Pension design and longevity

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Abstract

This study compares alternative designs of an unfunded pension system. Convex combinations between a fixed contribution rate and a fixed benefit rate are considered. The objective is to maximize the expected *ex-ante* welfare under stochastic old age mortality. The aim is to find the optimal design of the unfunded pension system *behind the veil of ignorance*. The model is a computable overlapping generations model where the effects on factor prices, labor supply, and human capital are accounted for. Individuals decision to enter and exit the labor force is endogenous. Results show that it is important to be able to alter the retirement period in response to a longevity shock. When this is the case then there is no crucial difference between the different pension designs. If not able to alter the retirement period then the preferred design depends on the longevity distribution. In this case the design of the pension system will also have an impact on the re-scaling of the life cycle.

Key Words: Pension schemes, demography, social security, education, longevity.

JEL classification: J13, H55, H52

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1 Introduction

Unfunded pension systems are sensitive to changes in the dependency ratios. Given the ongoing shift in the demographic structure, particulary the increase in old age dependency, several countries have re-designed their pension systems. While still mainly unfunded, they now have a *fixed contribution rate* instead of a *fixed replacement rate*. Is a *fixed contribution rate* the preferred design when considering changes in dependency ratios?

Changes in old age dependency ratios can arise from three main sources: fertility, mortality, and migration. This paper deals solely with changes that arise from mortality fluctuations, in particular old age mortality. The reason for this is the dramatic increase in life expectancy throughout the world during the past century. Moreover, nowadays most years of life are gained at post-retirement ages in the industrialized countries (e.g. Lee and Tuljapurkar (1997)).

The aim of this paper is to find the unfunded pension design that yields the highest expected *ex ante* welfare when old age mortality is uncertain. Welfare is measured according to a standard utilitarian welfare function. This corresponds to finding the pension scheme that the individuals would choose *behind the veil of ignorance*, i.e. before knowing the realization of the old age mortality. Besides finding the preferred pension design I will also investigate how the pension design affects the re-scaling of the life cycle (e.g. Lee and Goldstein (2003)).

From the pension literature it is known that alternative unfunded pension schemes have very different distributional properties (e.g. Hassler and Lindbeck (1997), Thøgersen (1998), and Wagener (2003)). In a unfunded system changes in the old age dependency ratio alter the contributions from the workers, the benefits to the retirees, or both. Which of these alternatives that occurs depends on how the pension system is designed. In this paper I analyze convex combinations between a pure fixed contribution rate (FC scheme) and a pure *fixed benefit rate* (FB scheme).[1] In the pure FC scheme the workers always pay a certain fraction of their income to the system, irrespective of the dependency ratio. In a pure FB scheme retirees are guaranteed a certain fraction of current workers' income. Convex combinations between these two extremes imply that both workers and retirees are affected by changes in dependency ratios.

Previous studies that have investigated preferred pension designs, deal with either uncertainty in mortality, fertility, or factor prices. Studies that

[1] This convexity approach is similar to the one used in Wagener (2004). Note that a *fixed benefit rate* is not the same as a *fixed replacement rate*.

focus on fertility fluctuations in a closed economy setup are for instance Smith (1982), Blomquist and Wijkander (1994), and Bohn (2001), while for instance Thøgersen (1998) and Wagener (2003)) focus on factor price uncertainty in a small open economy setup. None of these studies find justification for the fixed contribution rate design.[2] The study by Žamac (2005) finds that the FC design could be motivated *ex ante* for a small open economy that has a proper design for the education system. The question is then if the FC design could be preferred when considering mortality fluctuations.

The studies by Bohn (2001) and Andersen (2005) investigate intergenerational risk-sharing for different pension designs under mortality uncertainty. The difference is that they do not account for human capital formation. The study by Echevarra and Iza (2005) accounts for human capital accumulation but does not compare different pension designs and does not account for the transition after a shock. This study accounts for the transition, and human capital accumulation but with numerical methods, in the spirit of Auerbach and Kotlikoff (1987).

The computable general equilibrium model consist of overlapping generations with perfect foresight that maximize their expected life-time utility. The individual problem consists of choosing the optimal amount of human and physical capital, and the optimal retirement time. When the individuals exit the labor force, i.e. retire, they will receive pension benefits according to the benefit formula which is determined by the pension design. The pension design once chosen *behind the veil of ignorance* remains unaltered. The optimal pension design maximizes the expected *ex ante* life-time utility for all generations, taking account of individuals behavior. The analysis is conducted for a small open economy.

One finding is that the ability for the old to adjust their retirement period is very important. If they are able to do this then there is little difference between the pension designs. Irrespective of the pension system the old will increase their working period proportionally more then they increase their retirement period. The increase in working length is such as to leave the worker retiree ratio almost unaffected. Further the difference in labor distortions induced by the pension designs is quite small.

If the old cannot adjust their labor supply then the preferred pension design will depend on the longevity distribution. If there is a higher probability for a positive longevity shock, compared to a negative one, then the fixed contribution rate is preferred. The re-scaling of the life cycle will also differ between the designs. A fixed benefit rate will lead to variable tax rate which

[2] Thøgersen (1998) finds motivation for the FC design but as Wagener (2003) shows this is not valid from an *ex ante* perspective.

in turn will affect the entry into and the exit from the labor force.

2 The model

The model depicts a small open economy with 3 overlapping generations. In every period t there is a new generation born, which will be called the t generation.The size of generation t increases over time:

$$N_t = n_t N_{t-1}, \tag{1}$$

where n_t is the gross population growth between period $t-1$ and t. Since the focus in this paper is mortality and not fertility it is assumed that $n_t = n\ \forall t$. Agents are homogenous within generations and their objective is to maximize life time utility. Besides the individuals there is an exogenous unfunded pension system, that can operate under different designs. The individuals know which pension system that is in place and they have perfect foresight. The only shock that can occur in the economy is shocks to mortality in final period of life. The first two periods of life have a fixed length normalized to unity while the length of the final stage of life is ε_t and can vary.

2.1 Individuals

Individuals live through three phases: the young phase, the working phase, and the old phase. They can work in all three phases, but during the working phases they devote all their time to work. During the young phase, the agents invest a fraction of their one unit time to human capital accumulation while the remaining time is spent on work. The fraction, e_t, that the young generation in period t devotes to human capital accumulation is chosen endogenously. The first fraction of young time, e_t, is thus spent on education which produces human capital according to:

$$h_{y,t} = \varphi e_t^{\sigma},$$

where $h_{y,t}$ is the human capital of generation t during the young phase, φ is a scale parameter and $\sigma \in (0,1]$ is a measure the elasticity of scale. The human capital accumulated when young determines the stock of human capital during the last two phases of life, according to:

$$h_{w,t+1} = \eta_w h_{y,t}, \tag{2}$$
$$h_{o,t+2} = \eta_o h_{y,t}, \tag{3}$$

where $h_{w,t+1}$ and $h_{o,t+2}$ is the human capital of generation t during working phase and old phase, respectively. The parameters η_w and η_o allow for varying efficiency at different stages of the life-cycle. Besides choosing how much time to invest in human capital accumulation the individuals also choose how much to save when young.

In the second stage of life the individuals combine all their one unit of time with their human capital to receive wage income. The only choice during this phase is to decide on the amount of savings. During the last phase, when old, the generation t has a time endowment of ε_{t+2}. During this phase they choose to work a fraction $z_{o,t+2}$, implying that $p_{t+2} \equiv \varepsilon_{t+2} - z_{o,t+2}$ is spent in retirement. After ε_{t+2} they die with zero assets holdings.

The objective of the individuals is to maximize their life-time utility. I assume an additively separable utility function:

$$U_t = u(c_{y,t}) + \beta u(c_{w,t+1}) + \beta^2 \varepsilon_{t+2} \left(u\left(\frac{c_{o,t+2}}{\varepsilon_{t+2}}\right) + \upsilon\left(\frac{p_{t+2}}{\varepsilon_{t+2}}\right)\right), \qquad (4)$$

where β is the subjective discount factor, and c_i where $i = \{y, w, o\}$ is consumption during the different phases. The period utility during young and working phase is solely based on consumption. It is assumed that disutility from work and education is equal and hence disregarded to simplify the exposition. The utility when old comes both from consumption and retirement, and is scaled by the period length. The utility from retirement is similar to the specification in Andersen (2005) and implies that the individuals value longer lives but that this at the same time creates consumption and retirement needs. Retirement is viewed as a consumption good. It is assumed that $\upsilon' > 0,\ \upsilon'' < 0,\ \lim_{p\to 0} \upsilon' = \pm\infty$, which ensures that the individual always choose some retirement before they die. This captures the fact that in most cases it will become increasingly difficult to work when approaching the time of death. A simple functional form that satisfies the conditions above is:

$$\upsilon\left(p/\varepsilon\right) = \kappa \ln\left(\frac{p}{\varepsilon}\right), \qquad (5)$$

where κ is a scale parameter and will determine the marginal rate of substitution between consumption and retirement in the last period. The utility from consumption is specified according to:

$$u\left(c\right) = \ln c. \qquad (6)$$

The objective of the individuals is to choose e, c_i, and p as to maximize U_t under the constraints:

$$c_{y,t} = \left(1 - \tau_t\right) z_{y,t} w_t h_{y,t} - s_{y,t}, \qquad (7)$$

$$
\begin{aligned}
c_{w,t+1} &= (1-\tau_{t+1})\, w_{t+1} h_{w,t+1} + R_{t+1} s_{y,t} - s_{w,t+1}, && (8)\\
c_{o,t+2} &= R_{t+2} s_{w,t+1} + (1-\tau_{t+2})\, w_{t+2} h_{o,t+2} z_{o,t+2} + b_{t+2} p_{t+2}, && (9)
\end{aligned}
$$

where τ_t is the tax in period t devoted to finance the pension system, $s_{y,t}$ and $s_{w,t+1}$ are the savings of generation t during young and working phase, w_t is the wage rate per efficient labor unit, R_t is the gross interest rate on savings between period $t-1$ and t, and b_t are the benefits per retirement unit from the pension system. Given the small open economy assumption we have that $w_t = w$ and $R_t = R \;\forall\; t$. The first order conditions with respect to consumption gives the intertemporal Euler conditions:

$$
\begin{aligned}
c_{w,t+1} &= \beta R c_{y,t}, && (10)\\
c_{o,t+2} &= \varepsilon_{t+2} \beta R c_{w,t+1}. && (11)
\end{aligned}
$$

The first order condition with respect to education can, after some rearrangement, be stated as:

$$
e_t = \frac{\sigma}{1+\sigma}\left(1 + \frac{(1-\tau_{t+1})\,\eta_w}{(1-\tau_t)\,R} + \frac{(1-\tau_{t+2})\,\eta_o}{(1-\tau_{t+1})\,R^2} z_{o,t+2}\right), \tag{12}
$$

and comes from the equalization between the marginal return on investment and the marginal cost (in terms of opportunity cost of forgone labor income). The final first order condition with respect to retirement period links the marginal rate of substitution between consumption and retirement to the marginal product of labor, according to:

$$
\frac{p_{t+2}}{c_{o,t+2}\kappa} = \frac{1}{(w h_{o,t+2}\,(1-\tau_{t+2}) - b_{t+2})}. \tag{13}
$$

The equations (10), (11), (12), and (13) characterize the solution,

2.2 Pension system

Up to now it has only been stated that the individuals will contribute to the pension system, via wage taxes, and that they will receive benefits when retired. It was not specified how large the benefits and contributions will be. The only restriction on the pension system that was made is that the individuals cannot effect the size of the taxes, τ, and benefits, b, by their actions. They will, however, affect the total contributions and the total benefits by altering the time spent on work and the time spent in retirement. To impose some restrictions on the taxes and the benefits the period-by-period balanced budget restriction will be used.

For the pension system to be truly unfunded it is is necessary that the budget is balanced in each period, which implies that it is possible to state the transfers in period t as:

$$b_t N_t^r = d_t N_t^w, \tag{14}$$

where d_t is the mean contribution per worker, while b_t is the benefit per retired. N_t^w and N_t^r is the number of workers and number of retired respectively, which can be stated according to:

$$N_t^w = N_t z_{y,t} + N_{t-1} + N_{t-2} z_{o,t}, \tag{15}$$
$$N_t^r = N_{t-2} p_t = N_{t-2} \left(\varepsilon_t - z_{o,t}\right). \tag{16}$$

Further let m_t denote the worker retiree ratio in period t, i.e.:

$$m_t = \frac{n^2 z_{y,t} + n + z_{o,t}}{\varepsilon_t - z_{o,t}}, \tag{17}$$

which implies that $b_t = d_t m_t$. This is the direct effect that changing old age dependency will have on the pension system. We see that when m_t varies either benefits per retired, contributions per workers, or both need to adjust. Here the benefits and contributions are not related to anything, which does not make any sense.

From above we already know that the mean contributions per worker must be related to income of the workers, since the contributions are collected via taxes on wages.[3] Let $\bar{w}_t$ be the mean wage of the work force, which means that:

$$d_t = \tau_t \bar{w}_t, \tag{18}$$

where

$$\bar{w}_t = w \frac{h_{1,t} z_{y,t} n^2 + h_{2,t} n + h_{3,t} z_{o,t}}{m_t \left(\varepsilon_t - z_{o,t}\right)}. \tag{19}$$

Now it remains to relate the benefits to something. It is clear that the benefits also should be related to the income, but it is not as obvious to what income. Should it be to the mean income of current workers, mean income of one's own income over the life-cycle, or perhaps the mean income during x years of the working period? All these three approaches are equivalent in steady state (incentive motives put aside). It is during disturbances that it matters which system that is in place. Since this paper focuses on

[3]Regarding the contributions there seems to be more or less consensus that these should be related to the mean wage of the work force. There are however proposals such as to finance pension system by consumption taxes and the like.

disturbances to the worker retiree ratio, caused by mortality fluctuations, I will choose the first approach and base the benefits on the mean income of current workers. By doing so it is possible to abstract from direct effects from changes in wages, and thus focus on the worker retiree ratio.

Relating the benefits to $\bar{w}_t$ according to:

$$b_t = \gamma_t \bar{w}_t, \tag{20}$$

makes it possible to rewrite the budget restriction on the following form:

$$\gamma_t = \tau_t m_t, \tag{21}$$

where γ_t represents the benefit rate. The benefit rate is the fraction of the current mean wage that is given to each retiree. In steady state when $m_t = m$ it is possible to have both γ and τ fixed. During a disturbance however, it will not be possible to have both fixed at the same time. When facing a shock there are thus two extreme ways that the system can adjust: either keeping $\tau_t = \tau$, or keeping $\gamma_t = \gamma$. The first extreme will be refereed to as *fixed contribution rate*, FC, while the latter will be referred to as *fixed benefit rate*, FB. The FB system implies that the retirees will not bear any direct risk from fluctuations in m. If on the other hand the system operates according to FC then the retirees bear the whole direct effect while the workers are entirely sheltered. There is, however, a possibility for an indirect effect on benefits and contributions through changes to the mean wage. Changes to the mean wage are thus always shared between retirees and workers.

With only the extreme cases it is not possible to make the workers and retires share the direct effect from changes in m. To allow for this it is possible to construct the following benefit formula:

$$b_t = \bar{w}_t \gamma \left(\phi + (1 - \phi) m_t / m \right), \tag{22}$$

where ϕ indicates the mix between FC and FB, and thus how the risk is shared between the workers and retires. When $\phi = 1$ we have a pure FB system and when $\phi = 0$ we have a pure FC system. Choosing the design for the pension system amounts to choose the value for ϕ, while γ indicates the size of the system, and m stand for the expected value for m_t.

2.3 The intergenerational welfare function

To obtain a compact measure of how all generations are affected by a mortality shock, welfare is defined as:

$$W = E\left[\sum_{t=1}^{\infty} \psi_t U_t\right], \tag{23}$$

This is a pure utilitarian welfare function, implying neutrality towards the inequality in the distribution of utility.

There are different views on how the per capita lifetime utility of generation t should be weighted. The question is if the utility should be weighted by the generation size, and whether the utility of future generations should be discounted. It seems more or less necessary to account for the generation size, otherwise there would be an unequal treatment of individuals belonging to generations of different size. A social discount rate will be included and the weighting factor will be the following:

$$\psi_t/\psi_{t-1} = \beta_s n_t, \tag{24}$$

where β_s is the social discount rate. In the simulation the social discount rate will be set equal to the individuals discount factor, i.e. $\beta_s = \beta$. The formulation allows for varying the social discounting as long as $\beta_s \in (0, 1/n]$. If there is population growth then the discount rate should not exceed the inverse of the population growth; if it does, then the future generations would get an ever increasing impact on the welfare function, due to their larger number.[4]

3 Simulation and calibration

What will be simulated are shocks to m_t. There will be an assumption underlying the process of m_t and also an assumption about the knowledge about this process. The objective is to maximize the intergenerational welfare function in eq. (23) by choosing the the design of the pension system, i.e. ϕ. This corresponds to finding the optimal pension design in the Rawlsian sense, *behind the veil of ignorance.* This is an *ex ante* analysis similar to the one applied in Ball and Mankiw (2001). To be able to do this one first needs to calibrate the model.

3.1 Demography

What is considered are different shocks to the mortality rate in the last period. First, it is however necessary to define the process for ε_t. There is no doubt that longevity is increasing in the real life. The question of interest is however not the predicted part of of the increase but the unpredicted. All pension designs are identical as long as the realization of the outcome falls

[4]See for instance Blanchet and Kessler (1991) and Boadway et al. (1991) for a short comment concerning the weighting problem.

within the expectations. It is thus possible to abstract from the trend in longevity and focus on the uncertain part. For this reason it will be assumed that $\varepsilon_t = \varepsilon$ in steady state, i.e. there is no growth in longevity. With respect to demographics what needs to be calibrated is thus ε and n.

Choosing ε will be done as to obtain reasonable lengths between the different stages of the life-cycle. The first two phases are normalized to unity and should be of equal size in number of years. The old phase has a different amount of years by the factor ε. Assuming that children under 16 years of age cannot choose labor over education, while the ones above can, gives that the young phase starts at age 16. The young phase and the working phase should be of equal size. The number of years in each of these phases is thus half the period between the age 16 and the age at which the old phase starts. The age when the old phase starts is marked by the fact that labor work is not the only activity. To find this age it is possible to use the labor participation rate (LPR) at different ages, this is presented in table (1). During no age is the LPR 100 percent, which makes it necessary to choose a threshold value for LPR which will be considered as full time work. I choose this threshold value to 85 percent. This makes the old phase start at the age 55, while the young and the working phase will correspond to 20 years. Note that this does not imply that the agents retire at 55 or that they spend their time in education until 35. What it implies is that individuals do not spend time on education or retirement between the ages 35 and 55.

To obtain the value for ε we also need to know how long people live. Life expectancy at 55 will be used as a proxy for the number of years in the old phase. Ideally one would want to base this on cohort data. However this is not as available and instead the period life-tables will be used. Using the 2001 period life-table yields that remaining number of years at the age 55 are 23 for men and 27 for women.[5] Since the calibration has been done according to the male labor participation rates, I will choose the male life table and use 23 as the remaining years at 55. This implies that the last period comprises of 23 years while the first two phases have 20 years. Normalizing the first two to unity implies that $\varepsilon = 1.15$.

What remains to determine with respect to demographics is the growth of each new generation, n. The steady state gross population growth, n, will be set to 1.22, based on the annual average for the U.S. between 1910-2001.[6]

[5] See: Annual Statistical Supplement, 2004, to the Social Security Bulletin.

[6] The annual average, from National Vital Statistics Reports 51, no. 2, is approximately 1.01. This implies that per period $n = 1.01^{20}$, since one period corresponds to 20 years.

Table 1: Male labor participation rates at different ages in 2005 for the U.S.

Age	16-19	20-24	25-29	30-34	35-39	40-44	45-49
LPR	43	79	91	93	93	92	90
Age	50-54	55-59	60-64	65-70	70-74	75 +	
LPR	86	78	58	34	21	9	

Source: Bureau of labor statistics, Current population survey, Annual averages - Household data, 2005.

3.2 Preferences, wages, and the interest rate

Regarding preferences, β is the standard measure of the individual's impatience to consume. Using the one year estimate from Auerbach and Kotlikoff (1987) of 0.98 translates to $\beta = 0.7$, since the normalized period length represents 20 years. The parameter κ will be chosen so that share of retirement in steady state is half the time of the old phase, i.e. $z_o = 0.5\varepsilon$. This corresponds to the working share of the 55+ in table (1). This implies that the model retirement age is 66.5. This is somewhat higher then average real life retirement age, which is 63, but still within reason. What is important is that the resulting worker retiree ratio is reasonable.

Regarding the wage and the interest rate these will be set as to equalize the autarky prices with the world market prices in steady state. To obtain the autarky prices a standard Cob-Douglas production function is used with efficient labor and capital as factor inputs.[7]

3.3 The benefit rate and human capital

Choosing the size of the pension system amounts to calibrating the benefit rate in steady state, γ. According to the Social Security Office of the Chief Actuary the current benefit ratio, i.e.benefit to the average wage ratio in the same period, is 0.42. I will use the same value and set $\gamma = 0.42$.

Regarding the human capital process lets start by calibrating the relative efficiency during the three phases, i.e. η_w and η_o. Using the same efficiency profile over the life-cycle as in Auerbach and Kotlikoff (1987) leads to $\eta_w = 1.17$ and $\eta_o = 0.89$. Which means that the individuals are 17 percent more efficient during the working phase compared to the young phase, while they are 11 percent less efficient during the old phase compered to the young

[7] The exact specification and calibration can be found in Žamac (2005).

phase. The scale parameter φ will not affect the outcome and will be set to unity.

The rate of return on human capital, is determined by σ. Unfortunately it is very hard to get an accurate measure for σ. Card and Krueger (1992) investigate how the pupil teacher ratio affects future productivity. Translating their results, via assumptions on how the spending per pupil is related to the pupil teacher ratio, would yield $\sigma = 0.165$. The other approach how σ could be determined is by the use of table (1). Assuming that most of the individuals that do not work between the ages 16 and 35 spend time on education makes it possible get an estimate of z_y.[8] This value for z_y corresponds to 0.77, and implies that the model age at which individuals enter the labor market is 20.6. If the education length is fixed (as in this case, $e = 0.23$) then it is possible to back out the value for σ according to equation (12). Doing so leads to $\sigma = 0.17$, which is very close to the implied estimate by Card and Krueger (1992). That two vary different approaches of calibrating σ are almost identical is quite remarkable.

Table 2: Calibrated values for the exogenous parameters.

Parameter		Value
Time preference	β	0.7
Efficiency during working phase	η_w	1.17
Efficiency during old phase	η_o	0.89
Elasticity of scale in hum. cap. prod.	σ	0.17
Steady state benefit rate	γ	0.42
Population gross growth rate	n	1.22
$\text{MRS}_{c,p}$ parameter	κ	0.12
Longevity in steady state	ε	1.15

4 Results

To see how this stylized model performs it is possible to compare some variables in steady state with data. The steady state results, according calibration in table 3.3, are presented in table 4.

The model was calibrated as to obtain the first two values, regarding e and z_o. This was done by adjusting the free parameters κ and σ. More interesting

[8] What is meant by most individuals, is the excess out of labor force share during the young phase compared to the working phase.

Table 3: Steady state based according to calibration in table 3.3.

Working share of in old phase	z_o/ε	0.5
Education share of young phase	e	0.23
Worker retiree ratio	m	5.12
Pension tax	τ	0.08
Gross interest rate	R	2.41

is that the resulting worker retiree ratio is reasonable. The comparable value according to data in table 1 is 5.5. The resulting pension tax is 8 percent which is lower then the comparable OASDI pay-roll tax, which is 12.4 percent. The difference comes from the fact that I have calibrated the model to male data, which gives a higher worker retiree ratio compared to the ratio used for the OASDI pay-roll tax. Looking at the gross interest rate it might seem high but when adjusting for the models period length it is reasonable, since it implies a yearly rate of 4.5 percent.

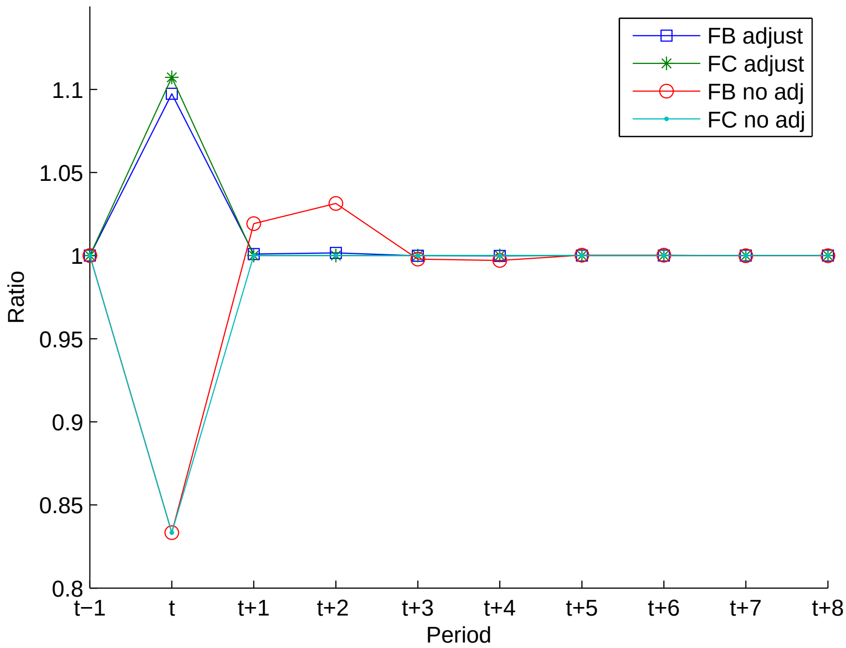
Considering that it is a highly stylized model, it seems as if the comparable variables match data.

4.1 Extreme cases

Before presenting the optimal value for ϕ, i.e. the pension design, the extreme cases are compared. They will be compared under the experiment that there is a positive longevity shock in one period. The experiment is the following $\varepsilon_{t+i} = \varepsilon \ \forall \ i \neq 0$ and $\varepsilon_t > \varepsilon$. This will give some insight when assessing the importance of the design and also some understanding of what effects that are at work.

It will important to distinguish between how quick individuals might react to a shock, or put differently, how long head notice do we have about the shock. To make this distinction two cases are considered. One, is to allow all individuals in period t to re-optimize fully when facing a longevity shock in period t. This case will be referred to as full adjustment. The other alternative is not to allow the old individuals in period t to alter their working length in response to the shock in period t. This corresponds to a scenario where the shock about time of death is not revealed until after the working period. This case will be referred to as no adjustment.

Figure 1: Working share of old phase time relative to steady state.



4.1.1 Labor length in last period

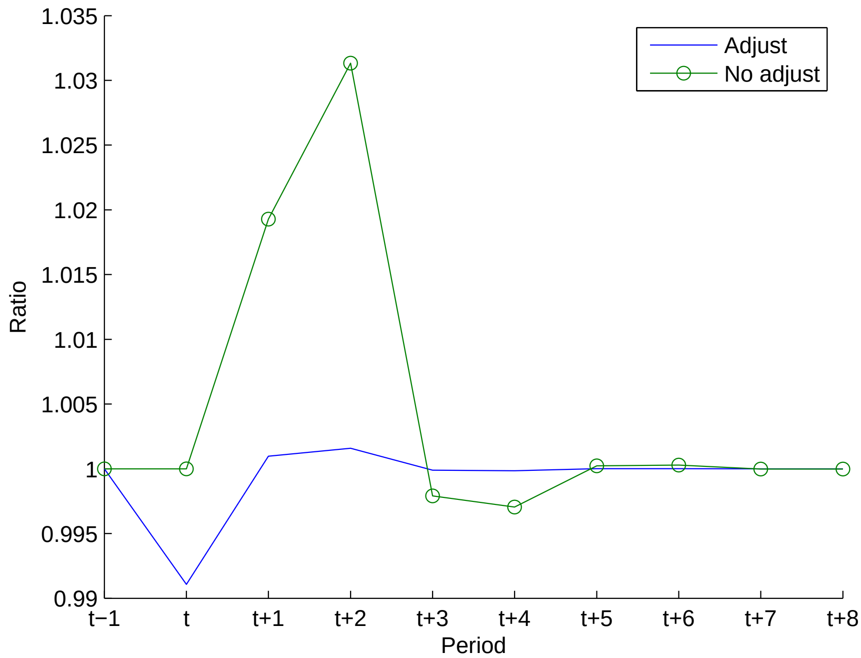
To understand how many of the other variables evolve lets first see how the old divide the longevity increase between retirement and work. Figure 1 presents the work share during old phase relative to steady state for the pure FB and the pure FC design under the two cases considered.

We see that it matters if there is prior notice about the shock or not. If the old cannot adjust then the entire increase in longevity will be in terms of retirement. In this case it is obvious that the pension design will not affect the old labor supply in the same period, but it might affect the labor supply later. The retirement length will increase by 40 percent while the working length will remain constant, when longevity increases by 20 percent.

If the old can adjust then we see that they increase their labor supply relatively more then they increase their retirement length. The work length increases by 32 (FB) and 33 (FC) percent while the retirement length increases by 8 (FB) and 7 (FC) percent. The design has a small impact the increase in labor. When the pension system is FB, the taxes increase which creates incentive to earlier retirement.

The difference between the designs is more clear from figure 2. This figure presents the ratio between the FB outcome over the FC outcome for the two cases. The largest difference between the cases is that the generation that

Figure 2: Working share of old phase time, ratio between FB outcome over the FC outcome, with and without adjustment.

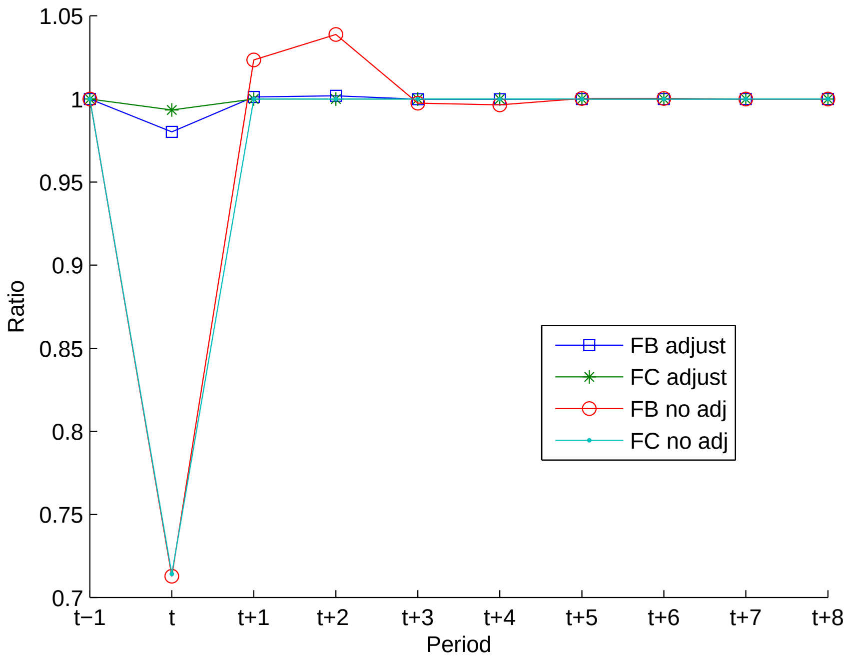


is young during the shock will work longer during the old phase. They do this since the tax increase makes them to invest more in human capital and thus they expand the working life in the other end to reap the return. The generation in working phase during the shock will also increase their working length during old phase under the FB design. This happens since the tax increase leads to less savings then predicted and to compensate for this they work longer during the old phase.

4.1.2 Worker retiree ratio

The worker retiree ratio will always go down, but the magnitude differs a lot. From a negligible decrease when the old can adjust to almost 30 percent reduction when there is no adjustment. This can be seen from figure 3, where the worker retiree ratio is presented as deviation from steady state. Once again we see that it is much more important if the old generation can adjust or not, then which pension design that is in place.

Figure 3: Worker retiree ratio, relative to steady state.



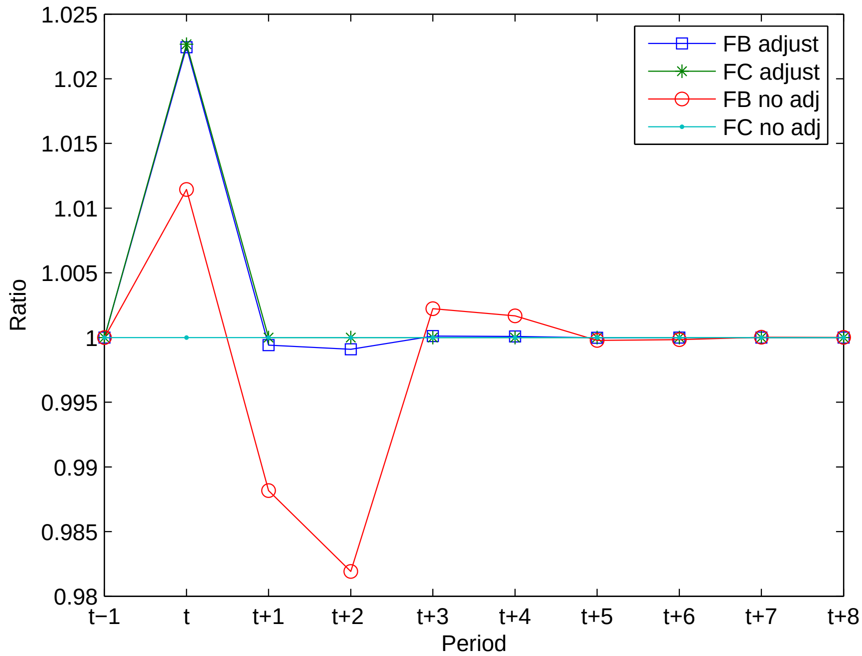
Comparing the pension designs with the FB outcome over the FC outcome, with and without adjustment, results in a very similar graph to figure 2. Once again the largest difference occurs for the generation that is young during the shock.

4.1.3 Life time consumption and utility

How life time consumption for different generations evolves after the shock is presented in figure 4. In the full adjustment case there is negligible difference between the different pension designs. When there is limited adjustment then the FC design leaves the total consumption unaltered. The retired during the shock get less pension benefits but get it during a longer time. If the FB design is in place then the retired get the same amount of benefit but during a longer period.[9] Even though the consumption stays unaltered in this case the utility of the old increases, since they enjoy longer lives.

[9]The benefit actually increase slightly due to increased human capital of the work force which increases the mean wage of the work force. Average human capital increases since

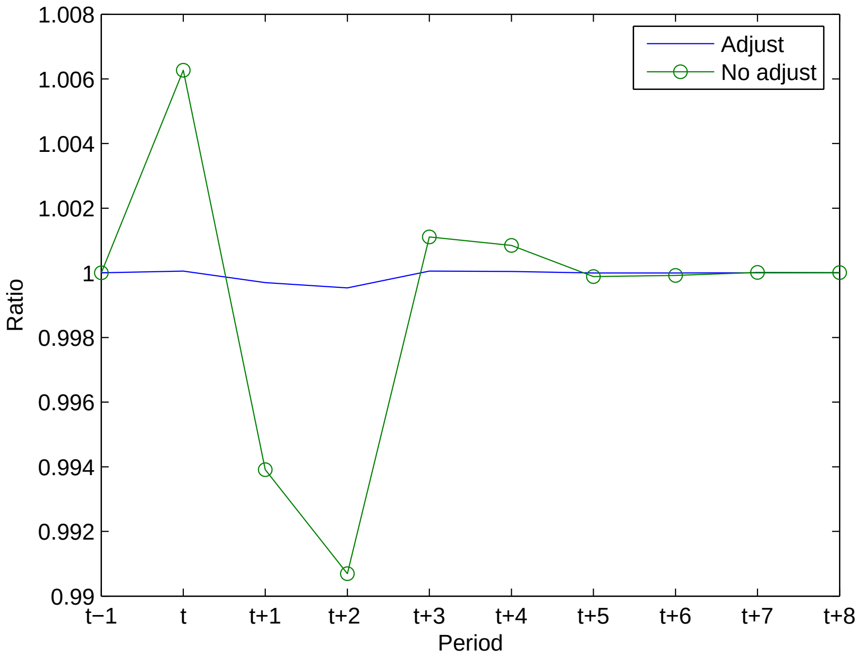
Figure 4: Life time consumption relative to steady state.



How life time utility differs between the pension designs is presented in figure 5. Here the ratio between the life time utility for the FB design over the life time utility for the FC design is presented. Once again the difference between full and limited adjustment is important. With full adjustment the design of the pension system does not matter. With limited adjustment the generation that is old during the longevity shock prefers the FB design while the following generations prefer the FC design. The generations prefer opposite designs since it is a matter of redistribution between them. This is, however, their preferred design *ex-post* when the uncertainty has been revealed. The shock could well have been negative instead of positive in which case the old would prefer the FC design while the young and working generations would prefer the FB design. The question is what design is the preferred *ex-ante*. This is analyzed in the next section.

less of the young are in the work force, and they have a lower human capital stock then those in the working phase.

Figure 5: Ratio between FB design over the FC design for life time utility, with and without adjustment.



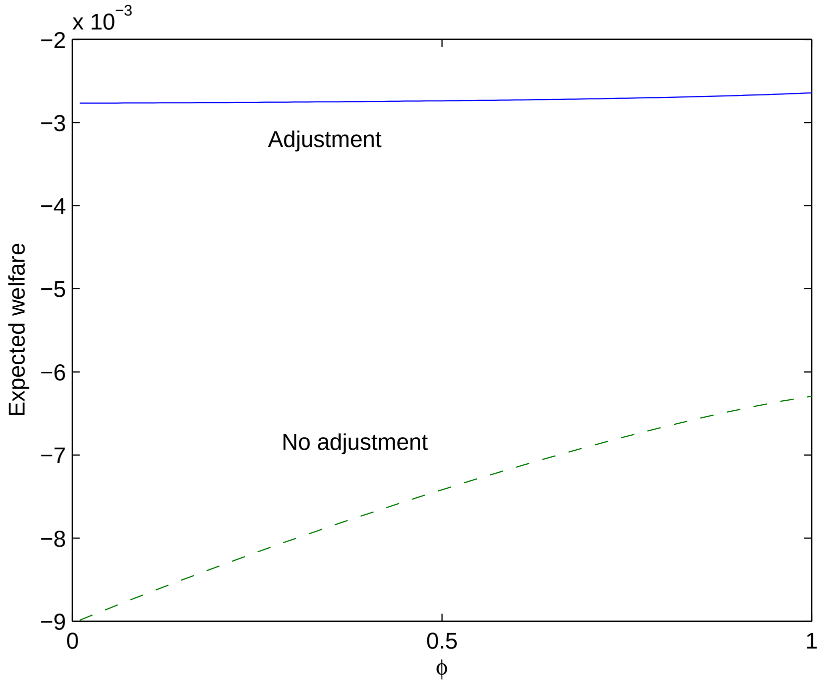
4.2 The *ex-ante* approach

The preferred *ex-ante* design will be identified by evaluating the welfare function specified in equation (23). The idea is that the generations have to decide on a pension design before being born, or at least before they know the longevity that will affect them. This corresponds to the choice *behind the veil of ignorance* a'la Rawls (1971). Once chosen, the pension design will be unaltered and it will respond to any longevity shock in the pre-determined manner according to the benefit formula.

For the generations to be able to decide they need to have some knowledge about the involved risk at hand. It will be assumed that they know the underlying distribution for the stochastic process. In general it is possible to analyze any stochastic process. Here I will choose a very simple process. There is only uncertainty about the longevity in period t. The longevity in period t will either be smaller or higher, by equal magnitude, and there is equal probability for both outcomes. Stated differently, the longevity sequence analyzed is: $\varepsilon_{t+j} = \varepsilon \; \forall j \neq 0$ and $\varepsilon_t = \varepsilon(1+x)$, where $x = \{-0.2, 0.2\}$ with equal probability. Here the disturbance is set to 20 percent but this could be changed without affecting the results.

Evaluating the welfare function at different designs for the two cases yields results according to figure 6. The preferred design is the pure FB design, i.e. $\phi = 1$. It also emerges that the difference between the designs when the old can adjust is small.

Figure 6: Expected welfare with and without adjustment.



The utilitarian welfare function implies a total willingness to substitute between generations utilities. Looking at the generation that is old during the shock we know that they prefer the FB design for a positive shock and the FC design for a negative shock. Since they are risk-averse this implies that they prefer the FC design from an *ex ante* perspective when there is equal probability and magnitude for positive and negative shocks. The overall welfare function ranked however the FB design over the FC design. This means that the gain in expected utility that the subsequent generations obtain from having a FB design is larger then the loss for the old generation during the shock.

If there only was a positive shock then the welfare function would rank the FC design highest while the opposite is true for a negative shock. When there was equal probability the FB design was ranked highest. This implies that increasing the probability for a positive shock eventually leads to the

FC design being ranked highest. The probability for a positive shock where this switch occurs is presented in table 4.

Table 4: Range for the probability for a positive shock where the FC design is preferred.

	No adjustment	Adjustment
Probability	0.59 - 1	0.54 - 1

If the probability for a positive shock is 0.59 or above then the FC design is ranked highest, irrespective if the old can or cannot adjust. Assessing the distribution for longevity is thus crucial when choosing pension design.

5 Conclusion

The preferred pension design with respect to mortality fluctuations is highly dependent on the old age mortality distribution. It is further important how well the old can adjust to the shock. Do assess the distribution of the old age mortality and how far into the future it is predictable is is thus of great concern.

If there is a somewhat higher probability for a positive shock than a negative shock then the a fixed contribution rate is the preferred design. If however the probability of a negative shock is almost as likely, or higher, then the FB design is preferred. This holds irrespective if the old can adjust to the shock or not. If the old can adjust then the difference between the designs is very small.

The re-scaling of the life-cycle is affected by the pension design. When the old can adjust then the fixed benefit rate leads to somewhat smaller labor supply, due to faster exit from the labor force, in the first period; the difference is, however, quite small. Irrespective of pension design the increase in labor supply is substantially higher then the increase in retirement length when the old can adjust. The increase in working length is so high as to keep the worker retiree ratio almost unaltered. Previous studies that have not accounted for the labor distortion effect off different systems can thus be viewed as a good approximations. Further, since the labor distortion effect is negligible the preferred design will for the most parts be on the extremes.

The largest difference between the pension designs impact on the labor supply occurs when the old cannot adjust. In this case the largest difference between designs occur a couple of periods after the shock. A longevity increase under the FB design leads to a tax increase which forces the coming generations to prolong their working length. The tax increase leads also to prolonged education and thus a delayed entry into the labor force. Accounting for human capital formation is thus important when trying to analyze how the economy will evolve after a mortality disturbance.

Regarding the recent shifts towards fixed contribution systems in some countries only one firm conclusion can be drawn at this stage. Introducing designs that allow for adjusting the retirement period is much more important then sole changes towards a fixed contribution rate. When the retirement period can be adjusted then the choice between a fixed contribution rate and a fixed benefit rate is not that crucial. The shift towards the fixed contribution rate is however not bad *per se*, especially if one attaches higher probability for positive longevity shocks. Further, considering previous results about the preferred design with respect to fertility fluctuations it seems as if there is a case for a fixed contribution rate in a small open economy especially when combined with the right design for the education system (see Žamac (2005).

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