained behind that for the rest of the studied countries. Table 3 shows that this is reinforced with an overestimation of the modal age at death in this country. The values for the age intervals and SDM in the United States are much higher than for the other countries. In other words, much work is needed in health matters in the US to achieve the shifting mortality era discussed in this article.

CONCLUSION

Modal age at death is an alternative measure in examining change over time in mortality. We have shown in this study that a shifting mortality scenario, where the compression of mortality has stopped, may be realistic.

Mortality models, adopted as good approximations of the force of mortality, show that the modal age at death increases over time. However, the number of survivors and deaths at modal age move toward constant levels. The distribution of deaths around the modal age and standard deviation from the mode also show a constant concentration–almost a reallocation–of the characteristic bell-curve distribution around the modal age at death towards more advanced ages.

Populations with historical mortality data support the early changes illustrated by the models. However, existing variations among the countries included in this analysis reveals the need to conduct detailed future analyses for each country. Further studies of the shape of distribution of deaths by cause of death could improve our understanding of the dynamics of mortality.

Finally, the rectangularization of the survival curve shows change from a wide dispersion of deaths to a concentration in the number of deaths. Although it describes the mortality changes observed in the last half of the century well, it might only be a temporary phenomenon. The shifting mortality hypothesis studied in this article might also be transitory, yet it brings to light alternative processes that might be expected if the current mortality changes maintain their pace.

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Table 1. Parameters of the Logistic Model for Adult Mortality and the Modal Age and Modal Value at Death in 14 countries.

	Females				Males			
	α	β	Modal Age	Modal Value	α	β	Modal Age	Modal Value
Austria	-11.7	0.117	81.3	0.0407	-10.4	0.106	77.1	0.0371
Canada	-11.1	0.106	83.3	0.0371	-10.1	0.100	78.3	0.0351
Denmark	-11.1	0.108	82.1	0.0377	-10.5	0.106	78.2	0.0371
England and Wales	-11.2	0.109	82.1	0.0380	-10.5	0.107	77.0	0.0374
Finland	-11.8	0.119	81.3	0.0413	-9.8	0.099	75.2	0.0347
France	-11.7	0.115	82.7	0.0400	-10.1	0.101	77.1	0.0354
Italy	-11.8	0.118	82.1	0.0410	-10.6	0.107	78.0	0.0374
Japan	-11.8	0.118	81.8	0.0410	-10.7	0.108	78.6	0.0377
Netherlands	-11.8	0.116	83.0	0.0404	-10.8	0.109	79.0	0.0381
Norway	-11.9	0.117	83.7	0.0407	-10.8	0.109	79.1	0.0381
Sweden	-11.9	0.117	83.2	0.0407	-11.1	0.112	79.7	0.0390
Switzerland	-12.0	0.120	82.3	0.0417	-10.9	0.111	78.6	0.0387
United States	-10.7	0.101	83.6	0.0354	-9.7	0.094	77.6	0.0331
West Germany	-11.7	0.116	82.1	0.0404	-10.4	0.105	78.0	0.0367
Average	-11.5	0.114	81.9	0.0397	-10.4	0.105	77.3	0.0367

Source: The logistic parameters derive from Bongaarts (2005) average of annual estimates for all available years from 1950 to 2000.

Table 2. Modal Age and Value at Death, and Standard Deviation from the Mode, SDM, for a Gompertz Mortality Change Model with Parameters α = -11, β = 0.1 and ρ = 0.01 and a Siler Mortality Change Model with Parameters α_1 = -1.6, β_1 = 1, ρ_1 = 0.015, α_2 = -5.8, α_3 = -8.5, β_3 = 0.1 and ρ_3 = 0.01

	Gompertz Mortality Change Model			Siler Mortality Change Model		
Year	Modal Age	Modal Value	Standard Deviation	Modal Age	Modal Value	Standard Deviation
0	87	0.037	14.02	86	0.021	50.07
100	97	0.037	14.02	97	0.031	33.26
200	107	0.037	14.02	107	0.035	22.98
300	117	0.037	14.02	117	0.036	17.94
400	127	0.037	14.02	127	0.037	15.72

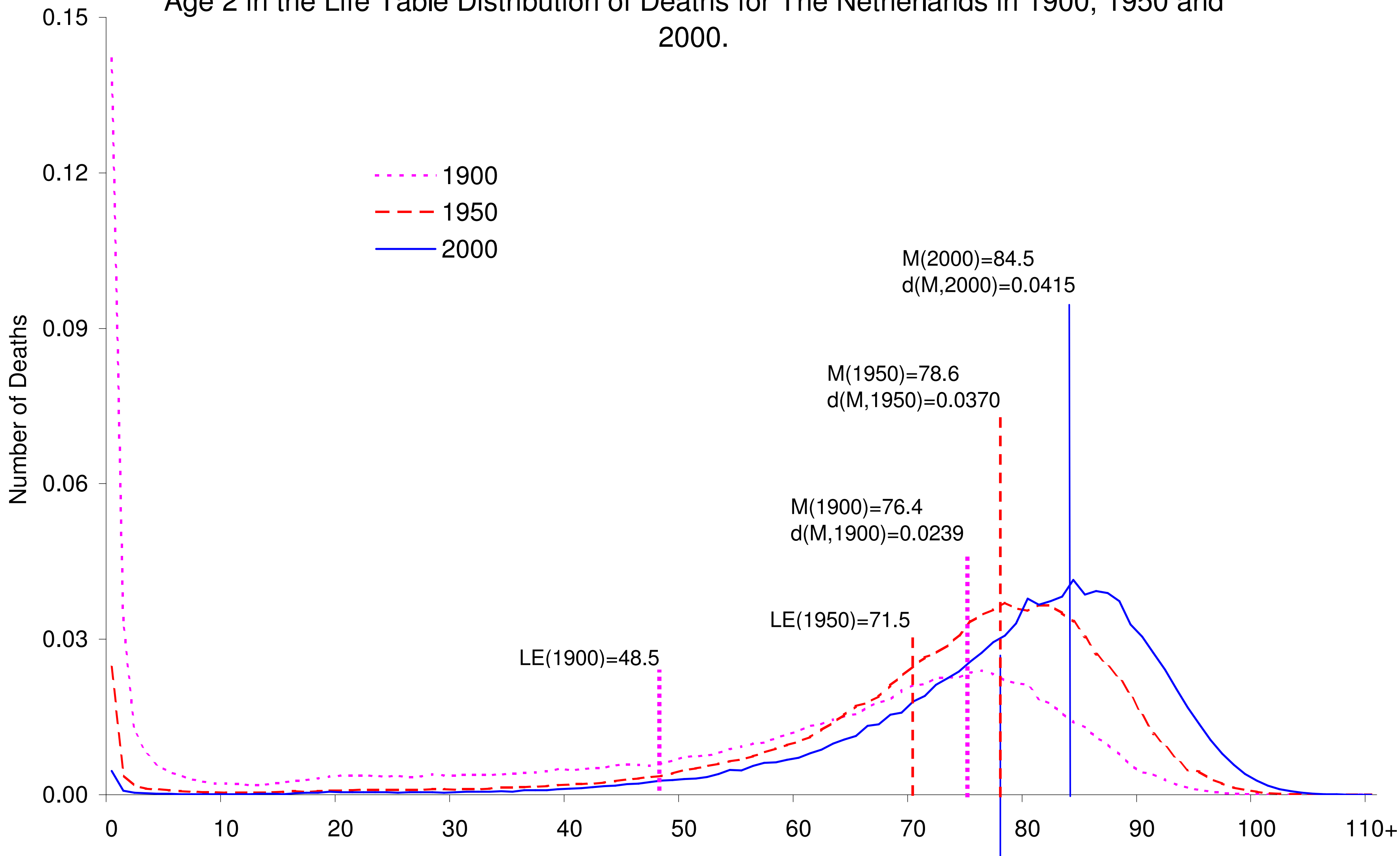
Table 3. Five Year Moving Average for the Modal Age and Highest Value at Death, Number of Years Before the Modal Age Needed to Obtain 90%, 75% and 50% Deaths, From 1970 to 1995 and 2000 if Available. For England and Wales, Japan, the Netherlands, Sweden and the United States.

				Number of Years Before the Modal Age to Have a Proportion of Deaths of			Standard Deviation
Year	Modal Age	Modal Value		90%	75%	50%	from the Mode
England and Wales		Estimated*	α =	-10.46	β =	0.096	
1970	79.8	0.0332		26.8	14.8	4.8	19.13
1975	80.5	0.0334		26.5	14.5	4.5	18.64
1980	81.2	0.0343		26.2	14.2	5.2	18.07
1985	82.3	0.0336		25.3	14.3	5.3	17.51
1990	81.8	0.0338		23.8	12.8	3.8	16.76
1995	83.1	0.0355		24.1	13.1	4.1	16.39
1995*	84.5	0.0354		23.4	13.0	3.8	14.55
Japan		Estimated*	α =	-10.57	β =	0.094	
1970	80.7	0.0370		27.7	14.7	5.7	19.35
1975	81.4	0.0392		25.4	13.4	4.4	17.76
1980	83.0	0.0400		25.0	13.0	4.0	17.04
1985	84.7	0.0406		24.7	12.7	4.7	16.74
1990	85.7	0.0412		24.7	12.7	3.7	16.39
1995	87.5	0.0403		25.5	14.5	5.5	16.96
1995*	87.4	0.0346		23.9	13.3	3.9	14.89
Netherlands		Estimated*	α =	-10.80	β =	0.099	
1970	81.9	0.0360		26.9	14.9	4.9	18.85
1975	82.0	0.0356		26.0	14.0	5.0	17.91
1980	83.7	0.0352		25.7	14.7	5.7	17.93
1985	83.7	0.0359		25.7	14.7	4.7	17.24
1990	84.7	0.0361		25.7	14.7	5.7	17.24
1995	84.2	0.0376		24.2	13.2	4.2	16.37
2000	85.1	0.0398		24.1	13.1	4.1	16.09
2000*	85.6	0.0365		22.7	12.6	3.7	14.12
Sweden		Estimated*	α =	-10.92	β =	0.099	
1970	82.7	0.0374		26.7	14.7	4.7	18.50
1975	81.8	0.0379		25.8	12.8	3.8	17.49
1980	83.4	0.0379		26.4	14.4	5.4	17.44
1985	84.8	0.0380		25.8	14.8	5.8	17.44
1990	84.6	0.0389		24.6	13.6	4.6	16.73
1995	85.4	0.0399		24.4	13.4	4.4	15.83
2000	86.4	0.0417		23.4	12.4	4.4	15.61
2000*	86.9	0.0365		22.7	12.6	3.7	14.14
United States		Estimated*	α =	-9.24	β =	0.079	
1970	79.8	0.0297		31.8	16.8	5.8	21.45
1975	82.7	0.0303		32.7	18.7	6.7	21.44
1980	82.7	0.0308		30.7	16.7	5.7	20.40
1985	84.4	0.0314		30.4	17.4	6.4	20.28
1990	83.5	0.0312		29.5	16.5	4.5	19.54
1995	85.4	0.0329		31.4	17.4	6.4	19.98
1995*	84.8	0.0291		28.3	15.7	4.6	17.47

Source: Human Mortality Database (2005).

* Values of the parameters α and β are estimated from the probabilities of deaths between ages 30 and 90 and used in equations (6), (9), (17) and (18).

Figure 1. Life Expectancy, LE, Modal Age, M, and Modal Number of Deaths, d(M), after Age 2 in the Life Table Distribution of Deaths for The Netherlands in 1900, 1950 and 2000.



Source: Human Mortality Database (2005).

Figure 2. Survivors Function and Distribution of Deaths for a Continuous Declining Mortality Model with Parameters: $\alpha = -11$, $\beta = 0.1$ and $\rho = 0.01$.

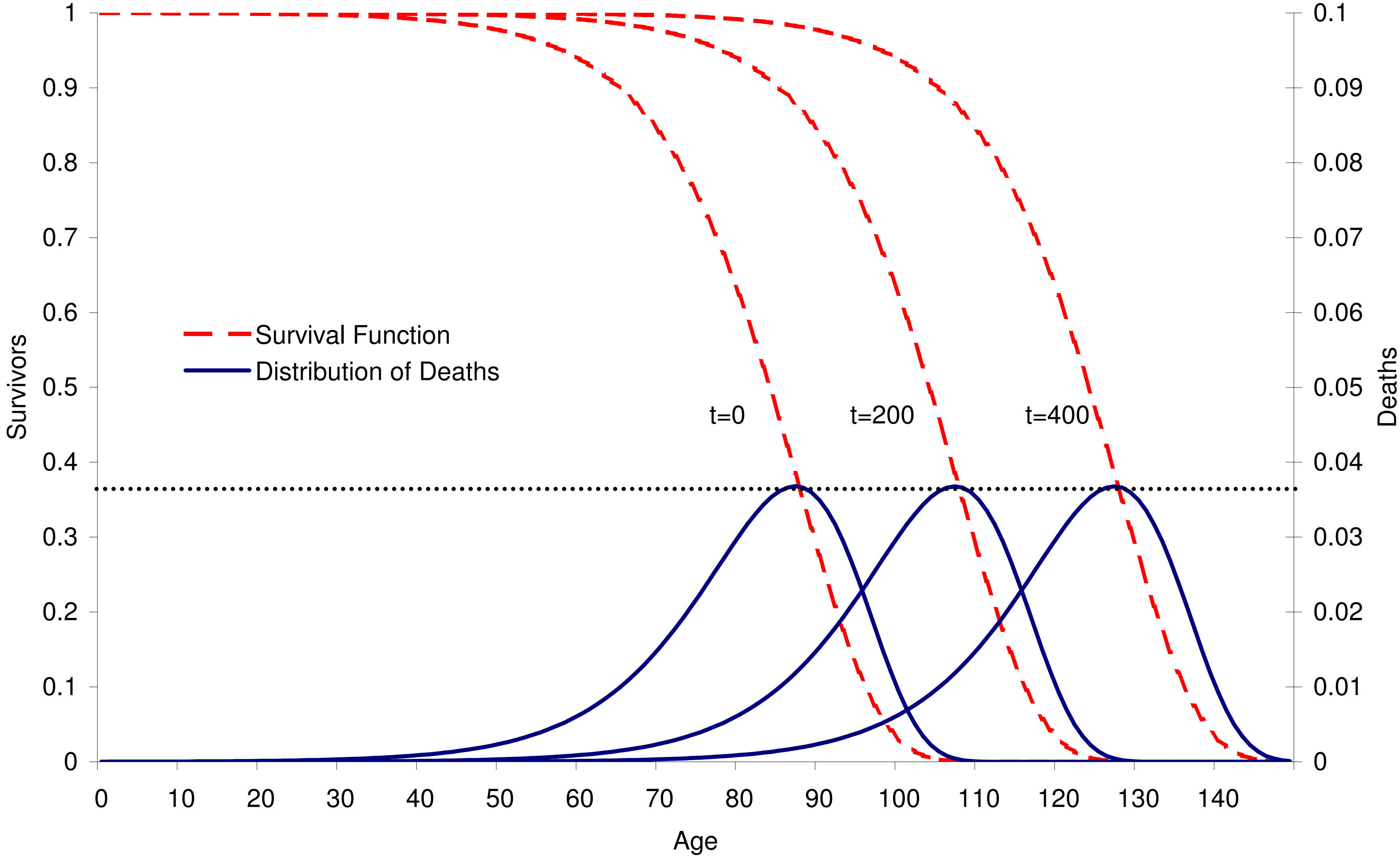


Figure 3b. Change over Time in the Modal Age and Modal Number of Deaths Under a Siler Mortality Change Model of Decline over Time: $\alpha_1 = -1.6$, $\beta_1 = 1$, $\rho_1 = 0.015$, $\alpha_2 = -5.8$, $\alpha_3 = -8.5$, $\beta_3 = 0.1$ and $\rho_3 = 0.01$.

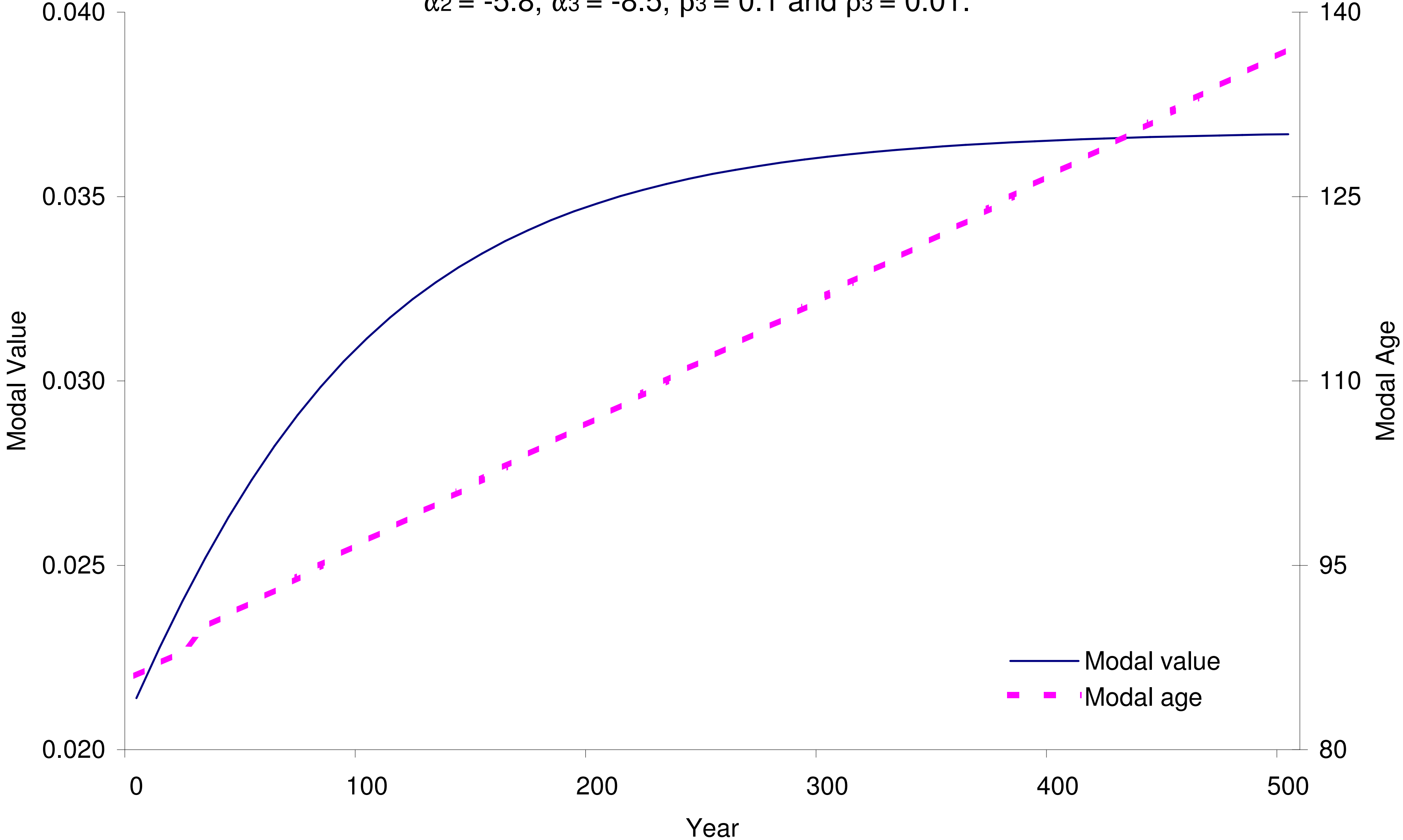


Figure 3a. Change over Time in the Survivors and Distribution of Deaths Under a Siler Mortality Change Model of Decline over Time: α_1 = -1.6, β_1 = 1, ρ_1 = 0.015, α_2 = -5.8, α_3 = -11, β_3 = 0.1 and ρ_3 = 0.01.

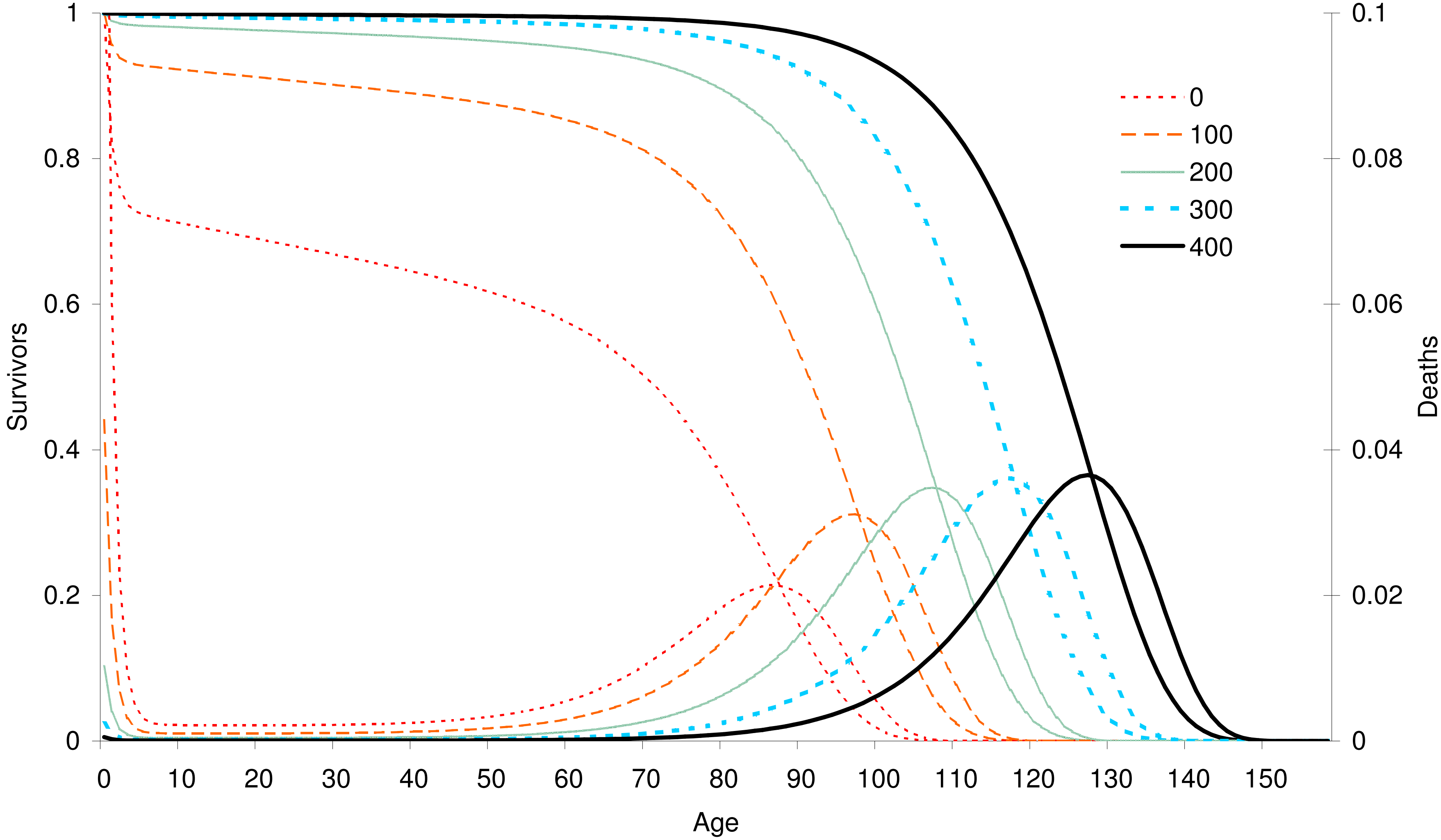
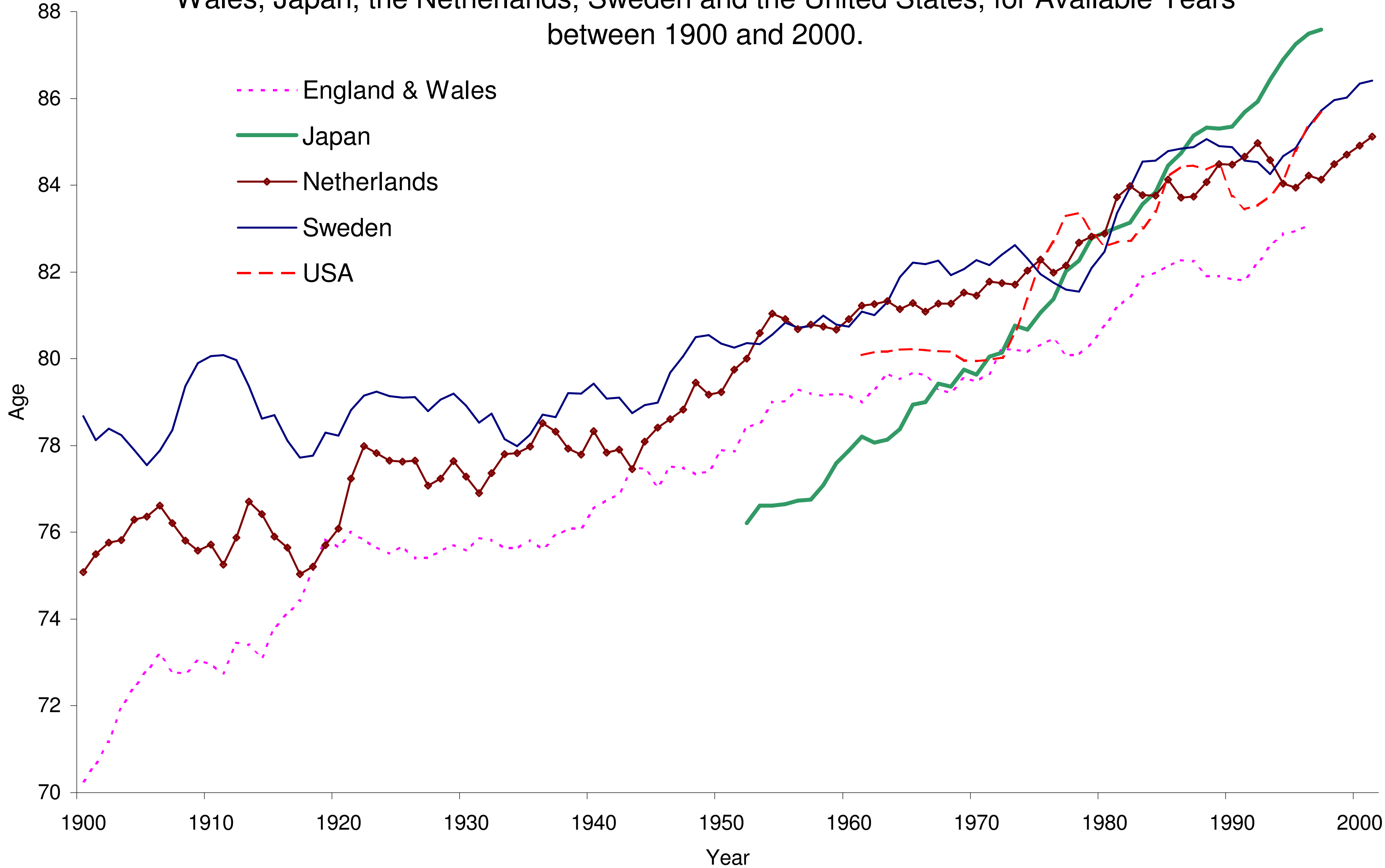
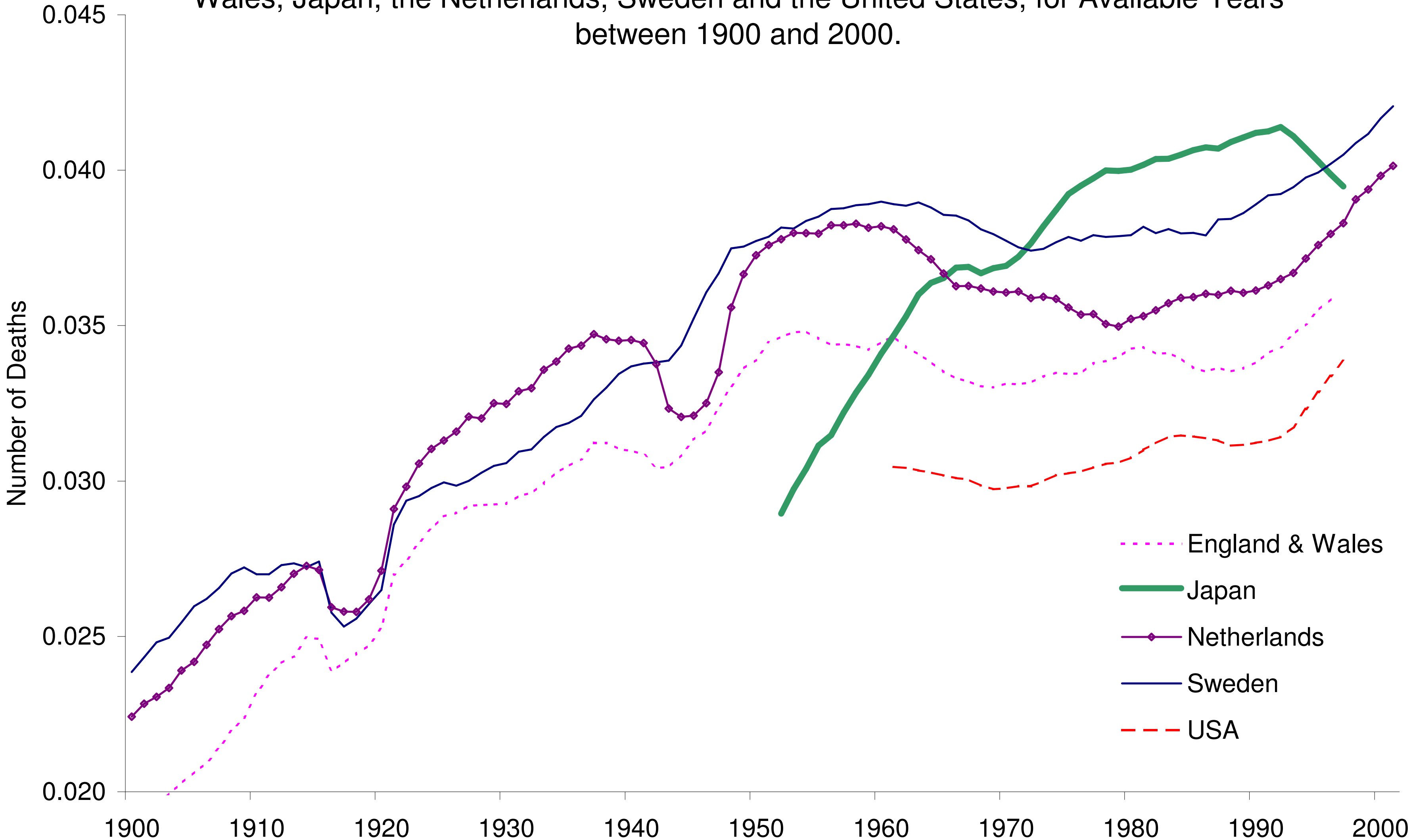


Figure 4a. Five Year Moving Average of the Modal Age at Death for England and Wales, Japan, the Netherlands, Sweden and the United States, for Available Years between 1900 and 2000.



Source: Authors' calcualtions from Kannisto (1996) proposal of modal age at death, based on Human Mortality Database (2005).

Figure 4b. Five Year Moving Average of the Modal Value at Death for England and Wales, Japan, the Netherlands, Sweden and the United States, for Available Years between 1900 and 2000.



Source: Human Mortality Database (2005).