Mife/draft 03.01.06

Migration-Fertility Trade-Off and Aging in Stable Populations

Juha M. Alho

Abstract

Fertility is below replacement level in all European countries, and population growth is expected to turn into decline in the coming decades. Increasing life expectancy will accentuate the concomitant aging. Migration has been seen as a possible means to decelerate aging. We introduce a stable open population model in which net-migration is proportional to births. In this case the migration-fertility trade-off can be studied with particular ease. We show that while migration can increase the growth rate, which tends to make the age-distribution younger, it also has an opposite effect due to its typical age pattern. We capture the effect of the age-pattern of net-migration in a migration survivor function. The effect of net-migration on growth is quantified with data from 17 European countries. It is shown that some countries already have a level of migration that will lead to stationarity. For other countries with asymptotically declining population migration still provides opportunities for slowing down aging. An extension for assesssing the proportion foreign-born is given.

Key words: growth rate, migration survivor function, proportion foreign-born, stationary populations

1. Introduction

The aging and forecasted decline of the populations of industrialized countries have lead to attempts at discovering to what extent increasing migration could alter the trends (e.g., Population Division 2000). One difficulty in such calculations is that one can make numerous alternative assumptions concerning the way migration occurs. It may not always be easy to check to what extent the results depend on the particular choices.

Despite its lack of realism, stable population theory[1] provides explicit formulas that can be used for insight (e.g., Espenshade, Bouvier and Arthur 1982). However, even the stable model can be formulated in different ways, and alternative formulations provide different insights. The assumption of a constant stream of immigrants has been the most popular choice (e.g., Mitra 1983, Schmertmann 1992, Schmidbauer and Rösch 1995, Bacaër 2003, Wu and Li 2003). However, as discussed by Pollard (1966) and Liao (2001), for example, a proportionality assumption also makes sense. In fact, in a full multi-state stable model net-migration would be proportional to the size of any age-group (e.g., Rogers 1995, 118).

In this paper we will assume that (the positive or negative) net-migration is proportional to births. This will lead to a particularly simple analysis in which the trade-off between fertility and net-migration can explicitly be studied, so our results complement those obtained earlier by, e.g., Schmertmann (1992), and Wu and Li (2003). Formulas for growth rates and age-distribution are similar to those of the closed population case.

Section 2 defines our "proportional" version of the stable population model and illustrates the key "migration survivor function" with data from the Nordic countries.

---

[1] By a stable population we mean a population that grows or declines exponentially and whose vital rates and age-distribution do not change (Keyfitz 1977, 51).

Mathematical arguments will be heuristic throughout and, in order not to disrupt continuity, some specifics are given in footnotes or postponed into an Appendix. Section 3 indicates the effect of net-migration on the growth rate. Numerical estimates are given for seventeen European countries. Section 4 discusses the fertility-migration trade-off by showing what levels of net-migration and fertility lead to a stationary population and give empirical estimates based on data from the Nordic countries. Similarly, using dependency ratio data from the Nordic countries we consider, in Section 5, the effect of migration on aging. Section 6 points out a possible extension for estimation of the proportion of foreign-born population. We conclude in Section 7 by noting some implications of the stable population analysis.

2. Stable Population with Net-Migration Proportional to Births

2.1. Births and Migration

Suppose the density of births at time t is $b(t) = be^{\rho t}$, where ρ is the (positive or negative) growth rate and $b > 0$ is the density at $t = 0$. We assume that $p(x)$ is the probability of surviving to age $0 < x \leq \omega$. Here ω is the highest possible age. Suppose $T > 0$ is the total fertility rate, $\alpha > 0$ is the lowest age of childbearing, and β is the highest age of childbearing with $\alpha < \beta < \omega$. We consider the female population only so it is numerically equal to the usual gross reproduction rate (cf., Shryock and Siegel 1976, 314). The age-specific fertility in age $\alpha \leq x \leq \beta$ is of the form $Tf(x)$, where $f(x) \geq 0$ integrates to 1 over childbearing ages. We assume that the same survival and fertility values apply to all members of the population of interest, as long as they stay in it.

Define $R(x,t)$ as the cumulative in-migration to the population of interest, by members of the cohort born at t, by age $0 < x \leq \omega$. Similarly, define $S(x,t)$ as the cumulative out-migration from the population of interest, by those born at t, by age $0 < x \leq \omega$. Their densities are $(d/dx)R(x,t) = r(x,t)$ and $(d/dx)S(x,t) = s(x,t)$. Write $R(\omega,t) = R(t)$ and $S(\omega,t) = S(t)$, for short. We assume that the age-patterns of the migration streams do not change over time, so $r(x,t) = r(x)R(t)$ and $s(x,t) = s(x)S(t)$, where $r(x) \geq 0$, $s(x) \geq 0$, and

$$\int_0^\omega r(x)\,dx = 1, \quad \int_0^\omega s(x)\,dx = 1. \tag{1}$$

Let $N(x,t)$ be net-migration to the cohort born at time t, so $n(x,t) = r(x)R(t) - s(x)S(t)$ is its (positive or negative) density in age $0 < x \leq \omega$.[2] Define $N(t) = R(t) - S(t)$.

2.2. Migration Proportional to Births

The *proportionality assumption* that leads to stability is that we take $R(t) = c_R b(t)$ and $S(t) = c_S b(t)$ for some c_R, $c_S \geq 0$. It follows that $n(x,t) = h(x)b(t)$, where $h(x) = c_R r(x) - c_S s(x)$. For notational simplicity we define a function

$$H(x) = \int_0^x h(y)\frac{p(x)}{p(y)}\,dy. \tag{2}$$

This generalizes a similar concept introduced by Espenshade et al. (1982, 128). They allowed immigration only. We show in the Appendix that the same formalism is applicable in the case of net migration also, so $b(t)H(x)$ is the contribution of net-migration to the density of population in age x. In the argument we assume that survival in the population of interest is independent of the propensity to leave. Using (2) we can then write the density of the population in age x at t as $b(t - x)(p(x) + H(x))$.

[2] Mathematically, $n(x,t)$ is the density of a signed measure (e.g., Friedman 1982, 25)

2.3. Population Renewal

From the argument of Section 2.2 it follows that earlier births generate the births at t via the basic renewal equation

$$b(t) = T \int_{\alpha}^{\beta} b(t-x)(p(x) + H(x)) f(x)\,dx. \tag{3}$$

This form of the equation could be applied in the subsequent analyses if information for both in and out-migration were available. However, in this note we will specialize to the case that assumes data on net-migration only. In order to simplify some of the following arguments, *we will assume that* $c_R - c_S = c \neq 0$.

We define first n(x) = h(x)/c, so n(x) integrates to 1. We then define G(x) = H(x)/c. This is the same as a normalized version of (2), when h(y) is replaced by n(y). Thus, both n(x) and G(x) are independent of the level N(t) of net-migration. Only the age-pattern matters. For lack of a better word, we call G(x) a *migration survivor function*. It is the positive or negative "fraction" of the cumulative net-migration surviving to age $0 < x \leq \omega$. Note that now H(x) = cG(x) in (3). Below, the survival probability p(x) and the migration survivor function G(x) are generally fixed so we will write v(x,c) = p(x) + cG(x), for short. Substituting the exponential form of the births into (3) we get the equation

$$1 = T \int_{\alpha}^{\beta} e^{-\rho x} v(x,c) f(x)\,dx. \tag{4}$$

This connects the three parameters ρ, c, and T.

In analogy with the closed population, the stable population experiencing proportional net-migration has an age-distribution proportional to $e^{-\rho x}$ v(x,c).[3]

For intuition, Figure 1 gives the graph of the migration survivor function G(x) for four Nordic countries with good quality migration data, in 2003.[4] We find that there is a somewhat irregular rise reflecting the particular age-pattern of the country's net-migration, and then a more regular decline determined by mortality. The non-monotonicity of G(x) demonstrates the phenomenon that stable populations that are open to migration do not have to have a monotone declining age-distribution. This has been observed earlier (e.g., Wu and Li 2003, p. 26) for the steady immigration populations.

3. Effect of Migration on Growth Rate

In a multi-state system the proportionality factor c would be determined as a part of the stable population calculation. Without a full model this is not available to us, and we will simply replace the theoretical rate by the empirically observed ratio of current net-migration and current births. Column c of Table 1 contains values obtained in this manner from seventeen European countries in 2003. We caution that for many of the countries reliable estimates of net-migration do not exist (e.g., Poulain 1993, Eurostat 2004) so for countries with poor migration data the estimates of c in Table 1 are illustrative only. Column ρ* contains the conventional intrinsic growth rate (or intrinsic rate of natural increase; cf.,

[3] Since $v(x,c) \geq 0$, the constant c must satisfy $p(x) \geq -cG(x)$ for $0 \leq x \leq \omega$.

[4] Our data are estimates of the so-called UPE Project for the year 2003. For some countries (e.g., the Nordic countries) data were available for 2003, but for others the values are one-year-ahead forecasts. In both cases the values were smoothed over age and time, so they represent approximately average net-migration after 2000. The values used are available at www.stat.fi/tup/euupe/.

Keyfitz 1977, 177) that is obtained from (4) by taking $c = 0$, $\alpha = 15$, and $\beta = 50$. The stable growth rate is the solution for $\rho = \rho(c,T)$ obtained using the values c and T from the first two columns.

Column T shows that all countries are far below replacement level.[5] Column c shows that all countries are in-migration countries. Less obviously, we see that countries with lower than average fertility have higher than average net-migration. Notably, according to the available data, in Spain net-migration exceeds births by 37 %. In fact, the correlation between T and c is -0.64 (with an approximate P-value of 0.004 for the hypothesis of zero correlation). Or, low fertility is associated with high net-migration, and vice versa. Note also that the dispersion across countries (as measured by standard deviation) is much larger in terms of migration c than in terms of fertility T.

Intrinsic growth is negative for all countries. With the exception of Ireland and France the values indicate a decline in population size of 0.6 - 1.7% per year. The average rate of decline is 0.93%. However, net-migration alleviates the effect considerably so the average stable rate of decline is 0.25%, with five countries showing positive growth. At the same time net-migration decreases the dispersion in the rate of growth across countries from 0.5% to 0.4%. Although concurrent correlation does not measure causality, the findings are consonant with the hypothesis that countries regulate growth via migration.

4. Trade-Off Between Fertility and Migration

[5] A two-sex total fertility rate of approximately 2.07 is required for the countries listed for renewal. Taking 2.05 as the sex-ratio at birth, we deduce that a gross reproduction rate of $2.07/2.05 \approx 1.01$ is required for stationarity.

For any value of ρ, we can consider (4) as defining a relation between T and c. Since we can solve for T > 0 in terms of c, and for c in terms of T, the relation is one-to-one. Taking $\rho = 0$ we have the special case of a stationary population. Table 1 indicates that the current level of migration already leads to positive growth in some countries, but it is of some interest to investigate the trade-off more closely.

Figure 2 displays pairs (T,c) that would maintain a stationary population in the four Nordic countries. As noted in above, we expect approximate stationarity at T = 1 in the absence of migration, but Figure 2 shows that should fertility decline to T = 0.5, net-migration would have to be approximately at the level it appears to be in Spain now. The current (T,c) values from Table 1 are marked with an asterisk: Denmark (0.84, 0.12), Finland (0.84, 0.10), Norway (0.86, 0.27), and Sweden (0.81, 0.33). In accordance with Table 1, Denmark and Finland are clearly below their respective curves, Sweden is almost on the curve, and Norway is slightly above. This comparison suggests that migration can have a substantial effect on population growth in the long run.

5. Aging via Dependency Ratios

5.1. Dependency Ratios

In a closed stable population, age-distribution is proportional to $e^{-\rho x}p(x)$ for $0 \le x \le \omega$, so a growing population with $\rho > 0$ is older than a declining population with $\rho < 0$. However, if we fix ρ in an open population, there is a trade-off between c and T. Thus, there can potentially be a second aging effect that derives from the relative values of c and T.

Age-dependency ratios are most often motivated by economic considerations. However, as we concentrate on reproduction, we will first define

$$I_1(c,\rho) = \int_0^{\alpha} e^{-\rho x} v(x,c)\,dx, \quad I_2(c,\rho) = \int_{\alpha}^{\beta} e^{-\rho x} v(x,c)\,dx, \quad I_3(c,\rho) = \int_{\beta}^{\omega} e^{-\rho x} v(x,c)\,dx. \qquad (5)$$

Then, we define the lower dependency ratio $L(c,\rho) = I_1(c,\rho)/I_2(c,\rho)$, the upper dependency ratio $U(c,\rho) = I_3(c,\rho)/I_2(c,\rho)$, and the overall dependency ratio $D(c,\rho) = L(c,\rho) + U(c,\rho)$. Using these measures we say that net-migration induces "aging", if $L(c,\rho)$ decreases or $U(c,\rho)$ increases with c.

5.2. Empirical Estimates

To illustrate the possible second aging effect we fix $\rho = 0$. In this case the population is proportional to $v(x,c)$ for $0 \le x \le \omega$. Figure 3 has a plot of L(c,0), U(c,0), and D(c,0) averaged across Denmark, Finland, Norway, and Sweden. From Figure 2 one might expect that an increase in net-migration and the concomitant decrease in fertility would lead to a decrease in L(c,0). From Figure 3 we see that this is, indeed, the case. Numerically, the decrease is from L(c,0) = 0.43 at c = 0 to L(c,0) = 0.32 at c = 0.75. Similarly, U(c,0) increases from 0.92 to 1.06. The net effect is that D(c,0) increases thus from 1.35 to 1.38. While the overall increase is small, the analysis suggests that in these European countries increased net-migration is positively associated with aging, when one controls for the rate of growth.

To see what the net effect on aging might be, let us consider Denmark and Finland that have equal total fertility rates, and negative stable growth with rates -0.0034 and -0.0045, respectively. Assuming these rates of growth and the level of migration of Table 1, we have for Denmark L(0.12, -0.0034) = 0.29, H(0.12, -0.0034) = 1.12, D(0.12, -0.0034) = 1.40, and for Finland L(0.10, -0.0045) = 0.30, H(0.10, -0.0045) = 1.27, D(0.10, -0.0045) = 1.57. Suppose then that these countries maintain their current level of fertility of 0.84 but boost their net-migration to the level leading to stationarity. From the data underlying Figure 2 we find that in this case we would have to have c = 0.29 for Denmark and c = 0.38 for Finland. Or, net-migration would have to be in Denmark 0.29/0.12 = 2.4 times, and in Finland 0.38/0.10 = 3.8 times as high as in 2003. For Denmark we get then L(0.29, 0) = 0.38, U(0.29, 0) = 0.93, D(0.29, 0) = 1.31, and for Finland L(0.38, 0) = 0.38, U(0.38, 0) = 1.04, D(0.38, 0) = 1.41. We see that for both countries increasing net-migration to reach stationarity would slow down aging and alleviate the burden or working/reproductive ages.

5.3. Practical Magnitude of the Effect

One can contrast the effects noted above to the effects that derive from increases in longevity. Finland has for several years had the lowest level of net-migration in the countries considered.[6] Using Finnish life tables compiled by Kannisto and Nieminen (1996) for five year periods, we analyze the situation as follows. During 1950-1990 female life expectancy at birth increased from 69.9 to 78.8. Using the corresponding stationary populations we find an increase of U(0,0) from 0.69 to 0.86. The data we use give the value 0.96 for Finland at 2003, when life expectancy at birth was 81.8. Thus, the increase in longevity by 81.8 - 69.9 = 11.9 years in 50 years' time, is accompanied by an increase in the upper dependency ratio by 0.96 - 0.69 = 0.27 units. Under the stable growth rate of $\rho = -0.0045$, another 1.27 - 0.96 = 0.31 units would be added, but by increasing net-migration to reach stationarity the added amount could be reduced to 1.04 - 0.96 = 0.08 units. Thus, Finland could reduce aging by almost the same

[6] In Table 1 the Netherlands has even a lower value, but this reflected the conditions of 2003 and earlier net-migration was higher.

amount that mortality in the past half a century has added to it, by boosting net-migration to the level of zero growth.

6. An Extension to Assessing the Proportion Foreign-Born

A frequent concern is that increasing net-migration will lead to an increase in the size of the population that is foreign-born. In order to identify the separate population stocks we need data by country of birth (e.g., Rogers 1995). This point of view is also helpful in providing more detail on the nature of the densities of in- and out-migration.

We decompose the measures of Section 2 by region of birth. First, we write $R(x,t) = R_I(x,t) + R_O(x,t)$, where $R_I(x,t)$ is the cumulative in-migration to the population of interest *by those who were born into it*, but migrated out, and subsequently returned. We call this population stock as the *in-born*. Correspondingly, $R_O(x,t)$ is the cumulative in-migration by the *out-born*, i.e., by those born outside the population of interest. Similarly, we will decompose the out-migration $S(x,t) = S_I(x,t) + S_O(x,t)$. Under the proportionality assumption we write their densities as $r(x,t) = c_{RI}r_I(x)b(t) + c_{RO}r_O(x)b(t)$, and $s(x,t) = c_{SI}s_I(x)b(t) + c_{SO}s_O(x)b(t)$. We will then assume that $c_{RI} - c_{SI} = c_{NI} \neq 0$, and $c_{RO} - c_{SO} = c_{NO} \neq 0$. Note that while above we had no reason to assume anything about the sign of c, now we must have $c_{NI} < 0$, because it is a net measure of out-migration of the in-born that acknowledges their possible returns. Similarly, $c_{NO} > 0$ as it measures the net number of out-born entering.

Define $h_I(x) = c_{RI}r_I(x) - c_{SI}s_I(x)$ and $h_O(x) = c_{RO}r_O(x) - c_{SO}s_O(x)$ as the densities of net-migration by the in-born and the out-born. Having these available, we can use (2) to define $H_I(x)$ and $H_O(x)$ for the in-born and out-born, respectively. Defining further $G_I(x) = H_I(x)/c_{NI}$ and $G_O(x) = H_O(x)/c_O$, we get the decomposition $v(x,c) = p(x) + c_{NI}G_I(x) + c_{NO}G_O(x)$. Since we now must have $cG(x) = c_{NI}G_I(x) + c_{NO}G_O(x)$, we have a decomposition of the effect of net-migration on age-distribution in terms of the net out-migration of the in-born and the net in-migration of the out-born.

The proportion out-born in the stable population is

$$\int_0^\omega e^{-\rho x} c_{NO} G_O(x)\,dx \Big/ \int_0^\omega e^{-\rho x} (p(x) \;+\; cG(x))\,dx. \tag{6}$$

This is the proportion "foreign-born" one would be interested in. To estimate this, information about the age-pattern of net-migration is required for both the in-born and out-born. However, in the special case in which the age-pattern of the net out-migration of the in-born is the same as the net age-pattern of the in-migration of the out-born, or $G_I(x) = G_O(x) = G(x)$, we have that $c = c_{NI} + c_{NO}$. Thus, the level of net-migration is the sum of the (positive) net-migration of out-born and the (negative) net-migration of the in-born.

The practical estimation of the proportion (6) depends on the availability of credible migration data. There are two aspects to this. First, many countries cannot provide acceptable estimates of c, let alone its decomposition into c_{NI} and c_{NO}. Second, even if data of acceptable quality are available, the observed pattern of migration may be too far from that expected under stationarity, to provide reasonable parameter values as input for a stationary population calculation. In the case of Finland in 2003 neither aspect is a problem. Thus, we present the following estimates.[7] For Finnish-born we have $c_{RI} = 0.1355$ and $c_{SI} = 0.1652$, so $c_{NI} = -0.0297$. For foreign-born we have $c_{RO} = 0.1888$ and $c_{SO} = 0.0545$, so $c_{NO} = 0.1343$. These yield the estimate $c = -0.0297 + 0.1343 = 0.1046$, which rounds to the value 0.10 of Table 1. Substituting c_{NO} and c into (6), together with the estimated p- and G-functions, and the growth

[7]The author would like to thank Timo Nikander of Statistics Finland for providing the migration data.

rate ρ of Table 1, yields the estimate 0.086 as the estimate of foreign-born that would obtain in a stationary population, under the current composition of net-migration. Comparing this to the current value of about 0.013 we see that the proportion foreign born would grow to about seven-fold.

7. Concluding Remarks

We assessed the migration-fertility trade-off using a simple stable population model. This is motivated by the fact that calculations under stability address directly the question of sustainability, or what combinations of fertility and net-migration could co-exist indefinitely. We have assumed an open population in which net-migration is proportional to births. This provides an alternative to earlier analyses that assumed a steady inflow.

Both the level of net-migration and the migration survivor function have identifiable effects on the age-distribution. Our findings partially support the earlier studies that have been skeptical about the possibility to combat aging via increased net-migration. However, the effects are subtle. Migration can boost growth rate substantially. This tends to make the age-distribution younger. However, if one controls for the growth rate, then increasing migration at the expense of fertility tends to make the age-distribution older.

Aging is, presumably, a desirable goal if it occurs via increases in longevity. However, it is less clear that it is desirable if it occurs via negative population growth. While one can argue that from a broad ecological point of view positive population growth can be non-sustainable, it is still possible that combating drastic population decline increases welfare by easing the burden of the working age population.

If there is room for growth, then increasing net-migration can markedly slow down aging. For a country with a low level of net-migration the effect can correspond in magnitude to aging caused by an increase of a decade or more in life expectancy. Note, however, that for most countries in our European data set the effect would be less, and for five of them have already "used up" the effect of growth rate as their stable growth is positive. Should they wish to reduce net-migration without accelerating aging, an increase in fertility is the only option.

We have assumed that immigrants have the same vital rates as the native born population. This seems reasonable since the heterogeneity induced by past migration is reflected in the estimated vital rates that form the basis of the stable population analysis. Yet, if the possibly higher fertility of immigrants persists and their share increases, then the aging effect of net-migration would be smaller than estimated here. A more refined analysis is possible in a multi-state context if vital rates are available by country of birth, for example.

References

Bacaër N. (2003) The asymptotic behavior of the McKendrick equation with immigration. *Mathematical Population Studies* 10, 1-20.

Çinlar E. (1975) *Introduction to stochastic processes.* Englewood Cliffs: Prentice Hall.

Espenshade T.J., Bouvier L.F., and Arthur B.W. (1982) Immigration and the stable population theory. *Demography* 19, 125-133.

Eurostat (2004) *Population statistics. Theme 3: Population and social conditions*. Luxembourg: European Commission.

Friedman A. (1982) *Foundations of modern analysis*. New York: Dover.

Kannisto V. and Nieminen M. (1996) *Revised life tables for Finland 1881-1990*. Population 1996:2. Statistics Finland.

Keyfitz N. (1977) *Introduction to the mathematics of population, with revisions*. Reading: Addison-Wesley.

Liao T. F. (2001) How responsive is U.S. population growth to immigration? A situational sensitivity analysis. *Mathematical Population Studies* 9, 217-229.

Mitra S. (1983) Generalization of the immigration and stable population model. *Demography* 20, 111-115.

Pollard J.H. (1966) On the use of the direct matrix product in certain population mathematics. *Biometrika* 53, 397-415.

Poulain M. (1993) Confrontation des statistiques de migrations intra-Européennes: vers plus d'harmonisation? *European Journal of Population* 9, 353-381.

Population Division (2000) *Replacement migration*. New York: United Nations.

Rogers A. (1995) *Multiregional demography. Principles, methods and extensions*. New York: Wiley.

Schmertmann C. (1992) Immigrants' ages and the structure of stationary populations with below-replacement fertility. *Demography* 29, 595-612.

Shryock H.S., Siegel J.S. and Associates (1976) *Methods and materials of demography*, Condensed Edition by Stockwell E.G. New York: Academic Press.

Schmidbauer H. and Rösch A. (1995) Population with constant immigration. *Mathematical Population Studies* 5, 341-358.

Wu Z. And Li N. (2003) Immigration and the dependency ratio of a host population. *Mathematical Population Studies* 10, 21-39.

Appendix. The Contribution of Net-Migration to Those in Age x.

Restrict attention to a fixed cohort, say those born at time t. We develop an expression for the number of those who are present in the population of interest in a fixed age $0 < x < \omega$. They have entered the population either via birth in age $y = 0$, or later as in-migrants in age $0 < y < x$. Suppose that in age $0 \le u < \omega$ the force of mortality is $\lambda(u)$ and the intensity of out-migration is $\gamma(u)$. To simplify the notation, define the corresponding cumulative hazards as

$$\Lambda(x) = \int_0^x \lambda(u)\,du, \quad \Gamma(x) = \int_0^x \gamma(u)\,du. \tag{A1}$$

Consider now an individual who enters in age $0 \le y < x$. The probability that the entrant stays in the population and survives to age x is $\alpha(x,y) = \exp(-\Lambda(x) + \Lambda(y) - \Gamma(x) + \Gamma(y))$. By a direct calculation one can show that this can also be expressed as the difference between $p(x)/p(y) = \exp(-\Lambda(x) + \Lambda(y))$ and the probability of leaving by those who would survive if they stayed, or

$$\beta(x,y) = \int_0^x \exp(-\Lambda(z) + \Lambda(y) - \Gamma(z) + \Gamma(y))\,\gamma(z)\,p(x)/p(z)\,dz. \tag{A2}$$

In other words, $\alpha(x,y) = p(x)/p(y) - \beta(x,y)$. In (A2) the first term of the integrand is the probability of surviving from y to z, $\gamma(z)dz$ is the probability of leaving during $[z, z + dz)$, and the last term is the fraction of those leaving who would survive further to x. Thus, the fraction $\exp(-\Lambda(x) + \Lambda(y)) - \Gamma(x) + \Gamma(y))\gamma(z)dt$ contributes to the density of exits at z. In this manner we can account for all times of entry and subsequent exits.

Entries come from the outside, and their density is $c_R r(y)b(t)$ in age $0 \le y < x$. Exits $c_S s(y)b(t)$ in age y satisfy, in terms of the previous entries, the relationship

$$c_S s(y) = \gamma(y)\{\exp(-\Lambda(y) - \Gamma(y)) + c_S \int_0^y r(z)\exp(-\Lambda(y) + \Lambda(y) - \Gamma(z) + \Gamma(z))\,dz\}, \quad (A3)$$

where the first exponential term on the right hand side comes from the births and the integral comes from later entries. Using the difference representation, both the density of entries and exits must be multiplied by the survival probability p(x)/p(y) to come up with the population in age x. Thus, the same holds for their difference, the density of net migration. The argument assumes that survival in the population of interest is independent of the propensity to leave. We assume that it is. Thus, b(t)H(x), where H(x) is given by (2), gives the contribution of net-migration to the number of those in age x.

Table 1. Stable Population Parameters in Selected European Countries: Female Total Fertility Rate (T), Ratio of Net-Migration to Births (c), Intrinsic Growth Rate (ρ*), and Stable Growth Rate (ρ).

Country	T	c	ρ*	ρ
Austria	0.68	0.45	-0.0136	-0.0035
Belgium	0.79	0.37	-0.0084	0.0000
Denmark	0.84	0.12	-0.0062	-0.0034
Finland	0.84	0.10	-0.0063	-0.0045
France	0.92	0.13	-0.0034	-0.0010
Germany	0.64	0.21	-0.0158	-0.0115
Greece	0.61	0.38	-0.0166	-0.0082
Ireland	0.95	0.48	-0.0020	0.0025
Italy	0.62	0.96	-0.0159	-0.0038
Luxembourg	0.80	0.47	-0.0081	0.0021
Netherlands	0.84	0.02	-0.0060	-0.0056
Norway	0.86	0.27	-0.0056	0.0003
Portugal	0.72	0.54	-0.0119	-0.0038
Spain	0.61	1.37	-0.0162	0.0016
Sweden	0.81	0.33	-0.0075	-0.0009
Switzerland	0.68	0.63	-0.0132	0.0011
United Kingdom	0.80	0.18	-0.0081	-0.0039
Average	0.78	0.39	-0.0093	-0.0025
Standard Deviation	0.11	0.34	0.0049	0.0037

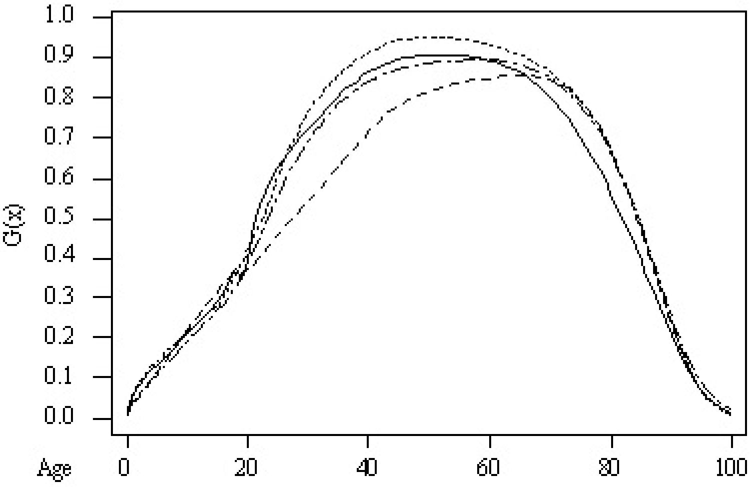

Figure 1. Migration Survivor Function G(x) for Denmark (Solid), Finland (Dashed), Norway (Dotted), and Sweden (Dash-Dotted).

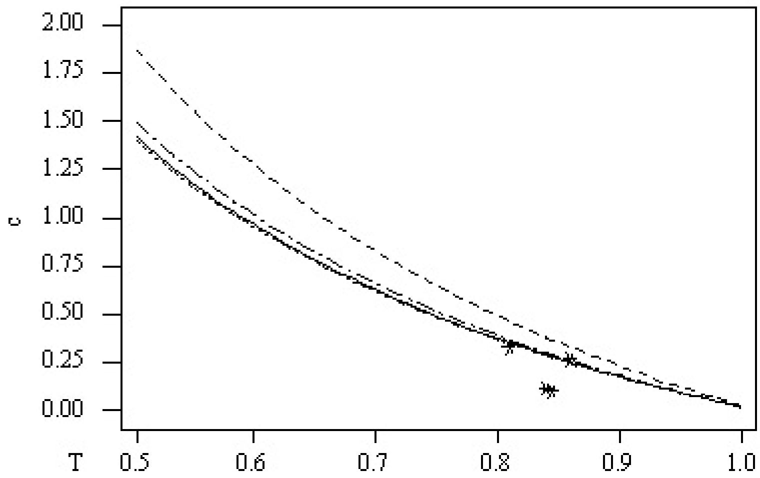

Figure 2. The Trade-Off Between Fertility and Net-Migration: Pairs of values (T, c) that Maintain a Stationary Population for Denmark (Solid), Finland (Dashed), Norway (Dotted), and Sweden (Dash-Dotted). Actual Values Marked with '*'.

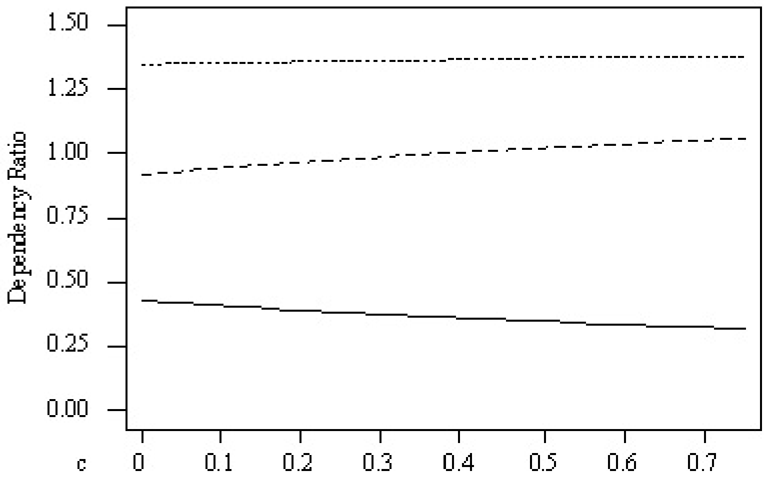

Figure 3. Average of Lower Dependency Ratio (Solid), Upper Dependency Ratio (Dashed), and Overall Dependency Ratio (Dotted) in Denmark, Finland, Norway, and Sweden, as a Function of c.