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Transfers, Capital, and Consumption over the Demographic Transition: An International Comparison

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Introduction

Nearly all countries in the world are at some stage of the demographic transition from initially high fertility and mortality to ultimately low, with the corresponding changes in population age distribution. Because the initial framing of the concept of demographic transition was guided by concern for the rapid population growth rates that occurred following mortality declines, attention was focused on the timing of the subsequent fertility decline and the longer term age distribution consequences were downplayed or ignored. Yet from the point of view of economic development and growth, the age distribution changes are fundamental and easily misunderstood.

A classic demographic transition begins with mortality decline while fertility remains high for a number of decades. Then fertility begins to decline towards replacement or below. In the end, fertility stabilizes at a low level while mortality may continue to decline. There are large changes in the rate of growth and size of the population. Increases in size raise concerns about pressures on natural resources, while rapid growth raises concerns about capital dilution. But our interest here is in the age distributional changes that accompany the demographic transition (Lee, 2003,181). At first the age distribution becomes younger (the median age falls and the youth dependency ratio rises), reflecting increasing survival of children as mortality falls, but once fertility begins to fall then youth dependency declines as well. During this phase of the transition, which may last about sixty years, the population of working ages (defined with fixed age-boundaries) grows as a fraction of the population and the total dependency ratio declines steadily. Other things equal, per capita income growth gets a boost of up to .6% per year, and amounts to an age standardized increase in consumption of 25% to 40% (Mason, 2005a, 2005b). This boost in per capita income growth due to a more favorable support ratio is known as the “demographic dividend”. We will call this the “first demographic dividend” to distinguish it from the age distribution effects that are the focus of this paper, here referred to as the “second demographic dividend”. Eventually, however, the fertility decline comes to an end and the effects of low fertility and longer life on the share of the elderly population come to dominate. At this point, the total dependency ratio rises and we enter the phase of population aging. Ultimately the total dependency ratio ends up close to where it began around 1900. The economically favorable changes of the dividend phase are offset by the unfavorable changes before and after. This sequence focuses attention on the possibility of taking advantage of the window of opportunity offered by the dividend phase.

While a number of cross-national studies have found that the population age distribution is associated with rates of income growth in a manner consistent with the dividend story, we must keep in mind that the pure support ratio effect is both relatively small (roughly a 35% cumulative boost in per capita income at its peak) and transitory (Bloom and Canning, 2001; Bloom *et al.*, 2002; Bloom and Williamson, 1998). We argue here that there are other consequences of these demographic changes over the transition that are far larger than the support ratio effect, and are permanent. While the total dependency ratio does return approximately to its original level, it initially reflects a high proportion of children and very few elderly, but after the transition reflects very few children and a high proportion of elderly. These changes in the cross-sectional distribution of the population correspond to fundamental changes in the economic and demographic life cycles of individuals, in addition to age compositional changes. It is from these permanent changes that we find lasting effects.

One centrally important aspect of these changes is that with low fertility, high child survival and long life, investment in the human capital of each child rises dramatically, in part as cause and in part as consequence. Here, however, we focus on another important aspect: as population ages, there is a large increase in the demand for wealth per capita relative to income or labor, and the capital-labor ratio rises.

In other work, we have shown the processes at work by simulating a model of life cycle savings applied to Taiwan and the US over the course of their demographic transitions. Many questions have been raised about the life cycle savings hypothesis, so here we take a different route. The life cycle savings hypothesis tells us that consumption and saving at each age are governed by the wish to smooth consumption over the life cycle including the period of old age. In a hypothetical society without inter-age transfers to the elderly, this implies that workers would save throughout their lives and dissave when elderly and no longer working. More realistically, the elderly in most societies receive transfers of income (or in-kind consumption goods) either from their families, with whom they typically live in Third World countries, or from the public sector through Pay As You Go (PAYGO) pensions and health programs. Also, they may wish to hold on to assets to pass on to their heirs or as insurance against living longer than expected.

For two countries, Taiwan in 1998 and the US in 2000, we have been able to measure the various components of support for old age consumption, for which the distribution is shown in Figure 1. In the US, income from ownership of assets makes up about 60% of the total, and public transfers are also very important, while familial (inter vivos) transfers play only a small role. In Taiwan, assets fund about 40% of old age consumption, while familial transfers are also very important at about 33%. In both countries, public and familial transfers combined provide finance between 40 and 50% of old age consumption, rivaling or exceeding the importance of assets (Mason, Lee, et al. forthcoming). Clearly, a focus on either life cycle saving and asset accumulation alone, or on transfers alone, would miss much of the story.

Elsewhere, we have constructed estimates of life cycle earnings and consumption, about which more will be said below. Our strategy in this paper is to assume that the cross-sectional age profiles of consumption and earnings estimated for a particular country in a particular year retain their shapes in the future, while their levels shift upwards over time. The labor earnings profile is assumed to shift at some exogenously given rate. Variations in the relative levels of the consumption and earnings profiles, as well as variations in the population age distribution, lead to different aggregate savings rates and therefore determine the trajectory of asset accumulation. The age profile of earnings shifts up rapidly over time when growth in labor productivity is rapid as it has been in Taiwan in recent decades (around 6% per year, in real terms), or more slowly when productivity growth is slower, as in the US in recent decades (1 or 2% per year).

Wealth is defined broadly as a net claim on future income, and it can take the form either of transfer wealth (PAYGO pensions, familial support, publicly provided health care) or capital (private savings, funded pensions, or a home). If these cross-sectional age profiles of labor earnings and consumption have unchanging shapes, then we can calculate the increment to life cycle wealth each period that is necessary to sustain them in the future. The calculation is not simple, and we will describe our estimation strategy later in the paper.

Our analysis relies on the assumption that the cross-sectional shape of the consumption age profile is fundamental and unchanging, and this requires some interpretation. In a strict life cycle savings model, the age profile of consumption would not be constant. Instead it would depend on the relative economic fortunes of each generation. For example, the young in Taiwan who may earn six times as much as their parents did at a comparable age (6 = exp(30*.06)) would consume correspondingly more at each age over their life cycle. But this is not what we see, and not what would emerge under a system of familial co-residence and income sharing. In fact, the cross sectional age profiles of consumption in the US and Taiwan have been fairly stable in shape over the period from 1980 to 2000 for which we have calculated them. This is what we would expect if individuals in families are altruistically linked. Differences emerging from different earnings histories would be offset by both familial and public sector transfers. This is our working hypothesis for the calculations reported later in this paper.

The key idea is that variation in consumption across generations at any point in time is a product of preferences or altruism that expresses itself through the host of transfer programs – both public and private – that permeate all modern societies. In the lifecycle model, the consumption of the elderly depends on their tastes and their lifetime earnings. In this model, the consumption of the elderly depends on general standards of living, the needs of the elderly, and social and familial preferences about the consumption of the elderly as compared with that of prime-age adults and children.

Here is a sketch of how the trajectories of consumption and assets are calculated; a more detailed explanation will be given later. Life cycle wealth can be decomposed into a portion that funds consumption in retirement and another portion that funds consumption by children. We will call the first "pension wealth" and the second "child wealth". Likewise, we will distinguish pension transfer wealth as the component of general transfer wealth that is used to fund retirement. We will project the effect of future demographic change on the future demand for pension wealth, which we expect to increase strongly as populations age. But how should this be translated into an increase in the demand for capital? We don't know to what extent the future consumption needs of an aging population will be met by unfunded transfer systems versus funded systems or private saving. This will depend on how policies and institutions develop over coming decades.

We will call $\tau(t)$ the share of total pension wealth that is held in the form of pension transfer wealth in some future year t. For our baseline simulations, we will assume that the value of τ observed for the most recent year of data remains constant throughout the simulation (for the US in 2000, tau was about .35). We will then calculate the growth in the demand for wealth in the society, and assume that this leads to a corresponding growth in the size of the capital stock. Evidently, faster growth in the capital stock will require lower consumption, other things equal.

Before turning more formally to the methods we use, a few comments are in order. First, the age boundaries assumed for dependency ratios, such as 20 and 65, are obviously arbitrary. Our calculations are based on the actual age profiles of labor earnings by age. Second, in any society, it is the elderly who have the highest ownership of assets, following a life time of accumulation. Holding the age profile of capital, or equivalently saving rates, constant, and multiplying it times the changes in age distribution over the demographic transition, would clearly imply rising capital to income ratios. We might call this a pure compositional effect. In our analysis, however, we do not hold the age profile of wealth constant. Rather, the demand for wealth by age will depend on fertility and mortality. Couples with fewer children assign a greater share of their life cycle earnings to their own consumption, and therefore have a greater demand for wealth to provide for higher consumption in retirement. People who expect to live longer have a greater demand for wealth to finance their longer period of post-work consumption. These changes associated with the demographic transition and changes in age structure are also reflected in our analysis.

The structure of our paper is as follows. First, we describe the data on which our calculations are based, including the estimated age profiles. Then we present a highly simplified analysis, ignoring children and population age structure, that asks how much wealth an individual would have to acquire over the life cycle to be able to fund the cross sectional consumption pattern for a given level of mortality. After this, we present the theoretical model and develop steady state results for the full-blown model, which enables us to carry out a simple comparative static analysis. Following that, we present dynamic results for consumption over time, subject to the constraint that τ, pension transfer wealth as a share of total pension wealth, be constant. We then summarize and conclude.

Data

Estimates of the economic lifecycle, consisting of age profiles of consumption and labor, are an essential feature of this analysis. The profiles used here have been constructed using a common methodology that is described in Lee et al. (2005) and in more detail in www.ntaccounts.org. The consumption profile consists of both public and private consumption. Public consumption has been allocated to age groups using administrative records and survey data. Private consumption has been allocated using household expenditure surveys. Consumption of public and private health, public and private education, and private housing services have been estimated separately. Labor income includes earnings of employees and self-employment labor income. All values have been adjusted to match National Income and Product Account estimates.

Estimates for Japan (Ogawa and Matsukura, 2005), Korea (An and Gim, 2006), and Thailand (Chawla, 2006) are compared to estimates for Taiwan and the United States presented in Lee et al (2005). The most extensive analysis emphasizes estimates for the United States for 2000 and Taiwan for 1977. These two sets of estimates are selected because they represent very different institutional settings. Like many other Western countries, the US relies on public transfer programs, such as Social Security, Medicare, and Medicaid, to provide support to the elderly. The family is relatively unimportant as a source of financial support for the elderly. In Taiwan, public programs are becoming more important, but in 1977 there were no public pension or health care programs of note, and familial transfers were and continue to be important.

Labor income is used in two ways: to construct an historical index of labor productivity to be used in simulations and to measure the relative size of economies necessary for constructing regional and global aggregates from national simulations. Labor income is not readily available in National Income and Product Accounts and other aggregate statistical sources, because the operating surplus from individual proprietorships, partnerships, and other unincorporated enterprise does not distinguish returns to capital from returns to labor. Hence, we use two-thirds of GDP measured in purchasing power parity adjusted US dollars as reported in the Penn World Tables.

The simulations make extensive use of demographic estimates and projections. The source of all demographic data used in this paper, except as noted, is the United Nations Population Division. Population by age for 1950-2050 and age-specific fertility rates for 1995-2050 are from the most recent edition of *World Population Prospects* (United Nations Population Division, 2005) Population by age for 2055-2300 are from *World Population to 2300* (United Nations Population Division, 2004). The UN Population Division provided detailed country information not available in the published version of the long range projections and unpublished historical estimates of age-specific fertility rates from 1950-1995.

Economic Lifecycles: A Comparative Perspective

A common feature of all human populations is an extended period of childhood dependency during which children produce much less than they consume. In some traditional economies in the past, adults produced as much as they consumed until their death (Lee, 2000). But as far as we know at this point, all present-day human populations also experience a period of dependency at older ages. Within these very broad parameters, however, there is considerable variation in the timing and intensity of economic dependency.

Two estimates of the economic lifecycle are presented in Figure 2A and 2B – the United States in 2000 and Taiwan in 1977. The labor income profiles incorporate and summarize, for men and women combined, labor force participation, hours worked, wages, and all of the factors that influence these variables. They are cross-sectional profiles and, hence, reflect the varied experiences of each of the age groups represented in the respective profiles. Despite the many ways in which Taiwan in 1977 differed from the United States in 2000, the labor income profiles are strikingly similar. There are some discernible differences, however. The US labor income profile rises somewhat more slowly with age and begins to decline at a somewhat later age than in Taiwan.

The consumption profiles shown in Figure 2 consist of both public and private consumption. Public consumption in the United States favors children, via spending on education, and the elderly, via spending on health care. Private consumption in the US rises steadily with age until around age 60 and then declines. Public and private consumption combined are highly favorable to the elderly. We estimate that average consumption by a 90-year-old was over \$40,000 in 2000 as compared with only \$25,000 by a young adult. The difference between them is essentially a consequence of health care spending.

The situation in Taiwan 1977 was very different. Consumption clearly favored young adults with total consumption declining from about NT\$40,000 per year for young adults to around NT\$30,000 per year for those who were 90 (or older). Public education programs were important in Taiwan in 1977, but public spending on health care was unimportant.

The key difference between the two cases, then, occurs at the older ages. In the US, per capita consumption of those 58 and older exceeded per capita labor income. In

Taiwan the cross-over age was 62. In relative terms the gap between consumption and production at older ages is much larger in the US than in Taiwan. In contrast, the dependency profiles at young ages appear to be quite similar in the two countries. The cross-over ages are the same – 26 years of age in both countries – and the magnitudes of the gap between consumption and production relative to labor income appear to be similar.

Simplified Calculations for Life Cycle Planning and the Demand for Wealth

Our full-blown calculation of consumption and asset trajectories is complicated, and to build insight and intuition we will begin with some simpler calculations based on the cross-sectional profiles. Here we calculate the demand for wealth by an individual over the life course, assuming that the only concern is one's own adult consumption, and that adults neither give nor receive any transfers. Support for children does not enter the picture.

Call the age at which earnings first drop below the level of consumption a*. Consider a situation in which both productivity and consumption age profiles shift upwards at rate g (that is, both are multiplied by $(1+g)^t$), leaving a* unchanged. Assume there are no transfers, and ignore the public and familial support costs of children. For a given level of mortality, and for an interest rate r, we can calculate how much wealth W an individual would have to accumulate by age a* to pay for the implied consumption after this age, net of any earnings after a*. We can also calculate the ratio of W to the average level of earnings in the five years preceding a*. Calculations are made on the assumption that there is perfect risk sharing for uncertain age at death. The resulting ratios of wealth to earnings will depend on the age profiles we start with, on the difference between r and g, and on mortality. With the same setup, we can also ask what constant level of saving would be required over the life cycle to accumulate the wealth necessary to fund consumption after a*.

For both cases, note that the consumption age profile includes all government spending, so that for the US it would include Medicare, nursing home care paid by Medicaid, food stamps, a pro-rated share of defense expenditures, and so on. Thus our calculation tells us what assets it would take to fund all this publicly and privately provided consumption without any taxes, reflecting a hyper privatization.

Results are shown in Table 1 for the age profiles and mortality of Japan, S. Korea, Taiwan (in 1977 and again for 2003), Thailand and the United States (in 1980 and again in 2000). The crossover age a* varies from 55 to 61, and given the mortality differences, the expected duration of old age dependency varies from 16 to 24 years. The required ratio of wealth to pre-crossover earnings varies from 6 in Japan, which is surprisingly the lowest, to 13.3 in the US. The required saving rate out of labor income over the life cycle varies from a low of 8% for Taiwan in 1977 to a high of 28% for the US in 2000.

We performed various experimental calculations to see the effects of altering the assumptions. To see the effects of mortality on these results, we did the calculations once with the life table of Taiwan in 1977 and once with that of Japan in 2003. In all cases, the expected duration of dependency increased by about 8 years when this was done. Depending on the age profiles, the required wealth to earnings ratio rose by 50% to 90%, and the required saving rate rose from 60% to 110%. Mortality clearly is important.

Table 1. Economic Lifecycles and their Implication for Wealth and Saving.

	Year	Old-age crossover a*	Expected age at death conditional on surviving to a*	Expected duration of old-age dependency	Net wealth / final labor income	Required saving rate out of labor income
Japan	1994	61	83.3	22.3	5.8	12.1
South Korea	2000	55	78.6	23.6	9.8	20.8
Taiwan	1977	62	77.8	15.8	6.4	7.9
Taiwan	2003	55	79.7	24.7	11.3	23.4
Thailand	1998	60	78.9	18.9	9.4	15.3
United States	1980	60	80.2	20.2	10.9	18.6
United States	2000	59	81.7	22.7	13.3	28

Source: For profiles of consumption and earning, see Ogawa and Matsukura (2005), Lee et al (2005), Chawla (2006), and An and Gim (2006).

In another experiment, we varied the difference between the rate of return *r* (discount rate) and the rate of productivity growth, g. Holding constant the rate of growth of wages and consumption, higher rates of return greatly reduce the needed amount of wealth and the needed savings rate. Put the other way round, a reduction in the rate of return on capital resulting from increasing capital labor ratios would have an important effect on the required rates of life cycle savings.

One leading option for policy and individual choice in the face of longer life and population aging is to extend the working years. We examined the effect of postponing retirement by inserting a flat five year segment in the age-earnings profile immediately following its peak and moving all subsequent point five years to the right. The necessary asset accumulation relative to earnings is reduced by 20% to 35% depending on the age profiles, and the required savings rate out of labor income is reduced by a third to a half.

These calculations reveal major differences in the need for saving and asset accumulation in the hypothetical case of full individual responsibility with no transfers, public or private. The profiles of different countries in different periods imply very different requirements, as do differences in longevity, discount rates, productivity growth rates, and age at retirement. Yet these calculations are seriously incomplete, for three reasons. First, they ignore transfers, which in all countries play a prominent role in supporting old age consumption. Second, they ignore the population age distribution, which assigns different weights in the aggregate economy to the behavior of individuals at different ages. We therefore miss much of the demographic change associated with the demographic transition, particularly changes in fertility. And third, we have ignored the financial costs of children, whose consumption is financed in part directly by familial transfers and in part through public sector transfers. We will now turn to calculations that fully reflect these additional influences.

Methods

The focus of analysis is the cohort of all adults, those who are a_0 years of age or older, in year t. The reason we focus on this group is a simple one – adults hold all assets. Children are important only indirectly as they influence the holdings of assets by adults. We will often refer to this cohort as "year t adults" and we follow it as it ages. In year t+1 all of its members are age $a_0 + 1$ or older, in year t+x its members are age $a_0 + x$ or older, and by the end of year $t + w - a_0$ all of its members will have died.[1]

Over the remainder of their collective existence, year t adults will consume and earn labor income in each year of the future. We assume that there are no bequests.[2] Thus, the lifetime budget constraint implies that the cohort's current lifecycle wealth must equal the present value of its consumption less the present value of its labor income. Note that elsewhere (Lee 1994a and b), lifecycle wealth includes the wealth of adults and children, but here it includes only the wealth of adults. Lifecycle wealth consists of assets – housing, land, fixed capital, credit extended to foreign nationals and governments, etc. – and transfer wealth. Transfer wealth is the present value of expected net transfers in the current and future years. This includes public transfers, such as, transfers received from public pension programs less taxes paid to finance those programs. But public transfers are not limited to pension programs. All public programs that shift resources from one age group to another are included. Transfer wealth also includes familial and other private transfers, including all spending by parents on their children and all support that adult children provide to their parents.

It is particularly important in this analysis to distinguish two exhaustive, mutually exclusive components of transfer wealth: child transfer wealth and pension transfer wealth. Consumption by children is financed entirely out of their modest labor income and out of public and private transfers. For year t adults, the present value of lifetime net transfers to children is equal to child transfer wealth. Note that this value will be negative.

Consumption by the elderly is financed from three sources: labor income, assets, and pension transfer wealth. Pension transfer wealth is equal to the present value of all net upward transfers, i.e., transfers from younger age groups to older age groups, for year t adults. Thus, it includes all transfers not just pensions. Pension transfer wealth may be positive or negative depending on the age of the cohort of adults. Young adults pay taxes or make familial transfers to support the old and receive transfers when they are old. Under steady-state conditions, the rate of return to pension transfer programs is the rate of growth of total income. If the return to capital exceeds the rate of economic growth, the typical situation, pension transfer wealth is negative for young adults. For older adults, who are the beneficiaries of transfers to older ages, pension transfer wealth is positive. In general, the combined pension wealth of all adults is positive.

These concepts are readily formalized. *W(a,t)* is the combined lifecycle wealth of all adults of age *a* in year *t*. It is equal to the present value of the consumption less the present value of the labor income of those adults over the remainder of their lives. Let PV[] be the present value operator. Then,

$$W(a,t) = PV[C(a,t)] - PV[Y(a,t)] \qquad (1)$$

where *C(a,t)* and *Y(a,t)* are vectors of current and future consumption and current and future labor income, respectively, for the cohort of age *a* in year *t*. Lifecycle wealth in

[1] Although we do not explicitly address migration in this paper, we include in our definition of year t adults any immigrants who were adults in year t irrespective of their country of residence at that time.
[2] Adults pool their assets to protect against uncertainty about mortality by participating in costless annuities.

year t for the cohort comes in three forms: assets (A), transfer wealth associated with childrearing (T_K) and pension transfer wealth (T_P), i.e.,

$$W(a,t) = A(a,t) + T_k(a,t) + T_P(a,t). \tag{2}$$

Pension wealth is defined as $W_P(a,t) = A(a,t) + T_P(a,t)$, i.e., assets plus pension transfer wealth.

Assets can be negative, but by assumption they can only be held by adults. Aggregate assets in year t is calculated by summing over all adult cohorts:

$$A(t) = \sum_{a=a_0}^{w} A(a,t) \tag{3}$$

where a_0 is the age of adulthood and w is the oldest age achieved. Summing transfer wealth variables over all adult ages:

$$A(t) + T_P(t) = W_p(t) = W(t) - T_k(t). \tag{4}$$

where $T_P(t)$ is pension transfer wealth, $T_k(t)$ is child transfer wealth, and $W_p(t)$ is pension lifecycle wealth equal to the sum of assets and pension transfer wealth.

The relative size of pension transfer wealth is captured by $t(t) = T_P(t)/W_p(t)$ and the relative size of child transfer wealth by $t_k(t) = T_k(t)/W(t)$. Substituting into equation (4) and rearranging terms gives the total assets of adults in year t and, because only adults hold assets, aggregate assets in year t:

$$A(t) = (1 - t(t))(1 - t_k(t))W(t). \tag{5}$$

In the analysis presented here, we assume that pension transfer policy, $t(t)$, is exogenous. The next two sections consider lifecycle wealth and child transfer wealth, the remaining variables that determine assets.

Lifecycle Wealth

Lifecycle wealth is the wealth that all adults must hold in year t in order to achieve a given path of consumption and labor income over the remainder of their collective existence. We assume that the productivity and, hence, the labor income of individuals varies by age reflecting a variety of factors – decisions about labor force participation, the effects of experience and aging on productivity, economic structure, institutional factors, etc. We assume that these factors do not change over the course of the simulation and that, hence, the cross-sectional age profile of productivity and earnings are fixed. The profile shifts over time reflecting general changes in wages that occur due to technological change. In the current analysis, we do not consider any feedbacks from capital deepening to wages. Hence, the model considered here is appropriate for a small open economy in which the rate of return on investment is determined by international capital markets and the shift in the wage profile is determined by exogenous technological change. We assume that the rate of technological change is constant and exogenous.

The effect of age on earnings is captured in the effective number of producers (L) where:

$$L(a,t) = g(a)P(a,t)$$
$$L(t) = \sum_{a=0}^{w} L(a,t), \tag{6}$$

and $P(a,t)$ is the population aged a at time t and $g(a)$ is an age-specific, time-invariant vector of coefficients measuring age variation in labor income. Similarly, the effective number of consumers (N) is:

$$N(a,t) = f(a)P(a,t)$$
$$N(t) = \sum_{a=0}^{v} N(a,t) \tag{7}$$

where $f(a)$ is an age-specific, time-invariant vector of coefficients measuring relative levels by age of cross-sectional consumption.

Total labor income in year t is determined by the total number of effective producers and the level of labor productivity as measured by the labor productivity index, $\bar{y}(t)$. Likewise, total consumption in year t is determined by the total number of effective consumers and the level of consumption as measured by the consumption index, $\bar{c}(t)$:

$$Y(t) = \bar{y}(t)L(t)$$
$$C(t) = \bar{c}(t)N(t) \tag{8}$$

The rate of growth of labor productivity (g_y) is exogenous and constant so that:

$$\bar{y}(t+x) = \bar{y}(t)G_y(x) \tag{9}$$

where $G_y(x) = (1+g_y)^x$. The rate of growth of the consumption index will vary over time and is endogenously determined. The consumption index can be represented as an annual series of endogenously determined growth rates:

$$\bar{c}(t+x) = G_c(t,x)\bar{c}(t)$$
$$G_c(t,x) = \prod_{z=0}^{x-1} (1+g_c(t+z)) \tag{10}$$

where $g_c(t+z)$ is the rate of growth in the consumption index between year t+z and t+z+1.

These general rules can be applied to year t adults to determine their labor income income and consumption over their remaining adult years and, hence, their wealth in year t. Let *NTOT(t,x)* denote the number of effective consumers in year t+x who were adults in year t. Similarly, *LTOT(t,x)* denotes the number of effective producers in year t+x who were adults in year t:

$$NTOT(t,x) = \sum_{a=a_0+x}^{w} N(a,t+x)$$
$$LTOT(t,x) = \sum_{a=a_0+x}^{w} L(a,t+x). \tag{11}$$

In a closed population NTOT and LTOT would depend only on survival rates, but in an open population they will include migrants who were adults in year t.

The labor income of year *t* adults at age $a = a_0 + t$ in year $t+x$ is:

$$Y(a,t+x) = \overline{y}(t+x)L(a,t+x) \tag{12}$$

and consumption by year *t* adults in year *t+x* is:

$$C(a,t+x) = \overline{c}(t+x)N(a,t+x). \tag{13}$$

The present value in year t of the current and future lifetime consumption of all adults is given by:

$$PVC(t) = \overline{c}(t)\sum_{x=0}^{w-a_0} D(x)G_c(x,t)NTOT(t,x), \tag{14}$$

and the present value in year t of the current and future lifetime production of all adults is given by:

$$PVY(t) = \overline{y}(t)\sum_{x=0}^{w-a_0} D(x)G_y(x)LTOT(t,x), \tag{15}$$

where $D(x)$ is the discount factor $(1+r)^{-x}$. Substituting into equation (1), the lifecycle wealth of all adults in year t is:

$$\begin{aligned} W(t) = {} & \overline{c}(t)\sum_{x=0}^{w-a_0} D(x)G_c(x,t)NTOT(t,x) \\ & -\overline{y}(t)\sum_{x=0}^{w-a_0} D(x)G_y(x)LTOT(t,x). \end{aligned} \tag{16}$$

Child Transfer Wealth

The final variable that determines assets in equation (5) is child transfer wealth which measures the costs to year t adults of providing resources consumed by children. If adults spend more on children in the current and future periods, then child transfer wealth is a larger negative value. Or as represented in equation (5), the ratio of child transfer wealth to adult transfer wealth is a larger negative value.

What determines child transfer wealth? In part, it depends on the difference between what children consume and what children produce in the current and in future periods. Production and consumption are determined in the same manner for children as for adults. The age profiles of production and consumption ($g(a)$ and $f(a)$) are held constant for all ages including children. The shifts of the profiles over time are governed by the shifts in the production and consumption indexes discussed above.

The cost of children to year t adults also depends on their share of the costs of children in future periods. By assumption all of the *current* costs of children are born exclusively by year t adults. Year t adults are responsible only for a portion of the cost of children in subsequent years, because some portion of the costs of children is shifted to persons who become adults after year t.

The share of child costs for year t adults depends on a host of factors, including the extent to which child costs are born by families as opposed to taxpayers, the system of taxation that is used to finance public transfers to children, and extent to which parents, grandparents, and other family members finance familial transfers to children. The model distinguishes two ways in which child costs are financed: familial transfers and public transfers. Adult parents are assumed to bear the cost of familial transfers. Public transfers are financed through a proportional tax on labor income. The relative mix of these two mechanisms is an exogenously determined policy variable.

Child transfer wealth is equal to:

$$T_k(t) = \bar{y}(t)\sum_{x=0}^{w-a_0} D(x)G_y(x)KLTOT(t,x) - \bar{c}(t)\sum_{x=0}^{w-a_0} D(x)G_c(t,x)KNTOT(t,x) \qquad (17)$$

where $KLTOT(t,x)$ and $KNTOT(t,x)$ are the effective numbers of child producers and consumers, respectively, in year $t+x$ for which year t adults are financially responsible. A detailed description of the methods involved in calculating these variables is provided in the appendix.

Lifecycle Pension Wealth: $W_p(t)$

Pension wealth is equal to lifecycle wealth less child transfer wealth. Combining the results from equations (16) and (17) and rearranging terms yields:

$$\begin{aligned} W_p(t) &= \bar{c}(t)\sum_{x=0}^{w-a_0} D(x)G_c(t,x)\left(NTOT(t,x) + KNTOT(t,x)\right) \\ &\quad - \bar{y}(t)\sum_{x=0}^{w-a_0} D(x)G_y(x)\left(LTOT(t,x) + KLTOT(t,x)\right). \end{aligned} \qquad (18)$$

Lifecycle pension wealth is the discounted present value of current and future consumption by year t adults and their dependent children less the present value of current and future production by year t adults and their dependent children.

Macro level and steady state

Total assets are governed by the lifecycle accounting just described, but also by a macroeconomic constraint: the change in assets from one period to the next must equal saving during the period. We assume that assets are measured at the beginning and that consumption and labor income accrue at the beginning of the period and, hence:

$$(1+r)A(t) + (1+r)\left[Y(t) - C(t)\right] = A(t+1). \qquad (19)$$

In steady-state, assets grow at the same rate as total labor income, g_Y. Substituting $(1+g_Y)A(t)$ for $A(t+1)$, substituting for income and consumption, and rearranging terms, assets in steady state must satisfy:

$$A(t^*) = \frac{(1+r)}{r-g_Y}\left[\bar{c}(t^*)N(t^*) - \bar{y}(t^*)L(t^*)\right]. \qquad (20)$$

From the analysis of the lifecycle the relationship between assets and lifecycle pension wealth is governed by exogenously specified pension transfer policy:

$$A(t^*) = (1-t(t^*))W_P(t^*), \qquad (21)$$

where $W_P(t)$ is given in equation (18). Combining the macro and lifecycle conditions, and noting that the growth rate of the consumption index must equal the growth rate of the production index in steady-state, the consumption index in steady-state must satisfy:

$$\frac{(1+r)}{r-g_Y}\left[\bar{c}(t^*)N(t^*) - \bar{y}(t^*)L(t^*)\right] = (1-t(t^*))W_p(t^*). \qquad (22)$$

Rearranging terms yields:

$$\frac{\bar{c}(t^*)}{\bar{y}(t^*)} = \frac{L(t^*)}{N(t^*)}\left[1 + (\tilde{r} - \tilde{g}_Y)(1-t(t^*))w_p(t^*)\right], \qquad (23)$$

where $w_p(t^*)$ is the ratio of lifecycle pension wealth to current labor income and $\tilde{r}$ and $\tilde{g}_Y$ are rates of interest and growth, respectively, discounted by $1+r$.

Equation (23) tells us the level of consumption that can be sustained in steady-state given any level of labor productivity. Age-structure determines the steady-state consumption ratio through two multiplicative factors – the economic support ratio and a second factor that captures the influence of age structure on lifecycle pension wealth and, hence, assets.

Under two conditions the effects of age structure on consumption are captured entirely by the economic support ratio, *L/N*. The first is golden-rule growth. In that case, $\tilde{r} = \tilde{g}_Y$. The second is the pure transfer economy. If all age reallocations are accomplished via transfers rather than capital accumulation, then $t = 1$. In either case, we have:

$$\frac{\bar{c}(t^*)}{\bar{y}(t^*)} = \frac{L(t^*)}{N(t^*)}. \tag{24}$$

Steady-state consumption is determined entirely by labor productivity and the economic support ratio.

The steady state equation is quite intuitive, and can be interpreted as follows. w_p is the aggregate demand for wealth by adults, needed to finance their intended net consumption in old age. A share τ takes the form of transfer wealth, which does nothing to increase the annual aggregate flow of income and therefore does not raise $\bar{c}/\bar{y}$. The remaining share, $1 - \tau$, is held as assets or capital. In steady state assets must grow at the rate g_Y, reflecting productivity growth and population growth, to equip new workers augmented by technological progress at rate g_y. The return earned by the total assets is $r(1-\tau)w_p$, but of this, an amount g_Y must be set aside as net savings, leaving $(r-g_Y)(1-\tau)w_p$ as net income gain (per augmented worker, $\bar{y}$). In golden rule steady state, all of asset income must be saved and invested, and $r = g_Y$ so consumption ($\bar{c}/\bar{y}$) is determined entirely by the support ratio. This seems to be identical to the situation in which all pension wealth is held in the form of transfer wealth. It is quite different, however, because in golden rule the capital raises labor productivity very substantially, and thereby has a strong positive effect on consumption, albeit not one that we are considering here. In a Cobb-Douglas economy with a capital elasticity of 1/3, income per augmented worker will be proportional to the square root of $(1-\tau)w_p$. Note that the concept of a golden rule steady state makes sense only in a closed economy since it requires that the rate of return on the asset, capital, fall to the level of g_Y, and in an open economy the asset has no effect on the rate of return it earns, so the golden rule steady state cannot be attained.

Under more realistic and interesting conditions, wealth effects are operative. In a dynamically efficient economy $\tilde{r} > \tilde{g}_Y$, and as an empirical matter rates of return to capital virtually always exceed the rate of economic growth. Likewise, some portion of lifecycle pension needs are financed through capital accumulation and, hence, $t < 1$. Pension wealth may be negative if young adults finance their own consumption or that of their dependent children by accumulating debt that exceeds the assets accumulated by other adults in anticipation of retirement. Imposing constraints on net indebtedness would preclude this possibility, but in the absence of these constraints populations with sufficiently young age structures may have negative pension wealth, negative assets, and consumption even lower than implied by the support ratio: $\bar{c}(t^*)/\bar{y}(t^*) < L(t^*)/N(t^*)$. The simulations presented below identify circumstances under which this condition occurs.

Except under this peculiar set of circumstances, adult pension wealth and assets are positive and the consumption level exceeds the level implied by the support ratio: $\bar{c}(t^*)/\bar{y}(t^*) > L(t^*)/N(t^*)$. How changes in the rate of economic growth, population growth, mortality, and age structure influence wealth (Arthur and McNicol, 1978; Lee, 1994a, 1994b; Willis, 1988) and saving (Mason, 1987; Modigliani and Brumberg, 1954) can be applied fairly directly to this model and will not be repeated here. It can be readily shown that an increase in the rate of economic growth or a decrease in interest rates will lead to an increase in lifecycle pension wealth and assets, as we saw earlier in the case of the simplified calculation. Demographic change – fertility decline or lower mortality at older ages – that leads to an increase in the population at older ages leads to an increase in lifecycle pension wealth, assets, and consumption relative to labor productivity.

The most important feature of this model is that changes in age structure influence consumption through two distinct paths – through the support ratio and through lifecycle pension wealth. The changes are reinforcing when changes at young ages are involved. An increase in the number of young dependents decreases both the support ratio and assets. The effect on sustainable consumption is unambiguous – higher youth dependency leads to lower consumption for any given level of labor productivity. In a closed economy, there would be an additional feedback effect of the asset/labor ratio on earnings.

In contrast, the effect of an increase in old-age dependency is ambiguous. Although the support ratio is reduced if the number of effective consumers rises relative to the number of effective producers, pension wealth is increased if the increase in effective consumers is concentrated at older ages. Which effect dominates will depend on features of the consumption and production profiles, interest rates, rates of economic growth, and transfer policy.

It is probably important to clarify an important point. Studies of the relationship between saving and age structure frequently advance the hypothesis that an increase in old-age dependency will lead to lower saving and some empirical evidence lends support to this view. Although this may seem to contradict the results presented here, this is not the case. A lower saving rate and a higher asset to labor income ratio are inconsistent only if the rate of population (labor force) growth is not changing. Population aging is accompanied by lower population growth so that lower saving rates and greater wealth are mutually consistent.

Model Dynamics

Model dynamics be explored using two approaches. In one approach we assume that economies reach steady-state in the distant future. We can solve for the steady-state consumption path directly using equation (23) and the ratio of assets to labor income using equation (21). Other macro-economic variables of interest are readily calculated. Backward recursion is employed to solve the model during the transition to steady-state. Given values for all relevant variables in year t, the dynamic model can be reduced to two equations in two unknowns. One equation follows from lifecycle conditions (equation (18) combined with exogenously specified transfer policy $t(t)$) and the other follows from macroeconomic constraints (equation (19)). The two unknowns are the consumption index in *t-1* and assets in *t-1*. Starting in steady-state, we solve for consumption and assets in year *t*-1* and repeat.

A second method employing forward recursion is currently under development. Results presented below rely on the backward recursion method and should be viewed as provisional at this point.

Steady-state Results

The steady-state implications of demographic variables, economic growth, interest rates, the economic lifecycle, transfer policy, and age at retirement are analyzed in this section. The analysis is carried out by constructing stable populations with varying assumptions about fertility and mortality primarily relying on data for 2000-2005 from World Population Prospects 2004. Fertility and mortality data from several countries were selected to represent the broad span of demographic conditions. Total fertility rates employed vary from 7.1, the value in Uganda, to 1.28, Italy's TFR. Life expectancy at birth ranged from 47.5, again in Uganda, to 82.5, the value for Japan. Data from Pakistan is used to represent a country more intermediate in its demographic transition and the United States which has a TFR near replacement. An additional set of calculations employ the UN long-range projection for the US to 2300.

Table 2. Demographic Indicators for Alternative Scenarios.

Population (ASFR/Lx)	Total fertility rate	Life expectancy at birth	Population growth rate	Mean age of Consumption	Mean age of Production	Percent under 15	Percent over 65
Uganda	7.10	47.5	0.026	28.1	39.2	52.6	3.3
Uganda/Japan	7.10	82.5	0.037	28.4	39.0	54.7	4.6
Pakistan	4.27	62.5	0.017	34.2	41.2	41.9	6.9
US/Uganda	2.04	47.5	-0.010	43.9	44.6	24.5	14.5
US 2000	2.04	77.5	0.000	45.8	44.6	25.5	18.7
Italy/Uganda	1.28	47.5	-0.022	49.2	46.5	16.8	20.9
US/Japan	2.04	82.5	0.000	48.4	44.8	24.2	22.6
Italy/Japan	1.28	82.5	-0.012	55.7	47.0	15.5	33.2
US 2300	2.07	100.6	0.000	57.5	45.6	19.9	35.2

Notes. Steady state populations were constructed using age-specific fertility rates and Lx values for 2000-05 from World Population Prospects 2004 (UN 2005). The "Population" label refers to the country or countries for which the age-specific fertility rates and Lx values were employed. The US 2300 scenario is the age distribution for the US population in 2300 from the UN long-range population projections. Mean ages of consumption and production are constructed using the US economic lifecycle for 2000.

Using combinations of fertility and mortality from these countries, we generated stable population age distributions. In these age distributions the percentage of the population under the age of 20 varied from 15.5% to 54.7% and the percentage over the age of 65 varied from 3.3% to 35.2%. The population growth rates also varied widely – from a low of -2.2% per annum which Italian fertility and Ugandan mortality would generate to a high of 3.7% per annum which Ugandan fertility and Japanese mortality would generate. The demographic characteristics are provided in Table 2 for the stable populations analyzed.

The first set of analyses explores the effects of fertility and mortality on the steady-state ratio of assets to labor income. The key conclusion is that either a decline in fertility decline or a rise in life expectancy leads to substantially higher steady-state assets. Moreover, these effects interact so that when both occur together, as they do over the demographic transition, the steady-state demand for assets rises particularly sharply.

The conclusion is based on results presented in Table 3, which uses the demographic scenarios presented above. Variation in life expectancy is explored using life table values for Uganda and Japan. Variation in fertility is explored using ASFRs for Uganda, the US, and Italy. Note that the medium fertility and low life expectancy combination yields a stationary population – one with zero population growth. Medium fertility and low life expectancy and both low fertility scenarios lead to populations that are declining in steady-state. The rate of labor productivity growth is 1.5% per year, the rate of interest is 3.0% per year, the share of transfer wealth in total pension wealth is 0.35.[3] We employ two economic lifecycles – the US 2000 lifecycle and the Taiwan 1977 lifecycle presented in Figure 2.

Table 3. Partial Effects on Steady-state A/Y of the Total Fertility Rate and Life Expectancy at Birth, Baseline Scenarios.

Life expectancy at birth	Total Fertility Rate: High (7.1)	Medium (2.0)	Low (1.3)
	US 2000 economic lifecycle		
Low (47.5)	0.72	1.44	1.67
High (82.5)	1.98	3.87	4.25
	Taiwan 1977 economic lifecycle		
Low (47.5)	0.15	0.53	0.80
High (82.5)	1.02	2.27	2.77

Notes. For details of demographic variables and sources see Table 2.

Early-transition demographic conditions are least favorable to asset demand. Total assets are about 70 percent of annual labor income given the US 2000 economic lifecycle, and only 15 percent of annual labor income given the Taiwan 1977 economic lifecycle when life expectancy is low and fertility is high. Compare this with assets for a TFR of 2.0 and life expectancy at birth of 82.5. For the US economic lifecycle, fertility decline produced an increase in assets of 0.72 and improved life expectancy led to a rise of 1.26 in assets relative to total labor income. Taking lower fertility and improved life expectancy together leads to an increase of 3.15 in the ratio of assets to labor income. In percentage terms, the rise is over 500%! The patterns using the Taiwan 1977 economic

[3] The assumed ratio of transfer to total pension wealth is somewhat less than the ratio of public and private transfers, excluding bequests, to consumption by the elderly in the US in 2000 (0.44). In alternative scenarios, we consider a transfer ratio of 0.65. This compares with a ratio of public and private transfers, excluding bequests, to consumption of 0.69 for Taiwan in 1998. See Mason *et al*. (forthcoming).

lifecycle are similar. The absolute changes are smaller, but the percentage changes are substantially greater. The absolute differences in assets are relevant if one is concerned, for example, with absolute differences in wages or income, but if one is interested in rates of economic growth, it is the percentage changes that are most relevant.

The final column in Table 3 addresses the implications of steady-state with sub-replacement fertility and, hence, population decline. A further reduction in fertility results in greater assets even if the reduction is below replacement level. With couples raising fewer children, they are consuming more at all adult ages increasing pension wealth and assets at every adult age. The compositional effects that arise with lower fertility – an older population – may or may not reinforce the behavior effect of smaller families, but the net effect here is to increase assets.

The final set of steady-state results presented analyze the effects of varying model parameters – the economic lifecycle, labor productivity growth, interest rates – and policy variables – transfer policy and the age at retirement. This analysis is confined to five realistic steady-state populations and excludes the unrealistic mix and match scenarios used to assess the effects of holding fertility or mortality constant while varying the other. The absolute effects are presented in Table 4 and the percentage effects in Table 5.

Table 4. Partial effects on assets of productivity growth, interest rates, transfer policy, and the age at retirement. Steady state populations.

			Partial effect of increase in			
Population (ASFR/Lx)	Economic lifecycle	Assets/ Labor Income, baseline	Labor Productivity Growth	Interest rate	Transfers	Age at retirement
Uganda	US 2000	0.72	0.23	-0.16	-0.10	-0.16
Pakistan	US 2000	1.33	0.38	-0.28	-0.20	-0.19
US 2000	US 2000	2.89	0.69	-0.52	-0.51	-0.22
US 2300	US 2000	7.34	1.69	-1.30	-1.33	-0.26
Italy/Japan	US 2000	4.25	1.05	-0.76	-0.83	-0.23
Uganda	Taiwan 1977	0.15	0.15	-0.10	-0.02	na
Pakistan	Taiwan 1977	0.43	0.25	-0.17	-0.06	na
US 2000	Taiwan 1977	1.54	0.48	-0.34	-0.27	na
US 2300	Taiwan 1977	5.01	1.31	-0.94	-0.89	na
Italy/Japan	Taiwan 1977	2.77	0.77	-0.53	-0.53	na

Baseline is for annual rate of growth of labor productivity of 1.5 per cent, an interest rate of 3.0 percent, and share of transfers in pension wealth of 0.35. The partial effects are calculated as the change in A/Y resulting from a one percentage point increase in productivity growth (measured from 1.0 to 2.0), a one percentage point increase in interest rates (measured over 3.0 to 6.0), an increase in transfer share of 0.1 (measured over 0.35 to 0.65), and an increase in "age at retirement" by one year measured as 0.2 times the effect of extending peak earnings by five years.

The rate of labor productivity growth has a substantial positive effect on steady-state assets. A one percentage point increase in the rate of labor productivity growth, from 1.0 to 2.0 per cent per annum, produced an increase in the asset-labor income ratio for the US 2000 steady-state population of about 0.7 for US 2000 economic lifecycle and about 0.5 for the Taiwan 1977 economic lifecycle. The magnitude of the effect is much smaller in for low asset steady-state populations, e.g., Uganda and Pakistan, and much greater for high asset steady-state populations, e.g., US 2300 and Italy/Japan. This pattern is true of the other effects reported in Table 4. They are consistently largest for the high asset steady-states. In percentage terms, however, the converse is often but not always the case. A one percentage point increase in the labor force productivity growth rate, given the US economic lifecycle, yields an increase in the asset ratio of between 25 and 31 percent with the largest effect in Uganda. Given the Taiwan 1977 economic lifecycle the percentage productivity growth effects are much greater for Uganda and Pakistan. This reflects the low baseline level of assets under these circumstances, but nonetheless it implies that an increase in productivity growth would have particularly strong effects on economic growth in such settings.

Table 5. Partial effects on assets of productivity growth, interest rates, transfer policy, and the age at retirement. Steady state populations.

			Partial percentage effect of increase in			
Population (ASFR/Lx)	Economic lifecycle	Assets/ Labor Income, baseline	Labor Productivity Growth	Interest rate	Transfers	Age at retirement
Uganda	US 2000	0.72	31.4	-22.6	-13.9	-22.5
Pakistan	US 2000	1.33	28.2	-20.8	-15.1	-14.0
US 2000	US 2000	2.89	24.0	-18.0	-17.7	-7.6
US 2300	US 2000	7.34	23.0	-17.7	-18.1	-3.6
Italy/Japan	US 2000	4.25	24.8	-18.0	-19.6	-5.4
Uganda	Taiwan 1977	0.15	102.1	-66.6	-14.0	na
Pakistan	Taiwan 1977	0.43	58.0	-38.7	-15.1	na
US 2000	Taiwan 1977	1.54	31.2	-21.9	-17.4	na
US 2300	Taiwan 1977	5.01	26.2	-18.8	-17.8	na
Italy/Japan	Taiwan 1977	2.77	27.7	-19.3	-19.2	na

Notes: See Table 4.

In models such as this one, the interest rate and the rate of labor productivity growth are opposite sides of the same coin and their effects will be equal, but opposite in sign. This is not quite the case numerically because the reported effects are for a one percentage point increase in the interest rate evaluated over the range of 3 to 6 percent. Moreover, the symmetry between the two effects does not hold because of the discrete form of the model. The qualitative observations about the rate of labor productivity growth hold for the rate of interest. The effects are negative, they are greatest in absolute terms for the high asset steady states, and they are greatest in percentage terms for the low asset steady states. Also the percentage effects are especially large for the low asset populations based on the Taiwan 1977 economic life cycle.

Public transfers to support old-age and assets are perfect substitutes in this model and, hence, an increase in transfers leads to a decline in assets. The effects reported here are for an increase in the share of transfers in pension wealth by 10 percentage points. The effect is evaluated by increasing the share from 35 percent in the baseline to 65 percent in an alternative scenario. This is roughly the current difference between the

United States and Taiwan. In Taiwan public transfers to the elderly are somewhat less than public transfers in the US, but familial transfers are substantially greater (Mason *et al.*, forthcoming). If that difference persists as assumed in the steady states calculations, transfer policy is much more supportive of capital accumulation in the US than in Taiwan. A ten percentage point increase in old-age transfer wealth leads to a decline in assets ranging from 14 to 20 percent for the values used in the illustrative calculations presented here.

Finally, we consider the implications of delaying retirement. This is implemented by stretching the labor income age profile by adding five years of earnings at the peak and shifting the profile after the peak to the right by five years of age. The effects reported here are for one year of delayed retirement, i.e., one-fifth of the total increase from five years of additional peak earnings. Working more at older ages is a substitute for accumulating pension wealth and, hence, delayed retirement leads to a decline in assets. The effect employing the US economic lifecycle is substantial for the lower asset populations, but rather small for the high asset populations. For the US 2000 steady state population, for example, the one year delay in retirement reduces assets by 7.6%.

Each of these factors operating in isolation plays a relatively significant role, and over the course of development many may operate in complementary fashion. To the extent that low income countries have economic lifecycles more similar to Taiwan's than the US, low productivity growth, high interest rates, strong reliance on familial transfer programs, delayed retirement, high fertility and short life expectancy, the lifecycle demand for assets is quite low or would even be negative in the absence of constraints on indebtedness. If development leads to an economic lifecycle that is closer to that found in the US, higher productivity growth, lower interest rates, an erosion of familial transfer programs, and earlier retirement, the demand for assets is fueled. It would be simplistic, however, to assume that development has some simple, uni-directional effect on each of these factors, however. The high level of consumption by US elderly, for example, is not a common feature of developed economies. Public transfer programs are substantial in many Western countries that rely less on familial transfer systems.

Simulations of Dynamic Results

The first set of simulation results we consider are for the United States. The results are obtained using backward recursion, the US consumption and labor income profiles estimated for the US and shown in Figure 2A and a real rate of interest of 3% per annum. The rate of labor productivity growth is set to 1.5% per annum. Two-thirds of the cost of children are met through familial transfers and on-third through public transfers values that are close to the estimated shares of consumption for 2000 (Mason *et al.*, forthcoming). Transfer wealth is held constant at 35% of pension wealth throughout the simulations, again a value similar to that estimated for the US in 2000. We rely on UN population estimates and projections as described above. Steady-state values are calculated for 2300, but results are reported only for 1950 to 2150. The assumption that the interest rate is constant and exogenous is more appropriate for a small open economy than for the United States. The implications of this assumption are important and discussed below.

The changes in US age structure that bear on the demographic dividends are summarized by comparing the rates of growth of the effective number of consumers and the effective number of producers charted in Figure 3. Between 1950 and 1970 the effective number of consumers grew rapidly relative to the effective number of producers as a consequence of US baby boom. As the baby boomers entered the workforce, the effective number of producers grew more rapidly than the effective number of consumers – a pattern that persisted until 2000. The first baby boomer turns 65 in 2011, but recall that in the US consumption exceeds production in the late fifties. The effective number of consumers is projected to grow more slowly than the effective number of producers from 2005 to 2035 as baby boomers swell the ranks of retirees. Another wave of aging is projected for the second half of the 21st Century.[4]

The difference between the rate of growth of the effective number of consumers and the effective number of producers is equal to the rate of growth of the support ratio (L/N) and defines the *first demographic dividend.* The first dividend period was relatively short in duration, 1970 to 1995 or 2000, and small in magnitude as compared with most developing countries, but the US experience was similar to that of other industrialized countries. For the remainder of this Century the first dividend is negative, producing a decline in income or consumption per equivalent adult of as much as 0.5% per year, but averaging less than half of that. We will defer a discussion of the rate of growth of $\bar{c}/\bar{y}$.

Life cycle wealth and its key components, child transfer wealth, lifecycle pension wealth, and assets are charted in Figure 4. Child transfer wealth is plotted as a positive value to improve the readability of the figure.[5] Child transfer wealth is the present value of all childrearing costs – direct familial support and indirect support through tax payments – that adults in year t will pay over the remainder of their lives expressed relative to current labor income. The US peak reached during the baby boom was roughly equal to 5 times annual labor income. As the baby boom ended child transfer wealth began to decline steadily to about 3 times annual labor income in 2050 and a little less in 2150.

Pension wealth, the net wealth required to finance old-age consumption in excess of labor income, was equal to twice labor income in 1950. As a consequence primarily of increasing life expectancy and changing age structure, pension wealth doubled between 1950 and 1970, but stagnated during the 1970s and 1980s – a consequence of the increase in the relative number of young adults. Pension wealth began to increase rapidly again starting around 1990 and is projected to reach 6 times labor income in 2025.

The total lifecycle wealth of adults, equal to the sum of child transfer wealth and pension wealth has gone from a large negative value, when the obligations to children substantially exceeded the value of resources for retirement, to a strong positive value.

The implication of the substantial rise in pension wealth for assets depends on the extent to which the elderly rely on transfers. Given the current transfer policy, assets rise from less than 2 times labor income in 1950, to about 3 times labor income in 2000, and about 4 times labor income in 2025. Looking into the more distant future, assets reach 6 times labor income in 2150 and a steady-state value of 7.3 (Table 3).

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[4] The spike at 2050 occurs at the point where we begin using the long-range population projections. These projections assume that there is no net migration which is a substantial shift from the current situation and the assumption prior to 2050.

[5] The simulation period is five years. The values are multiplied by 5 in this figure so that they will be expressed as assets per year of labor income rather than as assets per five-years of labor income.

In the current version of the model, the rise in assets leads to an increase in the value of assets held abroad. This leads to a rise in national income in the US and to capital deepening and higher labor income in the countries in which the US invests, but it does not lead to capital deepening or to an increase in US wages. Clearly a more satisfactory analysis would acknowledge that higher US saving will lead to greater US investment and to capital deepening. Moreover, population aging is a global phenomenon that will influence the global supply of capital. The projected rise of assets for the US is consequential. Given a standard Cobb-Douglas production function, for example, output per worker increases as a square-root of A/Y. A doubling of the capital-output ratio produces an increase in output per worker of about 40%. That would have been the magnitude of the second dividend's effect on wages in the US between 1950 and 1970 and, again between 1970 and 2080 given investment of the additional capital in the domestic economy.

The rate of growth of consumption index relative to the production index should be interpreted in this light. In the absence of domestic investment, the effect of the second dividend on consumption is quite modest. This can be seen by comparing the rate of growth of the support ratio *L/N* to the rate of growth of $\bar{c}/\bar{y}$ in Figure 3. If the growth of labor income is unaffected by increases in national saving, the second dividend is the difference between the rate of growth of the support ratio and the index of consumption to labor income. In most periods, the consumption index is growing relative to labor income faster than implied by changes in the support ratio, but the benefit is quite small. The important exception is during the US baby boom, when the second dividend was sufficiently large to reverse the effects of the first dividend.

More rapid accumulation of capital is possible only by reducing consumption. Thus, the effects of the second dividend on consumption will be smaller than the effects on per capita income. Although the literature on the demographic dividend has emphasized per capita income growth, welfare is more closely related to consumption.

The final results presented here are intended as a preliminary exploration of the implications of alternative demographic transitions for asset accumulation. For this purpose, we have simulated wealth using demographic data from three countries – Nigeria, Brazil, and China. Understand that only the demographic variables are tailored to these countries. We use the US lifecycle profiles for Brazil and the Taiwan profiles for China and Nigeria. We do not have estimates of the size of transfer wealth, but use a value of 0.35 for China and a value of 0.65 for Brazil and Nigeria. Nonetheless, the results are suggestive and interesting.

The simulated child transfer wealth for Nigeria was extremely high between 1950 and 2000 – more than 8 times labor income. Simulated pension wealth and assets are both negative until 2010. Why is this so? Adults are incurring debts to finance consumption at young ages and consumption by their children that exceed the value of assets accumulated to fund old-age support. Assets do turn positive in the Nigerian simulation and rise very slowly. Even in 2150, however, A/Y barely exceeds 1. These results suggest that low life expectancy and high fertility seriously undermine capital accumulation, particularly if transfer systems dominate old-age support.

In Brazil, child costs were also very high in 1950 but declined very rapidly as a consequence of fertility decline. Despite the high child costs, simulated pension wealth and assets were positive in 1950 and increased steadily. Assets exceed labor income by

2000 and approached 2 in 2050. This is well below the US level, in part a consequence of the much greater assumed reliance on transfer wealth, but much higher than in Nigeria.

In China child costs were relatively high in 1950 and rising, although they were well below the costs in Nigeria and Brazil. Although child costs were rising during this period, pension wealth and assets were rising as well reflecting improvements in life expectancy, but did not become positive. Starting in 1970 child costs dropped very rapidly reflecting the very rapid decline in fertility experienced in China. This did not produce an immediate increase in asset demand, but starting around 1980, the demand for assets turned positive and began to increase. The simulated A/Y value reaches 1.0 in 2005 and 2.0 in 2050.

Qualifications and Further Research

The results presented here are promising, but much remains to be done. First, further testing and evaluation of the simulation model are required. There are some features of this model, such as its ergodic properties, that we do not yet fully understand. Solving the model using forward recursion techniques that we are now working on and further analysis of the backward recursion approach should be very useful. Second, there are features of the theoretical model that require further development. The most obvious and important is to relax the small, open economy/exogenous interest rate feature of the model. Another is to relax the assumption that the cross-sectional consumption profile is fixed. We could, for example, explore the implications of a quantity-quality tradeoff for child expenditures or the implications of political economy models that might influence the consumption of the elderly. A third area requiring more work is empirical. Comparing results from the US and Taiwan shows that variation in the economic lifecycle across countries is important and, hence, the need for more estimates of the economic lifecycle and more analysis of how it varies and why. Of equal importance is improving estimates and analysis of transfers. Again, we have estimates for Taiwan and the US that can be employed here, but we know very little about transfer policy in the comprehensive sense of the term used here. In particular, estimates of familial transfers are not widely available.

CONCLUSIONS

Over the coming decades we will find ourselves in uncharted waters. The share of the elderly population will reach unprecedented levels and not just in the industrialized world. Many low- and middle-income countries are also far along in their demographic transitions. Even if adults begin to delay retirement, it is virtually certain that the number of retirees will rise relative to the number of workers – in most countries and in the world as a whole.

If labor were the only factor of production, the first order effects of population aging would be easily assessed. Per capita income and per capita consumption would vary directly with the economic support ratio. An increase in the number of retirees would add to the number to be supported but not to the number producing nor the amount produced. The economy would be fixed pie divided among more consumers, and thus, per capita consumption would decline in direct proportion to the decline in the support ratio.

Retirees do not, however, rely exclusively on the labor of others (and public and familial transfer systems). Retirees depend on pension funds, personal savings, homes acquired during their working years, and other assets to finance some part of their retirement. How much is a matter of some dispute and varies widely from place to place, but estimates we present show that assets are an important source of support for the elderly in Taiwan and especially in the United States. Thus, the lifecycle demand for assets, the size of the capital stock, and total production increase as populations age. The size of the pie increases with aging, but the important question is by how much.

This paper answers the question using a new conceptual approach. This model acknowledges the close ties and pervasive links across generations. Consumption at each age is not governed by an individualistic lifetime budget constraint as in the lifecycle model. Rather, consumption is governed by altruism and constrained by total production. The steady-state analysis and simulation results, although preliminary, indicate that fertility decline, increased longevity, and the accompanying changes in age structure have potentially large effects on the demand for assets. Early in the demographic transition, the demand for assets is near zero or even negative judging, for example, from the simulations for Nigeria. In a country like the United States, however, the demand for assets is over three times annual labor income and increasing.

These effects are large, but they are not certain. They depend on public and familial transfer policy. If the response to population aging were exclusively to expand public transfer programs and to increase the burden on adult children of providing support to their parents, then we will merely be dividing a fixed pie among more consumers.

APPENDIX

Child Transfer Costs

The cost of all children age z in year t+x is:

$$\begin{aligned} COST(z,t+x) &= Y(z,t+x) - C(z,t+x) \\ &= \overline{y}(t)G_y(x)L(z,t+x) - \overline{c}(t)G_c(t,x)N(z,t+x) \qquad z < a_0 \end{aligned} \tag{25}$$

A fraction of the cost of children of age z in year $t+x$ is financed through transfers by year t adults; the remainder is financed through transfers by persons who became adults between year t and $t+x$. Let $TAX_k(z,t,x)$ be the share of child costs paid by year t adults. Then, child transfer wealth in year t for year t adults is:

$$T_k(t) = \sum_{x=0}^{w-a_0} D(x) \sum_{z=0}^{a_0-1} \mathrm{TAX_k(z,t,x)COST(}z\mathrm{,t+x)} \tag{26}$$

Substituting for COST from equation (25) yields:

$$T_k(t) = \overline{y}(t)\sum_{x=0}^{w-a_0} D(x)G_y(x)KLTOT(t,x) - \overline{c}(t)\sum_{x=0}^{w-a_0} D(x)G_c(t,x)KNTOT(t,x)$$

$$KLTOT(t,x) = \sum_{z=0}^{a_0-1} \mathrm{TAX_k}(z,\mathrm{t,x})L(z,t+x) \tag{27}$$

$$KNTOT(t,x) = \sum_{z=0}^{a_0-1} \mathrm{TAX_k}(z,\mathrm{t,x})N(z,t+x)$$

where $KLTOT(t,x)$ is the total number of children in year t+x dependent on year t adults measured in equivalent production units and in year t+x and $KNTOT(t,x)$ is the total number of children in year t+x dependent on year t adults measured in equivalent consumption units.

Tax burden of year t adults

The share of year t adults depends on whether child costs are financed through public or private (familial) transfer programs. We assume that the shares of public and private transfers are constant and exogenous, i.e., they are a matter of public policy. Let the familial share be t^f and the public share be $1-t^f$. Then the share of cost paid by year t adults is a weighted sum of the taxes paid through a familial transfer system and the taxes paid through a public transfer system, i.e.,

$$TAX_k(z,t,x) = t^f TAX_k^f(z,t,x) + (1-t^f)TAX_k^g(z,t,x) \tag{28}$$

where $TAX_k^f(z,t,x)$ is the share of child costs paid by year t adults under a familial transfer systems and $TAX_k^g(z,t,x)$ is the share of child costs paid by year t adults under a public transfer system.

We assume that all public transfers to children are financed by a proportional tax on labor income. Thus,

$$TAX_k^g(z,t,x) = \sum_{a=a_0+x}^{w} Y(a,t+x)/Y_A(t+x) \tag{29}$$

where $Y_A(t+x) = \sum_{a=a_o}^{w} Y(a,t+x)$ is the total labor income of all in year t+x.

The tax share of year t adults is in year t+x is their share of labor income in year $t+x$. Note that the public tax share is independent of the age of the child, z. Henceforth, we drop the z argument.

We assume that familial transfers are determined by parentage. If we let F(z, t, x) equal the proportion of those aged z with parents (mothers) age $a_0 + x$ or older in year $t+x$, then

$$TAX_k^f(z,t,x) = F(z,t,x) \tag{30}$$

where F is calculated using the distribution of births to women:

$$F(z,t,x) = \frac{\sum_{a=a_0+x-z}^{AGEM} B(a,t+x-z)}{\sum_{a=a_0}^{AGEM} B(a,t+x-z)} \quad \text{for } x > z$$
$$= 1 \quad \text{for x} \leq \text{z.} \tag{31}$$

and $B(a,t+x-z)$ is births to women aged a in year t+x-z. Children who are x years or older are all the offspring of year t adults (mothers) and hence F has a value of 1. The value of F declines to zero as x increases. (Note that F can be represented as a function of t and x-z. It isn't really three dimensional.)

We can substitute into equation (28) and the share of year t adults is:

$$TAX_k(z,t,x) = t^f F(z,t,x) + (1-t^f) \sum_{a=a_0+x}^{w} Y(a,t+x)/Y_A(t+x) \tag{32}$$

Substituting into equation (27) yields child transfer wealth for year t adults. Note that the tax shares devoted to childrearing are determined exogenously by population age structure, fertility, the age profile of earnings – all exogenous factors. Thus, in the determination of child transfer costs, the only endogenous variable is the vector of the consumption index.

Backward Recursion

The backward recursion solution computes the consumption index and, hence, all other variables in period t-1 conditional on the values in period t. The steady-state values are known. Hence, we can begin in period t*, solve for period t*-1, and recursively solve for all periods t.

From lifecycle accounting, assets in period t-1 depend on pension policy and lifecycle wealth in year t-1. From equations (18) and (21):

$$\begin{aligned} A(t-1) = {} & \bar{c}(t-1)(1-t)\sum_{x=0}^{w-a_0} D(x)G_c(t-1,x)\left(NTOT(t-1,x) + KNTOT(t-1,x)\right) \\ & -\bar{y}(t-1)(1-t)\sum_{x=0}^{w-a_0} D(x)G_y(x)\left(LTOT(t-1,x) + KLTOT(t-1,x)\right). \end{aligned} \tag{33}$$

Pension policy may vary with year, but here we drop t to simplify notation. The right-hand-side variables include consumption in year t-1, consumption in year t and subsequent years, and labor income terms in year t-1 and later. Only the consumption terms in year t-1 are unknown and must be solved for. These are distinguished in:

$$
\begin{aligned}
A(t-1) = {} & \bar{c}(t-1)(1-t)N(t-1) \\
& +(1-t)\sum_{x=1}^{w-a_0} D(x)\bar{c}(t-1+x)\left(NTOT(t-1,x)+KNTOT(t-1,x)\right) \\
& -\bar{y}(t-1)(1-t)\sum_{x=0}^{w-a_0} D(x)G_y(x)\left(LTOT(t-1,x)+KLTOT(t-1,x)\right).
\end{aligned} \tag{34}
$$

From macro-accounting, we know that:

$$
A(t-1) = \frac{A(t)}{1+r} + \bar{c}(t-1)N(t-1) - \bar{y}(t-1)L(t-1). \tag{35}
$$

The gives us two equations in two unknowns, assets and the consumption index in period t-1. Substituting for *A(t-1)* yields:

$$
\begin{aligned}
& \bar{c}(t-1)(1-t)N(t-1) + (1-t)\sum_{x=1}^{w-a_0} D(x)\bar{c}(t-1+x)\left(NTOT(t-1,x)+KNTOT(t-1,x)\right) \\
& -\bar{y}(t-1)(1-t)\sum_{x=0}^{w-a_0} D(x)G_y(x)\left(LTOT(t-1,x)+KLTOT(t-1,x)\right) \\
& = \frac{A(t)}{1+r} + \bar{c}(t-1)N(t-1) - \bar{y}(t-1)L(t-1)
\end{aligned} \tag{36}
$$

Multiplying both sides by 1+r and rearranging terms yields:

$$
\begin{aligned}
& -t\,\bar{c}(t-1)N(t-1) \\
& = \frac{A(t)}{1+r} - (1-t)\sum_{x=1}^{w-a_0} D(x)\bar{c}(t-1+x)\left(NTOT(t-1,x)+KNTOT(t-1,x)\right) \\
& +\bar{y}(t-1)(1-t)\sum_{x=0}^{w-a_0} D(x)G_y(x)\left(LTOT(t-1,x)+KLTOT(t-1,x)\right) - \bar{y}(t-1)L(t-1).
\end{aligned} \tag{37}
$$

Further algebra gives the consumption index for t-1:

$$
\bar{c}(t-1) = \frac{\left\{\begin{array}{l} A(t)\big/1+r - (1-t)\displaystyle\sum_{x=1}^{w-a_0} D(x)\bar{c}(t-1+x)\left(NTOT(t-1,x)+KNTOT(t-1,x)\right) \\ +\bar{y}(t-1)\left\{(1-t)\displaystyle\sum_{x=0}^{w-a_0} D(x)G_y(x)\left(LTOT(t-1,x)+KLTOT(t-1,x)\right) - L(t-1)\right\} \end{array}\right\}}{-t\,N(t-1)} \tag{38}
$$

Assets in period t-1 can be calculated using either equation (34) or equation (35).

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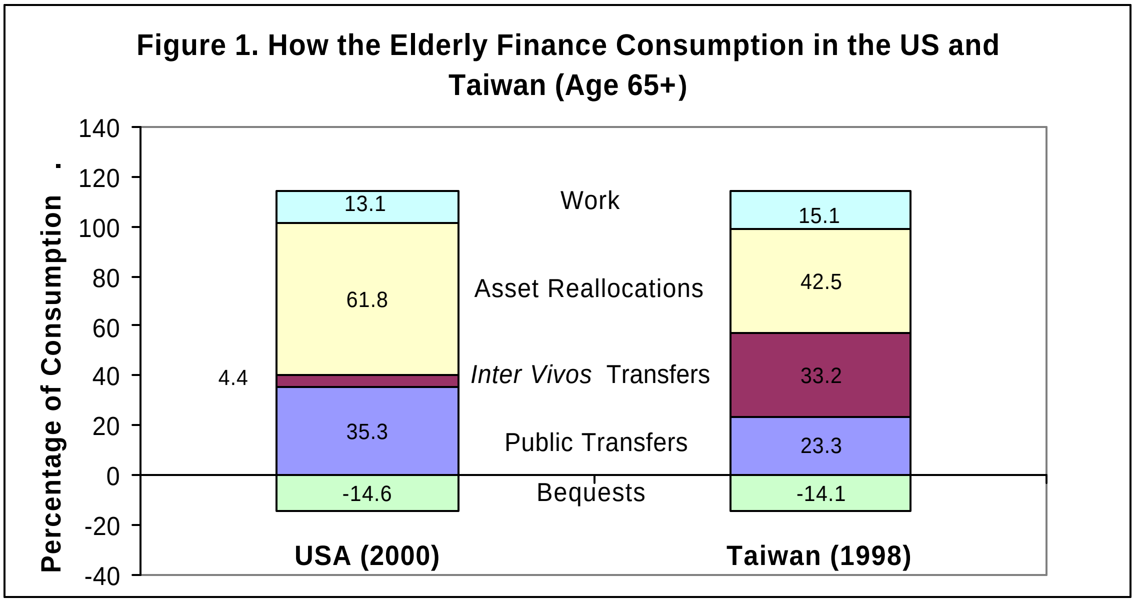
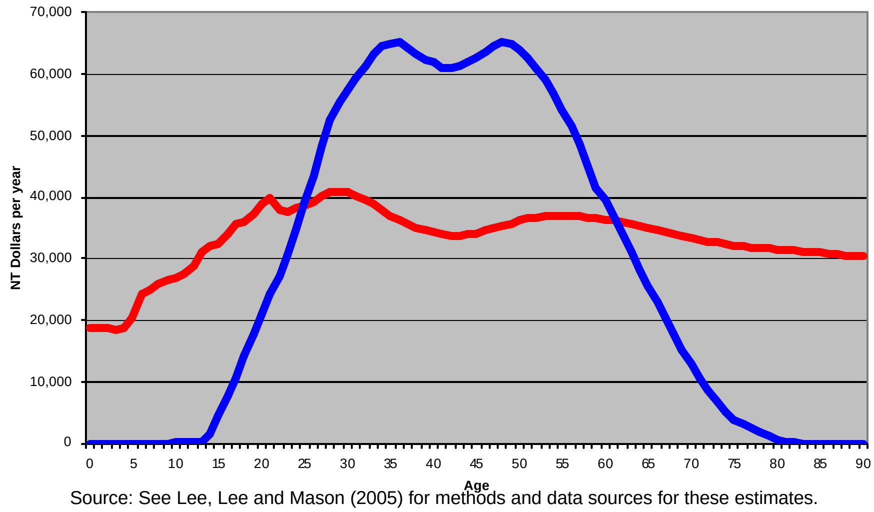


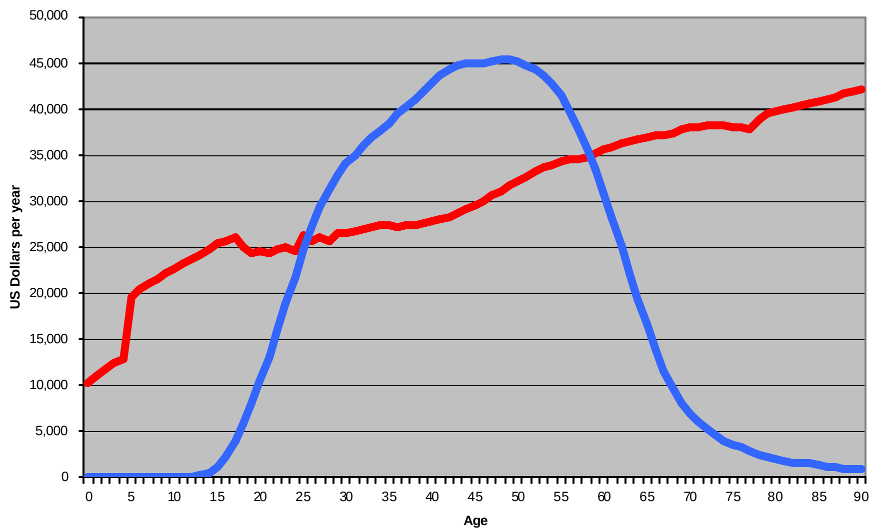
Source: Mason et al (forthcoming).

Figure 2A. Per Capita Labor Income and Consumption, Taiwan (1977)



Source: See Lee, Lee and Mason (2005) for methods and data sources for these estimates.

Figure 2B. Per Capita Labor Income and Consumption, US (2000)



Source: See Lee, Lee and Mason (2005) for methods and data sources for these estimates.

Figure 3

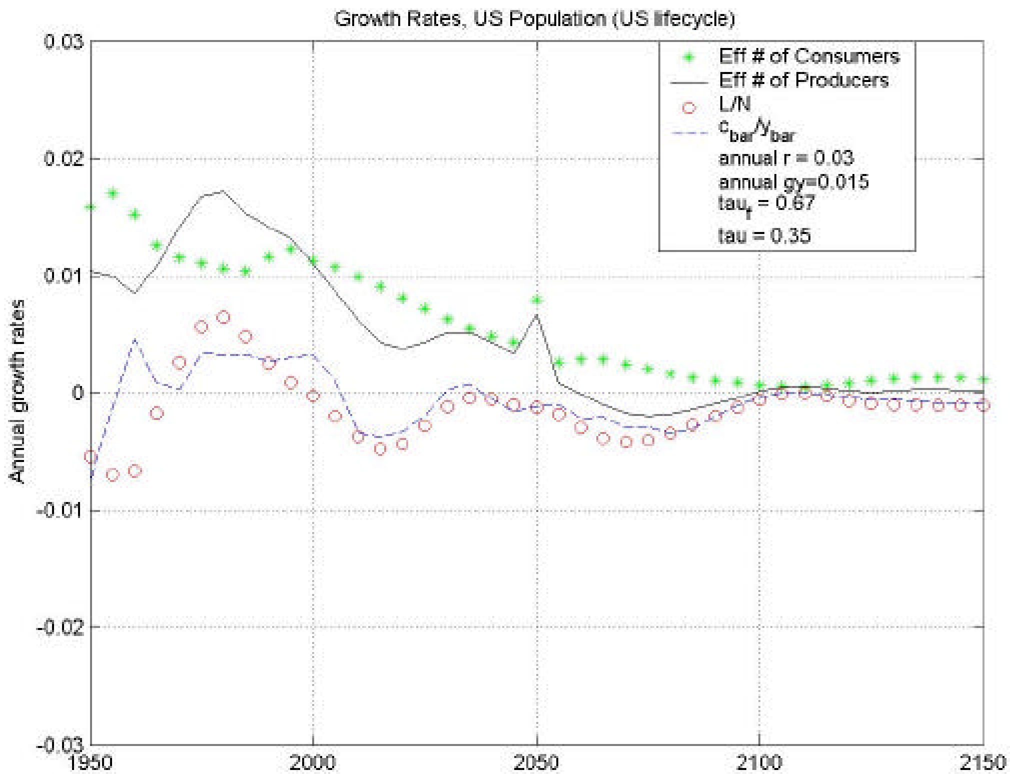
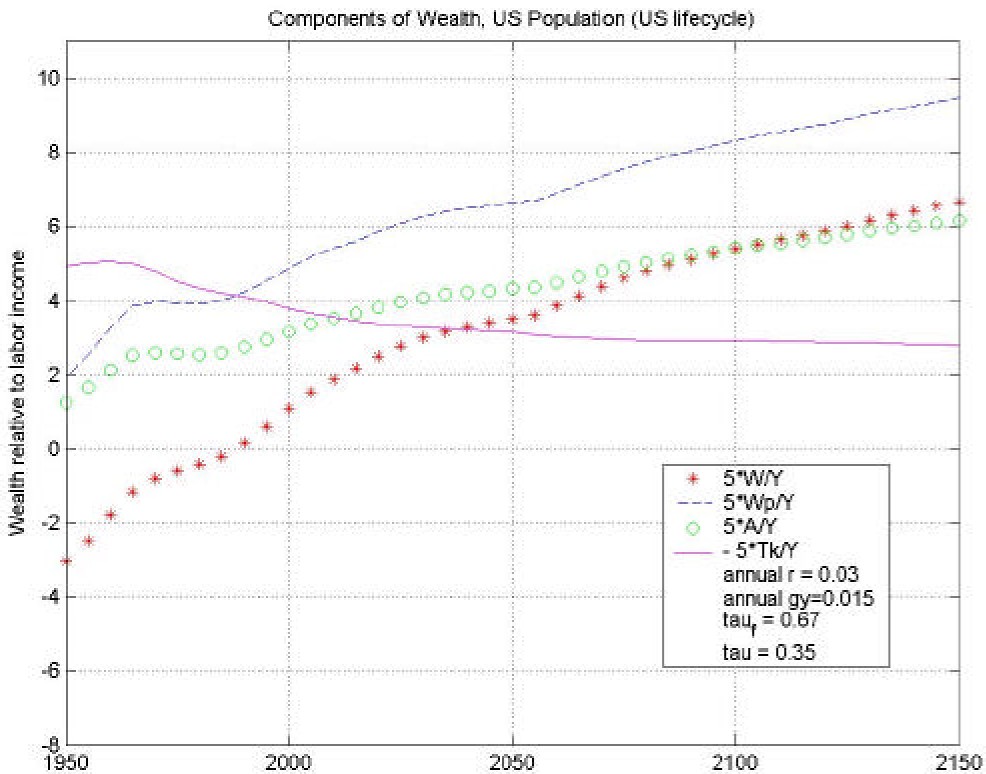
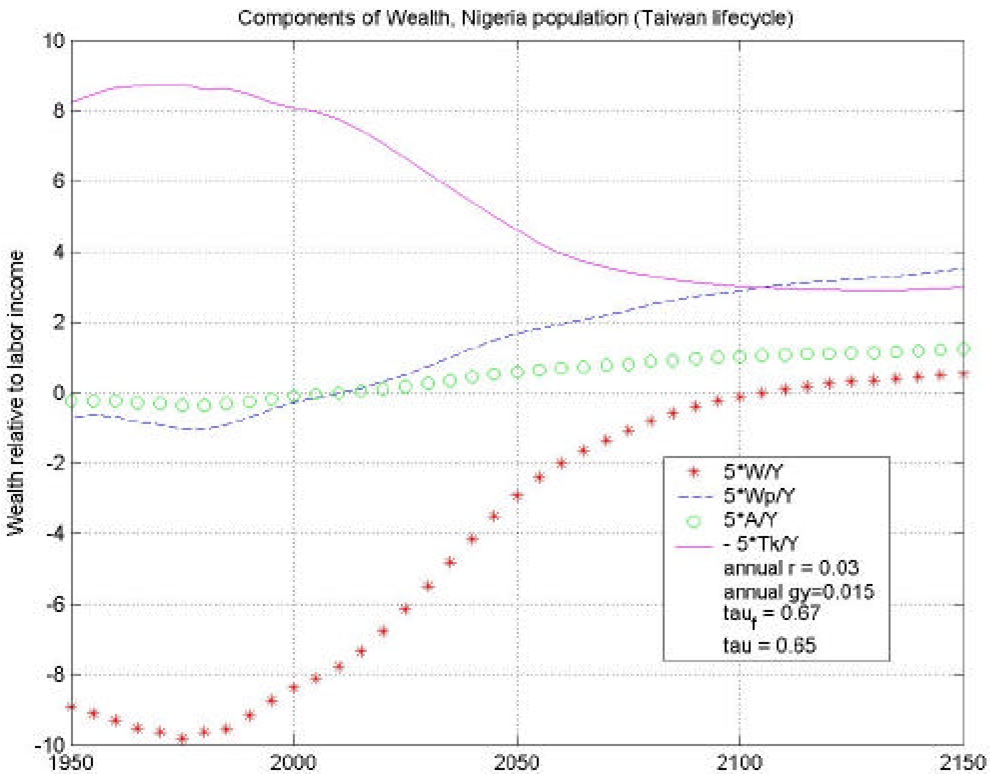

Growth Rates, US Population (US lifecycle)
Eff # of Consumers
Eff # of Producers
L/N
c_{bar}/y_{bar}
annual r = 0.03
annual gy=0.015
tau_f = 0.67
tau = 0.35
Annual growth rates
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
1950
2000
2050
2100
2150

Figure 4



Components of Wealth, US Population (US lifecycle)
Wealth relative to labor income
10
8
6
4
2
0
-2
-4
-6
-8
1950
2000
2050
2100
2150
5*W/Y
5*Wp/Y
5*A/Y
- 5*Tk/Y
annual r = 0.03
annual gy=0.015
tau_f = 0.67
tau = 0.35

Figure 5



Components of Wealth, Nigeria population (Taiwan lifecycle)
Wealth relative to labor income
10
8
6
4
2
0
-2
-4
-6
-8
-10
1950
2000
2050
2100
2150
5*W/Y
5*Wp/Y
5*A/Y
- 5*Tk/Y
annual r = 0.03
annual gy=0.015
tau$_f$ = 0.67
tau = 0.65

Figure 6

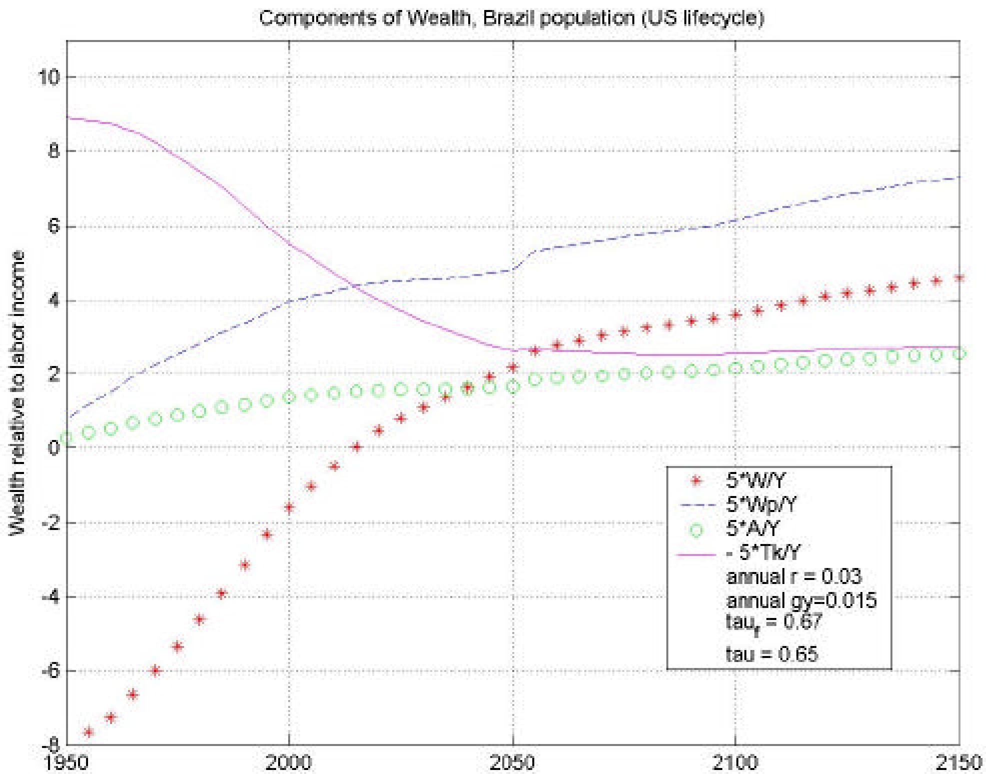
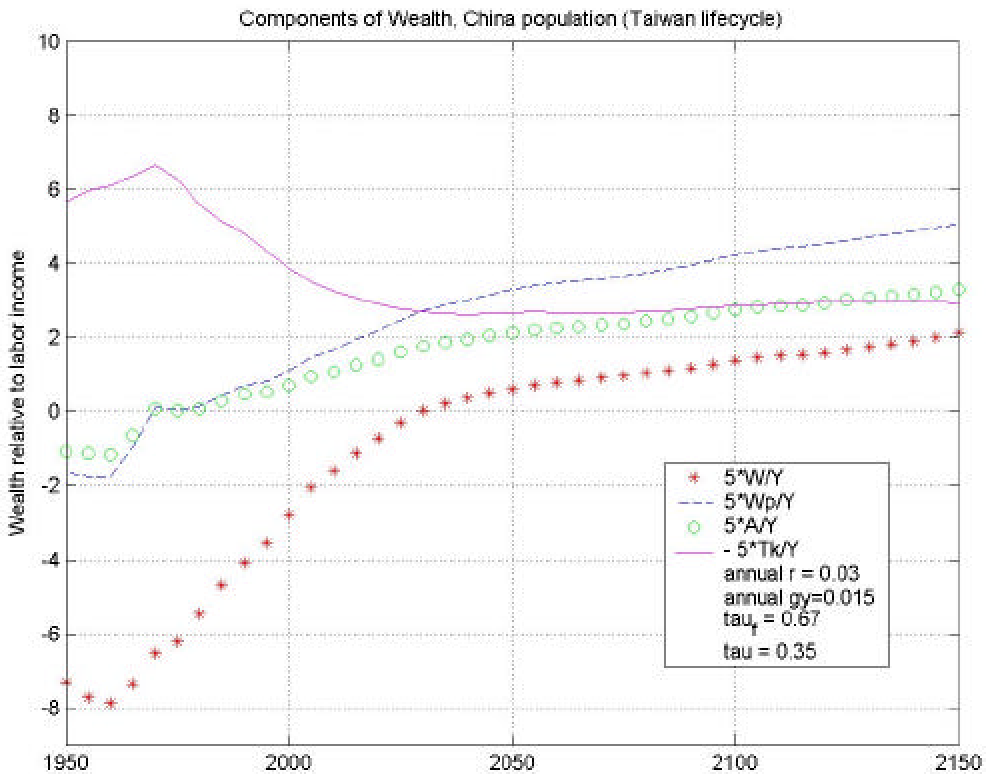

Components of Wealth, Brazil population (US lifecycle)
Wealth relative to labor income
10
8
6
4
2
0
-2
-4
-6
-8
1950
2000
2050
2100
2150
5*W/Y
5*Wp/Y
5*A/Y
- 5*Tk/Y
annual r = 0.03
annual gy=0.015
tau$_f$ = 0.67
tau = 0.65

Figure 7



Components of Wealth, China population (Taiwan lifecycle)
Wealth relative to labor income
10
8
6
4
2
0
-2
-4
-6
-8
1950
2000
2050
2100
2150
5*W/Y
5*Wp/Y
5*A/Y
- 5*Tk/Y
annual r = 0.03
annual gy=0.015
tau_f = 0.67
tau = 0.35