

# An IPAT-type Model of Environmental Impact Based on Stochastic Differential Equations

Emilio Zagheni  
UC Berkeley

Francesco C. Billari  
Bocconi University

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This work adds to the literature examining human driven climate change within the framework of IPAT-based models. Our contribution relates, on the one hand, to the modelling of the environmental impact, expressed in terms of carbon dioxide emissions, and, on the other hand, to the valuation of the costs that a country has to bear in order to reduce its emissions. Starting from the well-known IPAT equation, we develop a stochastic formulation of the relationship between population, affluence and technology alternative to the STIRPAT model. This first step leads to a stochastic differential equations model that describes the trend of carbon dioxide emissions of a country, on the basis of the dynamics of its population and affluence. As an example, we estimated the parameters of the model with regard to the United States. Our theoretical scheme has then been used to develop a model for the assessment of the costs that a country has to bear, if committed to respect an international agreement on emissions reduction, like the Kyoto protocol. In particular, we show that the adherence to an environmental treaty may be traced back to a problem of cost valuation in a risk situation: this allows us to exploit the mathematical tools that have been developed in the field of finance, specifically in the context of option pricing, to determine the expected investment that a country is supposed to make in order to reduce its emissions of a certain amount, within a well-defined temporal frame.

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**KEY WORDS:** Environmental Impact; IPAT; stochastic differential equations; costs of  $CO_2$  emissions reduction.

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Please address correspondence to Emilio Zagheni, Department of Demography, University of California, Berkeley, 2232 Piedmont Avenue, 94720-2120, Berkeley, USA;  
e-mail: emilioz@demog.berkeley.edu

## INTRODUCTION

We live in a period characterised by unprecedented global changes and by the consciousness that transformations that are taking place may have consequences on our daily lives. Global climate change is felt as a serious threat in several regions of the world, where local ecosystems risk to disappear or dramatically change. In addition, the consequences of climate alterations may have an impact not only on the presence of ecosystems, but also on the economy of several countries and, in extreme cases, on their survival, as it is the case for certain pacific islands that risk to be submerged.

The awareness of environmental concerns is reflected the on the debate taking place within the international community and the effort that the United Nations put in the scientific analysis of the transformations under way.

At the institutional level, the debate on climate change is centred on the Kyoto Protocol, that has been recently ratified, and the future of agreements on climate change, in particular concerning the regulation of carbon dioxide emissions. The limitations of the Protocol and the need to extend rules concerning carbon dioxide emissions to developing countries are some of the relevant topics discussed at the international level.

The United Nations are involved not only at the political level, but also in scientific issues: as an example we can cite the Millennium Ecosystem Assessment (2005) that reports, for instance, on ecosystems and human well-being. United Nations put a great effort into monitoring the state of the earth and in providing relevant scientific-based information about environmental issues, involving a great number of institutions and scholars from all over the world.

However, the role of population into the explanation of global environmental impact is still unclear and needs further investigation. Our paper stems from the necessity of improving the modelling of environmental impact and the comprehension of the consequences of different population trends on both environmental disruption and the effort required to the countries to counteract it. Our starting point is the debate that took place in the seventies and that led to the formulation of models, like the so-called IPAT equation, that reserved a prominent role to the demographic factors.

## ENVIRONMENTAL IMPACT WITHIN THE FRAMEWORK OF THE IPAT EQUATION

A proper awareness of environmental issues, in the academic world and at the level of institutional policies and international organisations, is quite recent and dates only back to the early 1970s. It was during those years that demographers, biologists and ecologists started to put a great effort

into the comprehension of the relationships between population dynamics, human welfare and environmental impact. Within this framework, Ehrlich and Holdren (1971) devised a simple equation, in dialogue with Commoner (1972), identifying three factors as the key determinants of environmental impact. Its mathematical representation, the so-called ‘IPAT equation’, simply states that environmental impact ( $I$ ) is the product of population ( $P$ ), affluence ( $A$ ), and technology ( $T$ ):

$$I = P * A * T \quad (1)$$

The IPAT equation embodies a simple scheme that has been chosen by many scholars (e.g. Dietz and Rosa, 1994, 1997; Mackellar et al., 1995; York et al., 2003; Aufhammer et al., 2004) as a starting point when investigating interactions between population, economic growth, and technological development.

The original formulation, presented by Ehrlich and Holdren in 1971, was intended to refuse the notion that population was a minor contributor to environmental crisis. Rather, this formulation makes population central to the equation by expressing the impact of a society on the ecosystem as:

$$I = P * F. \quad (2)$$

where  $I$  is the total impact,  $P$  is the population size and  $F$  is the per capita impact. As the authors explain, impact increases as either  $P$  or  $F$  increases, or if one increases faster than the other declines.

To show that the equation is non-linear and the variables are interdependent, Ehrlich and Holdren then expanded their equation as follows:

$$I = P(I, F) * F \quad (3)$$

Commoner (1972) played an important role in the formulation of the IPAT equation as well: much of his work and scientific analysis, during the period 1970-1972, was concerned with measuring the amount of pollution resulting from economic growth in the United States during the post-war period. To do so, he and his colleagues became the first ones to empirically apply the IPAT concept with mathematical rigor. In order to make operative the three factors that influence  $I$ , environmental impact, Commoner further defined  $I$  as “the amount of a given pollutant introduced annually into the environment”. His equation, published in a 1972 conference proceedings paper is:

$$I = Population * \frac{Economic\ good}{Population} * \frac{Pollutant}{Economic\ good} \quad (4)$$

where *Population* is used to express the size of the population of a region in a given year or the change in population over a defined period. *Economic good*

is used to express the amount of a particular good produced or consumed during a given year or the change over a defined period. *Pollutant* refers to the amount of a specific pollutant and it is a measure of the environmental impact.

Commoner compared the relative contribution of the three IPAT variables in explaining pollutants such as detergent phosphate, fertiliser nitrogen, synthetic pesticides, tetraethyl lead, nitrogen oxides, and beer bottles. He concluded that the contribution of population and affluence to pollution levels, calculated at the same period, is much smaller than that of technology of production. His conclusions generated an intense debate with Ehrlich and Holdren who, on the other hand, emphasised the importance of the demographic aspects instead of the technological ones.

The IPAT model, that stems from these debates, has become famous in its tautological formulation: the equation 1 is simply an identity if one chooses the per capita income ( $Y/P$ ) as a measure of the affluence term  $A$ , and the environmental impact per unit of income ( $I/Y$ ) as a proxy for the technology level  $T$ :

$$I = P * A * T = P * \frac{Y}{P} * \frac{I}{Y} \quad (5)$$

This identity equation may be exploited for accounting or decomposition purposes, but it is not immediately useful for statistical analyses, since the identity is not suitable for parameter estimation and hypothesis testing.

In order to overcome these limitations, Dietz and Rosa (1994) reformulated the IPAT equation as STIRPAT, meaning “Stochastic Impacts by Regression on Population, Affluence and Technology”. The specification they used to perform a regression analysis is the following:

$$I = aP^bA^cT^de \quad (6)$$

where the variables  $a-d$  can be either parameters or more complex functions estimated using standard statistical procedures and  $e$  is an error term. This functional form allows for the presence of non-linear relationships between the driving forces and the environmental impact; moreover, the logarithmic transformation

$$\ln I = \ln a + b \ln P + c \ln A + d \ln T + \ln e \quad (7)$$

makes it easy to compute the elasticity of the environmental impact with respect to each of the anthropogenic factor.

Dietz and Rosa mainly used the STIRPAT model in studies of global climate change. For instance, in a subsequent paper (Dietz and Rosa, 1997), they estimated, from a panel data set, the effects of population, affluence, and technology on national  $CO_2$  emissions.

Researches in the field of human driven climate change, specifically energy-related carbon emission studies, have been a successful application

of the IPAT approach. Our work stems from these contributions and it is intended to deepen the IPAT-based stochastic approach, with reference to global climate change issues.

## A STOCHASTIC MODEL OF ENVIRONMENTAL IMPACT BASED ON THE IPAT EQUATION

In this section, we develop a stochastic approach, alternative to the STIRPAT formulation, that will prove to be useful to model the economic cost that a country has to bear to reduce its environmental impact.

Our starting point is the IPAT scheme: consider the environmental impact  $I$ , expressed for example in terms of  $CO_2$  emissions, evolving in time as a function of  $P$  (the population size of a country or a geographical region),  $A$  (the affluence, expressed in terms of per capita income,  $Y/P$ ) and  $T$  (the effectiveness of technology, expressed as the ratio between environmental impact and income,  $I/Y$ ):

$$I(t) = f(P(t), A(t), T(t)) \quad (8)$$

It holds that:

$$\frac{dI}{dt} = \frac{dI/I}{dP/P} \frac{dP/dt}{P} + \frac{dI/I}{dA/A} \frac{dA/dt}{A} + \frac{dI/I}{dT/T} \frac{dT/dt}{T} \quad (9)$$

Therefore we can write the expression 8 in terms of growth rates:

$$\frac{\frac{dI}{dt}}{I} = \frac{\dot{I}}{I} = \mu_{I,P} \frac{\dot{P}}{P} + \mu_{I,A} \frac{\dot{A}}{A} + \mu_{I,T} \frac{\dot{T}}{T} \quad (10)$$

where  $\mu$ s represents the elasticities of  $CO_2$  emissions with regard to the driving forces  $P$ ,  $A$  and  $T$  respectively.

It also holds that:

$$\frac{\dot{T}}{T} = \frac{\frac{d(I/Y)}{dt}}{I/Y} = \frac{\dot{I}}{I} - \frac{\dot{Y}}{Y} \quad (11)$$

Consequently we can rearrange the expression 10 and we can rewrite it as:

$$\frac{\dot{I}}{I} = \left( \frac{\mu_{I,P}}{1 - \mu_{I,T}} \right) \frac{\dot{P}}{P} + \left( \frac{\mu_{I,A}}{1 - \mu_{I,T}} \right) \frac{\dot{A}}{A} - \left( \frac{\mu_{I,T}}{1 - \mu_{I,T}} \right) \frac{\dot{Y}}{Y} \quad (12)$$

According to this formulation, when data on growth rates of  $I$ ,  $P$ ,  $A$ ,  $Y$  are available, it is possible to estimate the elasticities: in particular, it is easy to estimate the value of  $\mu_{I,T}$ .

The expression 12 may be further simplified to

$$\frac{\dot{I}}{I} = \left( \frac{\mu_{I,P} - \mu_{I,A}}{1 - \mu_{I,T}} \right) \frac{\dot{P}}{P} + \left( \frac{\mu_{I,A} - \mu_{I,T}}{1 - \mu_{I,T}} \right) \frac{\dot{Y}}{Y} \quad (13)$$

and the parameters may be estimated by means of the following stochastic formulation:

$$\frac{\dot{I}}{I} = a \frac{\dot{P}}{P} + b \frac{\dot{Y}}{Y} + \epsilon \quad (14)$$

where  $\epsilon$  is a zero mean error term distributed according to a gaussian distribution and with the property of no serial correlation.

Equation 14 represents an alternative IPAT-based stochastic model of environmental impact with respect to the equation 7. As an example, we estimated the coefficients  $a$  and  $b$  with reference to the United States and the period 1975-2000<sup>1</sup>, obtaining a value of  $-1,59$  for  $a$  and  $0,86$  for  $b$ . The coefficients of our regression are statistically significant at the 5% level, moreover the residuals show no serial correlation and are distributed according to a Gaussian distribution (Jarque-Bera p-value=0,25).

Under our assumptions, in particular that the elasticities remain constant over the time period we consider, the equation 13 may be seen as:

$$dI = (a \frac{\dot{P}}{P} + b \frac{\dot{Y}}{Y}) I dt \quad (15)$$

This equation may be expressed in stochastic terms basically for two reasons: the first one is related to the possibility that other factors than those included in the model might intervene in the explanation of environmental impact. In that sense we can rewrite the 15 as

$$dI = (a \frac{\dot{P}}{P} + b \frac{\dot{Y}}{Y}) I dt + \sigma_1 I dB_1(t) \quad (16)$$

where  $B_1(t)$  stands for Brownian motion (e.g. Øksendal, 2000) .

A second reason that makes the relationship aleatory is due to the possibility that population and income growth rates do not evolve in a deterministically way, this meaning that their trend might show a random component. We can assume that population and income evolve as a stochastic process regulated by the equation of the Brownian geometric motion:

$$dP = cPdt + \sigma_2 P dB_2(t) \quad (17)$$

and

$$dY = eYdt + \sigma_3 Y dB_3(t) \quad (18)$$

where  $B_2(t)$  and  $B_3(t)$  are mutually independent Brownian motions, also independent of  $B_1(t)$ .

Starting from these assumptions, we can rewrite the 15 in its stochastic version:

$$dI = (ac + be) I dt + \sigma_1 I dB_1(t) + a\sigma_2 I dB_2(t) + b\sigma_3 I dB_3(t) \quad (19)$$

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<sup>1</sup>Data are provided by the World Bank (2004).

Therefore our model is described by a system of three stochastic differential equations:

$$\begin{cases} dI = (ac + be)Idt + \sigma_1 IdB_1(t) + a\sigma_2 IdB_2(t) + b\sigma_3 IdB_3(t) \\ dP = cPdt + \sigma_2 PdB_2(t) \\ dY = eYdt + \sigma_3 YdB_3(t) \end{cases} \quad (20)$$

This representation implies that the carbon emissions growth rate is a normally distributed random variable, with mean  $(ac + be)$ . This model is consistent with our data about carbon dioxide emissions in the United States, where the emissions growth rate shows a gaussian distribution with mean 0,01 for the period 1975-2000.

In order to have an intuitive grasp of the trend followed by the environmental impact under these assumptions, it is useful to run a numerical simulation of the solutions of our model: in Figure 1 we represent three possible trajectories of the stochastic process according to which environmental impact evolves. This representation has been obtained by choosing values of parameters that are consistent with the evidence shown by our estimates, and by writing equation 19 in discrete-time terms:

$$I(t_{i+1}) - I(t_i) = (ac + be)I(t_i)h + \sigma_1 I(t_i)\sqrt{h}Y_i + a\sigma_2 I(t_i)\sqrt{h}Z_i + b\sigma_3 I(t_i)\sqrt{h}K_i \quad (21)$$

where  $i = 0, 1, 2, \dots, 1000$ ;  $h = 1/365$  and  $Y_i$ ,  $Z_i$  and  $K_i$  are random variables with a standard gaussian distribution.

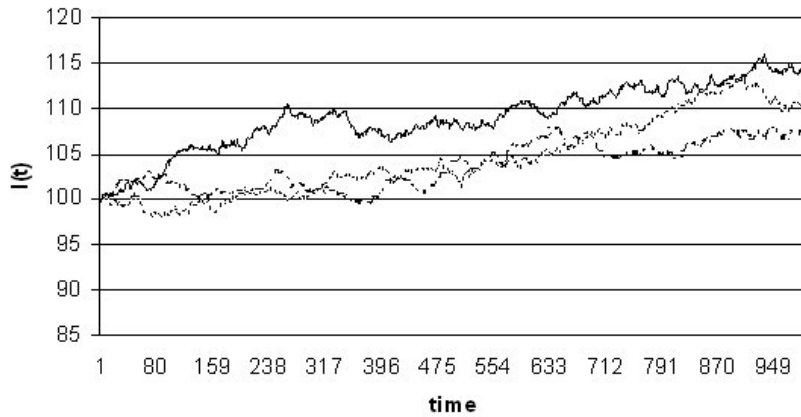


Figure 1: Three simulated trajectories of the environmental impact under the assumptions of our model and with the following choice of parameters:  $\mu_{I,P} = 0,7$ ;  $\mu_{I,A} = 1$ ;  $\mu_{I,T} = 0,8$ ;  $a = -1,5$ ;  $b = 1$ ;  $c = 0,01$ ;  $e = 0,03$ ;  $\sigma_1 = 0,03$ ;  $\sigma_2 = 0,002$ ;  $\sigma_3 = 0,02$ .

An analytical solution for equation 19, that describes the dynamics of environmental impact, can also be found. Consider  $I(u)$  as the amount of carbon emissions at time  $u$  and  $i$  as the initial level of emissions, at time  $t$ . The dynamics of emissions may be written as:

$$\begin{cases} dI(u) = rI(u) + \sigma_1 I(u)dB_1(u) + a\sigma_2 I(u)dB_2(u) + b\sigma_3 I(u)dB_3(u) & u > t \\ I(t) = i \end{cases} \quad (22)$$

where  $(ac + be) = r$ .

In order to derive an analytical solution, consider the transformation  $g(t, I(u)) = \ln I(u)$ . By applying the Itô formula for the differential of a compound function to  $\ln I(u)$  (e.g. Øksendal, 2000), it is simple to derive the solution of 22:

$$\begin{aligned} I(u) = & i \exp \left\{ \left[ r - \frac{1}{2}(\sigma_1^2 + a^2\sigma_2^2 + b^2\sigma_3^2) \right] (u - t) + \sigma_1 (B_1(u) - B_1(t)) + \right. \\ & \left. + a\sigma_2 (B_2(u) - B_2(t)) + b\sigma_3 (B_3(u) - B_3(t)) \right\} \end{aligned} \quad (23)$$

Thus the solution may be expressed as:

$$I(u) = i \exp \{k\} \quad (24)$$

where

$$k \sim N\left(\left(r - \frac{1}{2}\sigma^2\right)(u - t); \sigma^2(u - t)\right) \quad (25)$$

with  $\sigma_1^2 + a^2\sigma_2^2 + b^2\sigma_3^2 = \sigma^2$ .

Finally, it is easy to show that  $I(u)$  has a lognormal distribution. Let  $\phi_{I(u)}(y)$  be the probability function of  $I(u)$ , it results that:

$$\phi_{I(u)}(y) = p(t, i; u, y) = \frac{1}{\sigma y \sqrt{2\pi(u - t)}} \cdot \exp \left\{ -\frac{[\ln y - \ln i - (r - \frac{\sigma^2}{2})(u - t)]^2}{2\sigma^2(u - t)} \right\} \quad (26)$$

## AN ANALYSIS OF THE COSTS RELATED TO A REDUCTION OF THE ENVIRONMENTAL IMPACT

In this section we make use of the model presented in the previous section to analyse the costs that a country has to bear in order to reduce its environmental impact, expressed in terms of carbon dioxide emissions.

This is a relevant problem that has been brought to light by some international agreements on the environment: in 1992, for example, the international community adopted the Climate Change Convention, the main purpose being the stabilisation of atmospheric concentrations of greenhouse



gases at safe levels. The Convention commits all countries to limit their emissions, gather relevant information, develop strategies to adapt to climate change, and cooperate on research and technology. It also requires developed countries to take measures aimed at returning their emissions to 1990 levels.

In 1997, the international community gathered in Kyoto and agreed on a Protocol that required developed countries to accept a legally binding commitment to reduce their collective emissions of six greenhouse gases by at least 5%, compared to 1990 levels, by the period 2008-2012.

Our analysis relies on the assumptions that we made in the previous section and our approach stems from the literature on stochastic processes and their applications to economics and finance. In particular, we have to mention the well-known *Black and Scholes model* for the determination of the fair price of an option, from which we derived the conceptual scheme for our modelling (see, for instance, Øksendal, 2000 or Cifarelli and Peccati, 1998).

Before starting our analysis, we recall our assumptions about the trends of demographic, economic and environmental variables that we made explicit in the system 20 and that we can write in a more compact way as:

$$dX(t, \omega) = u(t, \omega)dt + V(t, \omega)dB(t) \quad (27)$$

where:

$$X(t, \omega) = \begin{bmatrix} I \\ P \\ Y \end{bmatrix} \quad (28)$$

$$u(t, \omega) = \begin{bmatrix} (ac + be)I \\ cP \\ eY \end{bmatrix} \quad (29)$$

$$V(t, \omega) = \begin{bmatrix} \sigma_1 I & a\sigma_2 I & b\sigma_3 I \\ 0 & \sigma_2 P & 0 \\ 0 & 0 & \sigma_3 Y \end{bmatrix} \quad (30)$$

$$dB(t, \omega) = \begin{bmatrix} dB_1(t) \\ dB_2(t) \\ dB_3(t) \end{bmatrix} \quad (31)$$

We consider a country, like the United States, for which our assumptions are reasonable, and we suppose that this country is engaged in the reduction of a certain amount of its emissions, within a deadline set by an international agreement. The finances of this country have to bear a cost, say  $C(t, X(t))$ , that is directly related to the amount of national emissions ( $I$ ) and that does not explicitly depend on  $Y$  and  $P$ : this means that it is reasonable to let the cost assume functional forms such as  $C(t, X(t)) = \sqrt{I}$  or  $C(t, X(t)) = \ln(I)$ .

On the other hand, the country will benefit from the economic aid of an international organisation: we assume that it will receive an amount of money proportional to its emissions, say  $\alpha I(t)$ ; conversely, if the country did not meet its commitments, it would be obliged to pay a fine proportional to the difference between its actual emissions, at the deadline, and the target. The national spending on environment may be written as:

$$S(t) = C(t, X(t)) - \alpha I(t) \quad (32)$$

By recalling the Itô formula for the differential of a compound function (e.g. Øksendal, 2000), we can write the stochastic differential for the national spending:

$$\begin{aligned} dS(t) = & [C'_t + (ac + be)I(C'_I - \alpha) + \frac{1}{2}(\sigma_1^2 I^2 + a^2 \sigma_2^2 I^2 + b^2 \sigma_3^2 I^2)C''_{II}]dt + \\ & + (\sigma_1 dB_1(t) + a\sigma_2 dB_2(t) + b\sigma_3 dB_3(t))I(C'_I - \alpha) \end{aligned} \quad (33)$$

A crucial assumption that we have to make is that the International Organisation might and would like to choose  $\alpha = C'_I$ , this meaning that the economic aid that the country receives,  $\alpha I(t)$ , is proportional to national emissions, with a coefficient of proportionality that is set equal to the marginal costs of emissions that the country has to bear. In this way, it is possible to cancel out the stochastic component from the differential. Moreover, it is reasonable to assume that the share of national income spent on environmental quality is constant over time, that is:  $dS = S e dt$ , where  $e$  is the expected value of the income growth rate. These assumptions lead to the following equation:

$$C'_t + eC'_I I + \frac{1}{2}(\sigma_1^2 I^2 + a^2 \sigma_2^2 I^2 + b^2 \sigma_3^2 I^2)C''_{II} - eC = 0 \quad (34)$$

with the condition that the cost  $C$  at time  $T$ , the deadline for the reduction of emissions to a level  $\bar{I}$ , is expressed as:

$$C(T, I(T)) = \max[\gamma(I(T) - \bar{I}), 0] \quad (35)$$

This means that the national spending on emissions reduction at time  $T$  will be zero if the country attains its goal; otherwise the country will have to pay a fine proportional to the difference between its actual emissions and the target. The Feynman-Kac formula (e.g. Øksendal, 2000) makes it possible to represent the cost  $C$  as:

$$C(t, I) = E^{t,i} \{ \max[\gamma(I(T) - \bar{I}), 0] \exp \{ - \int_t^T e du \} \} \quad (36)$$

Therefore the problem may be seen in terms of evaluation of an expected value. We obtain:

$$C(t, i) = \gamma i \Phi(N_1) - \gamma \bar{I} \Phi(N_2) \exp \{ -r(T - t) \} \quad (37)$$

where

$$N_1 = \frac{\ln i + (r + \frac{\sigma^2}{2})(T - t) - \ln \bar{I}}{\sigma \sqrt{T - t}} \quad (38)$$

and

$$N_2 = \frac{\ln i + (r - \frac{\sigma^2}{2})(T - t) - \ln \bar{I}}{\sigma \sqrt{T - t}} \quad (39)$$

with  $\Phi$  being the cumulative function of a  $N(0, 1)$ ,  $r = (ac + be)$ ,  $\sigma^2 = \sigma_1^2 + a^2\sigma_2^2 + b^2\sigma_3^2$ .

As an example, consider a country whose emissions dynamics is described by our model, where the parameters are those chosen for the simulation shown in Figure 1, but with an initial emissions level of  $5 \cdot 10^8$  (kilos). Suppose that this country is engaged in the reduction of its emissions by 5% within five years: in the case the country were not able to diminish its emissions level to the target, it would have to pay a fine proportional to the difference between its actual emissions at the deadline and the target, with a coefficient of proportionality set equal to 0,5. Under the assumptions we made, the cost that this country has to bear to reduce its emissions is equal to 30144 thousand dollars.

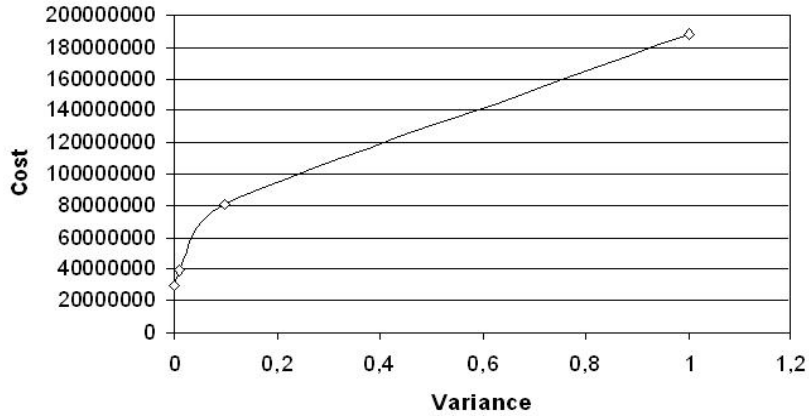


Figure 2: The relationship between  $\sigma^2$  and the costs that the country has to bear, according to our model, with the other parameters used in the example held constant.

An observation that we can make is that in this model the costs that the country has to bear are positively related to the variance of emissions, the amount of the reduction required, and the deadline set (see Figures 2 and 3): larger values of  $\sigma^2$  mean higher uncertainty and, consequently, increasing costs; the larger is the amount of reduction required, the greater is the effort that national finances have to bear; finally, the assumption of

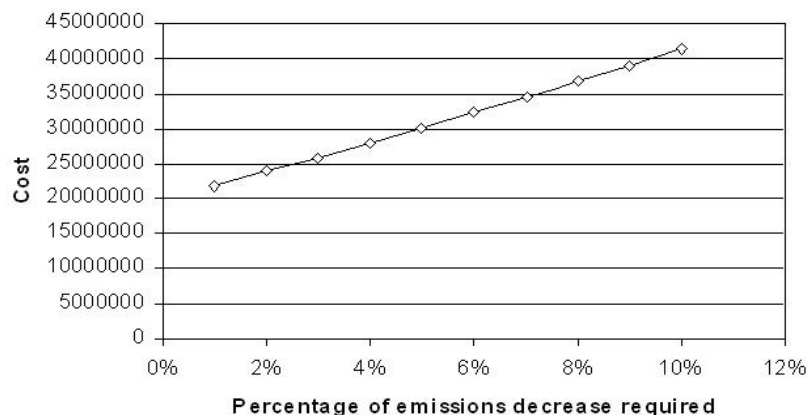


Figure 3: The relationship between the percentage of emissions decrease required by the agreement and the costs that the country has to bear, according to our model, with the other parameters used in the example held constant.

positive economic and demographic growth rates makes it more difficult for a country to reach the same target after a longer period of time.

## CONCLUSIONS

In this paper we proposed an IPAT-type model, based on stochastic differential equations, in order to grasp the trend of carbon dioxide emissions and to estimate the costs that a country has to bear, when committed to reduce its emissions within the framework of an international agreement. We estimated the parameters of our model with respect to the United States and we showed the mechanics behind the assessment of the costs that a country has to bear, once an international treaty has been signed.

Our suggestion represents, on the one hand, a stochastic modelling alternative to the STIRPAT one, and it may be considered a generalisation of IPAT-based models, whose parameter-estimation procedure results to be easy. On the other hand, the cost-valuation model represents an attempt to introduce, in a straightforward way, economic and demographic factors into the process of cost valuation that a country may implement before committing to an international agreement. The processes that govern the evolution of both population and income are considered stochastic and, as a consequence, the country has to take a decision in a risk situation.

The model we suggested is based on reliable assumptions, however further research should be accomplished in order to take into account also

populations that are not exponentially growing, and to include the effects of structural changes that affects several populations, especially in the developed countries, like the reduction of family size and the population aging.

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