Degree Distributions of Sexual Networks:
Evaluating Social Process?

Deven Hamilton MPH
University of Washington
Department of Sociology

Mark Handcock PhD
University of Washington
Department of Sociology
Department of Statistics

Introduction

The emergence of HIV and the reemergence of other STI over the last 30 years have forced researchers and scientists to look for new ways of conceptualizing disease transmission dynamics as well as intervention strategies. Early work in this area resulted in the development of core group theory. The fundamental principle of core group theory is that a small group of people with large numbers of partners and repeated infections can maintain endemic infection within a population even when the overall reproduction rate within the population is below 1 (Hethcote and Yorke 1984) (Thomas 1996). Such insights directed both policy makers and health care providers to focus on identifying and treating members of core groups, a strategy that can successfully curtail disease spread without the cost and coordination challenges of broad population based interventions.

HIV and other incurable STI are not effectively conceptualized within the core group framework because there is no subgroup wherein re-infection is common. Prevention and treatment strategies predicated on the core group framework may therefore be ill-suited to meet the challenges posed by incurable STI. New alternative methods have therefore been developed. Network analysis in particular shows great promise for understanding both the dynamics of transmission and the consequences of intervention strategies (Morris 2004). However, some of the early work on sexual networks has been somewhat misleading and potentially detrimental to the development of effective health care policy. Here we re-evaluate some of the early claims about the structure of sexual networks. In particular we focus on the degree distributions which have been characterized as scale free. To determine the best course of action it is essential that we develop a more complete understanding of the dynamics of transmission.

An often used approach to developing our understanding of the affect of network structure on transmission or diffusion is to focus on key general characteristics of a network and try to interpret the consequences of network structure based on theoretical arguments regarding these individual features rather than focus on more complex features of empirical networks like nodal attributes, or the processes that generate the network structure. One such focal feature is the degree distribution. In a sexual network the degree distribution is the number of sexual partners each member of the network has.

This relatively simple attribute of the network has been identified as a potential key concept for understanding disease transmission across sexual networks (Liljeros et al. 2001; Schneeberger et al. 2004). In particular the degree distribution has been characterized as "scale free" (Schneeberger et al. 2004) (Amaral et al. 2000) (Liljeros et al. 2001). Scale free distributions are a set of distributions which follow a power law and are characterized by infinite variance. Power law models have the general form $P(k) \propto k^{-\alpha}$.

The relevant characteristic of these models for understanding disease transmission is the scaling parameter (α) which is the slope of the line described by the model when it is represented on a log-log scale. If this parameter is between 2 and 3 the variance of the degree distribution is infinite. When the degree distribution has infinite variance the level of infectivity requisite for a disease to generate an epidemic approaches 0. Concordantly, the critical vaccination fraction under such circumstances is unity – all members of the population must be successfully vaccinated to eradicate the disease. Under these conditions efforts to reduce transmissibility such as condom distribution and use promotion, mass media campaigns and antiretroviral therapy will fail to curtail disease spread to a sufficient degree to ultimately eradicate the disease. Only selective interventions directed at high degree "hub" nodes will effectively restore the epidemic threshold and curtail the diffusion process (Denzo and Barabasi 2002). Given the public health implications of infinite variance in the degree distribution of sex partners, it may be premature to classify these distributions as scale free.

Attempts to classify sexual network degree distribution have not been definitive in determining there form but the empirical evidence shows that degree distributions for sexual partnership networks are highly right skewed (Jones 2002). For partners over a 12 month period the modal degree is $k = 1$ for nearly all large representative surveys (e.g (Laumann et al. 1994) (Lewin 1996) (Hubert, Bajos and Sandfort 1998) (Aral 1999) (Youm 1999) (Davis, Smith and Peter V. Marsden 2003) (Tanfer 1993) (Tanfer 1995)). In the 5 sets of survey data used for these analysis (described later) 63.9% to 76.7% of respondents report $k=1$ partners. The histograms of the degree distributions are shown in figure 1a and 1b. The extreme right skew of sexual network degree distributions has been likened to that of a variety of physical systems suggesting the possibility of power-law scaling (Lloyd and May 2001).

The similarities between the degree distributions of physical systems and the degree distributions of sexual networks has led several researchers to apply power law models to a multitude of populations including Rakai, Sweden, the United States, England, and Zimbabwe (Schneeberger et al. 2004) (Handcock and Jones 2003) (Amaral et al. 2000) (Liljeros et al. 2001). Schneeberger et al reported the fit of power law models to the degree distribution of heterosexual males in Britain in 1990 and 2000. In both years the scaling parameter was between 2 and 3 and the reported 95% confidence interval was also within those bounds. Handcock and Jones also reported that the scaling parameter of a power-law model fit to the degree distribution of heterosexual males in the United States was between 2 and 3. These finding would suggest that these population have a high potential for rapid epidemic spread, but the empirical evidence does not indicate that a strong epidemic exists in these populations.

In population that do show evidence of a sever epidemic such as the heterosexual population of Rakai (Handcock and Jones 2003)and men who have sex with men in Britain (Schneeberger et al. 2004)the scaling parameters were shown to be well outside the range of 2-3. These counter intuitive findings indicate that power law models may not accurately reflect the social processes generating the observed distribution. The contradictory conclusions of these investigations serve as the motivation for the present investigation.

It is our contention that the methods previously employed in the evaluation of sexual network degree distributions have been inadequate in three ways. First, the amount of error in the tail of the distribution has been underestimated. Second, social process has been bypassed in favor of simple curve fitting despite the existence of myriad alternative social process models. Finally, random mixing has been assumed despite overwhelming evidence that there is a high level of heterogeneity in the partner selection process. As a consequence of these errors and omissions some have drawn the erroneous conclusion that power law models adequately fit the degree distributions of empirical sexual networks.

When using power law models to make inferences about network structure the scaling parameter ρ of a power-law model is typically estimated by fitting a regression line through the apparently linear region of a plot of the survival function of the degree distribution plotted against the distribution on double logarithmic axes (Amaral et al. 2000; Liljeros et al. 2001) and the standard error of the estimated slope is the estimate of uncertainty in the model. There is an obvious appeal to such models as they are mathematically simplistic and well described in the physics literature. There is however, an implicit assumption that uncertainty is not correlated with degree. The skewed distribution and the manor by which respondents report sexual network data suggest that this assumption is not met. Even for very large surveys the number of observations in the tail of the distribution is very small. In the data sets we are analyzing > 3 partners were reported by 4.4% to 8.6% of respondents and > 10 partners reported by 0.2% to 1% with sample sizes ranging from 3,282 to 44391. The information contained in the tail of a degree distribution is therefore minimal and as a consequence the precision of inferences on the tail is extremely low. As the number of partners increases beyond 10 there is also a tendency for respondents to report in round numbers (Huttenlocher, Hedges and Bradburn 1990) (Morris 1993b) and as a consequence the precision of the model approximation becomes more dependent on less and less accurate data as it fits the higher degree values. This methodology is therefore, inappropriate for the inference problem, yielding biased estimates of the scaling parameter, and greatly underestimates model uncertainty (Jones 2002).

More recently, attempts have been made to fit power law models to specific portions of the degree distribution or portions of particular populations (Schneeberger et al. 2004). This too brings into question the validity of any inferences made with regard to the existence of an epidemic threshold or the properties of sexual networks more generally. Fitting models to the tails of the distribution, those components with >10 or >15 partners, disregards the majority of the data where the statistical power lies and places undue reliance on the least reliable data. We use three different cutoff points in these analyses which preserve progressively less information from the available data to show how model fit changes as we ignore more of the left side of the distribution. The cutoffs we employ are 1,2 and 3. All three cutoffs are at low number of partners compared to the highest numbers of reported partners but they are very high in terms of the amount of data they exclude. A cutoffs as low as four partners ignores more than 95% of the data from most surveys of sexual behavior.

The duration over which we measure partnership number must also be considered before drawing conclusions about the epidemic potential of a network. The degree distributions of individuals over their lifetimes suggest a network that is far more dense than the sexual network that might exist at any given moment. The network of interest is the network over which a disease must travel in order to create an epidemic and it is likely that the relevant network is better approximated by more restricted time frames, but even truncated approximations to a sexual network ignore the dynamic nature of sexual networks. The degree distribution of a sexual network is a consequence of the social processes that drive partner selection. The behavior of the individual actors that generates the distribution is dynamic and to consider such a network from a cross-sectional perspective is misleading. At any time point it is probably best thought of as representing a dynamic equilibrium of underlying social and epidemiological processes, rather than simply a static pattern of sexual behavior (Kendall 1961). Given the limits of data collection, static networks derived from survey data representing relatively short time frames may however, function sufficiently as a proxy for network a disease may traverse.

Because behavior of individuals is the force that generates the degree distribution the stochastic processes that generate the observed data should not be ignored in favor of mathematical distributions with weak proximate mechanisms. There are several plausible stochastic process models that should be considered. The equilibria of stochastic process models of network formation can be fit to empirical data using likelihood techniques, allowing both the estimation of parameters and the assessment of goodness-of-fit to the data.

We use maximum likelihood estimation to estimate the model parameters of 11 different models and compare their relative fit to the empirical data. Maximum likelihood estimation has been regarded as optimal based on asymptotic arguments (Casella and Berger 2002).

Like any sample from a population, the samples obtained in the sexual surveys we analyze here are imperfect representations of their populations. Errors accrue due to sampling frame misspecification, informant misreport and non-response (Rubin 1987) (Morris 1993a) (Thompson 1992). Likelihood methods enjoy the tremendous advantage that the sampling design is "ignorable" for many standard (and nonstandard) designs under the likelihood framework (Thomas 1996). That is, the likelihood only depends on data in the sample, and not on unknown missing data.

In this paper we use 5 different surveys of the US population to test the fit of power law models to the degree distribution of sexual networks. By using multiple surveys of the same population we can determine if the goodness of fit of the power law models relative to alternative models is robust to subtle changes in the survey sample. We can also determine if the scaling parameters are robust to these same changes.

In addition to testing model fit across surveys we also test the consistency of model fits within survey. One of the surveys we use has three different measures of sexual partners in the last year.

The inclusion of several alternative models also allows us to gain leverage on the question of social process. If one model consistently fits the empirical distribution better than the others, we may glean some information about the process that drives partner acquisition. If, on the other hand, the best fit model is different in every case and appears to be almost a random process, the indication would be that the partnership acquisition process is far too complex for a relatively simple model to capture and that reported "fits" of individual models are most likely idiosyncratic and not particularly informative.

Stochastic Models for Network Formation

We test the fit of 11 different models to five sets of data collected in the United States. The models we fit are drawn largely from (Handcock and Jones 2003)but also include the log-normal model described by (Perline 2005). The models evaluated are the

Yule, Waring, Discrete Pareto, Poisson, Negative Binomial, Geometric, Discrete Pareto Exponential, Geometric Yule, Negative Binomial Yule, Non-Zero Negative Binomial and the Poisson Log normal. Briefly, these models fall into three general classes of models: (1) Non-homogenous Poisson, (2) preferential attachment, and (3) "vetting" models.

Briefly, Poisson models consider the population of individuals with at least one partner in a given time period. The number of additional partners K -1 that the person has in the time period follows a Poisson distribution with expected value λ. There are many proximate mechanisms for this (for example, the partners are accumulated at a constant rate in time). But, because the assumption that all individuals have identical propensities to form partnerships is unrealistic (individuals differ by gender, age, marital status, attractiveness, and other fundamental characteristics that greatly influence partnership formation) we include heterogeneity in individual propensities. To model within-population heterogeneity, we can represent the individual expected values λ as independent draws from a distribution P(λ).

In preferential attachment models, the probability that a contact is made with any particular individual is a function of that individual's current degree. Two models for preferential attachment are (1) the Yule distribution (Simon 1955) (Jones 2002) and (CDC 2001) the Waring distribution (Irwin 1963). These are often referred to as "the rich get richer" models.

In vetting models people form sexual partnerships based on a two-stage process. First, they generate an acquaintance list that serves as the eligible population. People then choose their sexual partners from the acquaintance list. This class of model is extremely flexible in that practically any probability distribution can be specified for both of these processes. This process focuses attention on the stopping rules that people employ when forming sexual partnerships. The Yule-vetting models are generalizations of the Yule distribution that recognize that the formation of sexual partnerships is not costless.

This vetting process may represent social networking, geographic, temporal or other processes. As before, the assumption that all individuals have identical propensities to form sex partnerships is unrealistic due to the individual characteristics such as gender, age, marital status, attractiveness. We model this within-population heterogeneity, by independently drawing the individual expected values from a distribution $P(\lambda)$. The acquaintance list distribution can be modeled as Yule, Waring, or negative binomial.

The vetting process can also be interpreted as a selection process designed to satisfy multiple criteria. Individuals may choose partners from their acquaintances that independently satisfy some criterion with a fixed probability. The accumulation process continues until they meet a quota of people that satisfy the criterion. An example of this would be people looking for a number of long-term partners among many casual partners. Many other interpretations are possible and interesting within this context.

The discrete Pareto distribution lacks a plausible stochastic mechanism for network formation, which limits its appeal as a model of sexual contact networks. While this lack of a motivating stochastic mechanism constrains the utility of the discrete Pareto distribution as a model for sexual networks, it can nonetheless be used as an acquaintance-list generator in a vetting model.

For a more in depth description of the models see (Handcock and Jones 2003) and (Perline 2005).

Data

In this analysis we use five different probability samples collected in the United States during the 1990's including the National Health and Social Life Survey (NHSLS), the General Social Survey (GSS), the National Survey of Men (NSM) and the National survey of Women (NSW) and the Behavioral Risk Surveillance Survey. A quick overview of the data is provided in table 1.

The GSS is conducted by the National Opinion Research Center (NORC) and was designed as part of a program of social indicator research to gather repeated measures on a broad range of data. The GSS uses the NORC national probability sample, which includes all non-institutionalized English-speaking persons eighteen years of age or older living in the United states. The samples are designed to give each household in the United States an equal probability of inclusion. Respondents report estimates of sexual partners via SAQ. The GSS asked questions about the number of sexual partners in the last year in 1988-1991, 1993, 1994, 1996, 1998 and 2000. There are a total of 16,159 respondents who reported the number of sexual partners in the last year (Davis et al. 2003). GSS response codes for the number of sex partners in the last year are categorical and topcoded (1, 2, 3, 4, 5-10, 11-21, 21-100, 100+).

The NHSLS is a survey conducted in 1992 by NORC. It was designed to be a comprehensive survey of the sexual behavior of adults 18-59 in the United States (Laumann et al. 1995). Respondents were selected using a multistage area probability sample designed to give each household an equal probability of inclusion. A cross sectional sample of 3,159 respondents was collected as well as an over-sample of 273 black and Hispanic respondents. The study used FTFI as well as SAQ to collect data on sexual experiences. The number of partners in the last year is asked multiple times in the survey using both modes of data collection. Laumann et al. have constructed a categorical aggregate measure for partners in the last year that is based on both the FTFI and SAQ data. The three different distributions that are derived from the three variables allow us to determine if the fit of power law models or alternative social process models are robust to small changes in reporting even within the same survey.

The NSM was designed to examine sexual behavior and condom use among men. The study population consisted of 20-39 year old non-institutionalized men. The sample was based on a multi-stage, stratified, clustered, disproportionate-area probability sample of households within the contiguous United States and included an over-sample of Blacks. The data were collected in 1991 using FTFI (Tanfer 1993). Respondents are asked to report the number of vaginal sex partners and anal sex partners separately. There is no way to ascertain how many partners are represented in both categories, so we define the number of partners as the maximum of the two categories, which may be lower than the actual number of unique partners. A total of 586 (19%) of the men reported anal sex. Of these, 18 report no vaginal sex partners, and 35 report more anal than vaginal sex partners.

The NSW was also conducted in 1991 and was designed to examine sexual, contraceptive, and fertility behaviors, and the factors associated with these behaviors. The sample includes 1669 cases from two sub-samples. The first sub-sample (n=929) consisted of follow-up cases from the 1983 National Survey of Unmarried Women, which surveyed 1314 never-married women between 20 and 29 years of age. The second sub-sample (n=740) is from a different probability sample of 20 to 27 year old women of unspecified marital status selected in 1991. Data were collected using FTFI (Tanfer 1995). The NSW uses a very similar instrument to the NSM, so the same adjustment strategy is used.

The BRFS is a part of the state-based Behavioral Risk Factor Surveillance System initiated in 1984 by the Center for Disease Control (CDC) to collect data on risk behaviors and preventative health practices (National Center for Chronic Disease Prevention and Health Promotion 2003). The BRFS uses telephone surveys and the questions regarding sexual behavior are part of a supplement started in 1996. States make the decision whether to include the supplement in each year. We use data for the five years from 1996 to 2000. The number of states electing to include the supplement during this period varied from a high of 25 in 1997 to a low of 2 in 1996. The variation makes it impossible to aggregate these data into a true national probability sample. We did not want to exclude the BRFS entirely however, because of the amount of data it provides (n=72,280). The variable for the number of sex partners in the last year is top coded at 76+ which decreases the right skew of the distribution, but there are only three respondents reporting the topcoded value.

In order to make these data comparable each of the data sets are re-sampled with an adjusted weight that combines both the post stratification weight and a normalizing factor so that all of the data sets have the same age distribution as the United States in 2000. The age range of the different studies vary so our analysis focuses only on a selected range (20-39) included in all five of the surveys. Restricting to just this range allows us to compare the observed degree distribution across study. It is unfortunate that a broader age range was not possible but the 20-39 range should include a relatively high level of heterogeneity in sexual behavior.

Methods

When using MLE the natural comparison between two models is a likelihood ratio test. The log likelihood of the two models is set as a ratio and the value is compared to a Chi-square distribution with degrees of freedom equal to the difference in the number of parameters in the first and second model being compared. The ratio of the log likelihoods must be used rather than a direct comparison of log likelihoods because a model with a greater number of parameters will always fit the observed data more closely than a parsimonious one. Likelihood ratio tests can only be used when the smaller model is nested within the larger model. A model is nested in any other model that contains all of the same parameters plus some additional complexity. The different models we consider have different numbers of parameters, but they are not nested, so for our analysis the likelihood ratio test is an inappropriate method of determining the relative goodness of competing models. There are several alternative methods for comparing the fit of non-nested models, we adopt two different approaches: (1) the Akaike Information Criterion (AIC) (Akaike 1974) (Burnham and Anderson 2002) and (2) the Bayesian Information Criterion (BIC) (Raftery 1995). For a simple random sample of n people with data K1, . . . ,Kn, the AIC is defined as $AIC = -2L(\hat{\theta}|K1 = k1, . . . ,Kn = kn) + 2d$, and $BIC = -2L(\hat{\theta}|K1 = k1, . . . ,Kn = kn) + \log(n)d$. The two approaches are very similar but the BIC has the benefit of incorporating model uncertainty and sample size into the decision. The AIC has the advantage of efficiency. That is, for large sample size, it is the best approximation to the "true" model (Burnham and Anderson 2002). If the complexity of the true model does not increase with the size of the data set, the BIC is usually preferred, otherwise AIC is preferred. However, both criteria should be used for guidance and not used to unilaterally exclude models solely based on ranking. Regardless of which criterion is used, smaller is better

Results and conclusions

Looking first at the degree distribution of the male populations at a cutoff of one, we see some consistency in the fits both within and between surveys. Within the NHSLS three of the four best fit models are common to all three distributions. The order of fit based on the BIC for the four best fit models is shown in table 2. They are the Discrete Pareto, Waring and Negative Binomial Yule. The rank of fit based on the AIC and BIC is not consistent across all three distributions, but rather it is dependent on whether or not the data is continuous or categorical. The Discrete Pareto is the best fit model to the data from males reporting partners as a continuous variable. This is the only time the Discrete Pareto is the best fit model at a cutoff of one. For both the categorical responses the four best fit models are the same with the Negative Binomial fitting the best followed by the Negative Binomial Yule, Waring and Discrete Pareto. In both cases the BIC associated with the Negative Binomial fit to the distribution is more than 65 points lower than the next best model. The AIC, BIC and log-likelihood associated with the four best fit models to each distribution at the three different cutoffs are shown in table 3a-f.

Comparing the distributions derived from the two categorical variables from the NHSLS to the distribution from the GSS, we find that the order of the four best fit models is the same, and again the Negative Binomial is a much better model than the next best alternative with a BIC more than 270 lower than the next best alternative. The GSS data is also categorical and it was collected using the same sampling frame as the NHSLS.

When the two other data sets are considered, both of which used continuous responses, we find the same three best fit models, but the top two are in reverse order. For both the BRFS and NSM the Negative binomial Yule is the best fitting model followed by the Negative Binomial.

Once we increase the cutoff point to 2 partners the consistency with witch the best fitting models fit across the range of the distributions ebbs. In three cases the Negative Binomial model remains the best fitting model. As is the case when the cutoff is one, the distributions generated from the two categorical variables from the NHSLS and the distribution from the GSS are best fit by the Negative Binomial. But despite the similarity in the best fit model for these three distributions, the remaining models in the top four are neither in the same order nor even the same models. Among the models that qualified as the four best fitting to these three distributions, three models, the Discrete Pareto, Discrete Pareto Exponential and Yule each appear once while the Waring model is twice the fourth and once the third best fit model. The distribution from the continuous response question on the NHSLS is best fit by the Yule, but the difference between the associated BIC and the BIC of the next best fit model is < 2. For the two remaining data sets, the NSM and BRFS, the four best fit models are all the same with the Poisson Log-Normal being the best fitting model followed by the Waring, Negative Binomial Yule and Discrete Pareto Exponential. The Poisson Log-Normal models has BICs 64.58 and 18 less than the next best alternative fit to the distributions from the BRFS and NSM respectively.

Finally, when we focus our analyses to a more extreme portion of the right tail by increasing the cutoff to three, the results become even more variegated. The Negative Binomial remains the best fitting model to the GSS data and one of the NHSLS categorical variables, but it no longer is the best fitting model to the distribution from the second categorical variable from the NHSLS. That distribution is best fit by the Geometric model. The Poisson Log-Normal continues to be the best fit to both the BRFS and NSM. The Discrete Pareto best fits the distribution of the NHSLS continuous response variable as it does when the cutoff is 1. It was the second best fit at cutoff 2.

Comparing the model fits to the observed data we find a high level of consistency across cutoffs. For three of the six distributions the best fit model at cutoff 1 is also the best fit model at cutoff two. Similarly the best fit model at cutoff two is often the same as the best fit model at cutoff three, four out of the six distributions. In only one case is the best fit model at cutoff one and three the same when a different model fit the distribution best at cutoff two. There are two distributions for which the Negative Binomial was the best fit model at all three cutoffs. These are the distributions for the GSS and the NHSLS constructed categorical variable. Certain data sets also seemed to be more similar than others. The distributions from the BRFS and NSM are fit by the same models, often in the same order. Likewise the distribution from the GSS and the categorical variables on the NHSLS are very similar. This is particularly true of the constructed variable. There is not however a clearly best fitting model across all four data sets within or across the three different cutoff points. The Negative Binomial is the best fitting model more often than any other model we tested. It is the best fit in 8 out of 18 analyses. The power law models are rarely the best fit to the data. The Yule model is the best fit model only once and the Waring is never the best fit model. This indicates that power law models do not generally fit the sexual network degree distributions among males in the United States.

For the females there is almost no consistency in the goodness of fit of the 11 models across or within the different surveys. At cutoff one the Waring is the best fit model to three of the distribution while the Negative Binomial is the best fitting model to two of the distributions and the Negative Binomial Yule is the best fit to just one. Interestingly, within the NHSLS each of the three distributions is fit best by a different model. There is even less consistency as the cutoff is moved to the right. At cutoff two there are three different models that are the best fit models, each one fitting two of the six distributions, and at cutoff three there are four different models.

In general the set of models that we find better fit the female distributions are not the same as those that better fit the male distributions. The degree distributions for the females are much more likely to be well fit by a power law model. The Yule is the best fit model in three of 18 analyses and the Waring is the best fit in three of 18. The three instances in which the Waring is the best fit the cutoff is one, while the instances when the Yule is the best fit occur when the cutoff is two or three. The data sets fit by the power law models are not the same across cutoffs with the exception of the GSS which is best fit by the Waring at cutoff one and the Yule and cutoff two and cutoff three.

In instanced when the best fit model is a power law model for either the males or the females the scaling parameter generally does not fall between two and three, the interval within which the variance is infinite. The scaling parameter for the Yule model fit to the NHSLS continuous variable data from the males is 2.915 with a standard error of .139. The other case were the scaling parameter is close to the 2-3 interval is the

Waring model fit to the NHSLS constructed categorical variable. In that case the scaling parameter is 3.003 with a standard error of .233. In both of these cases the fit of a regression line to the degree distribution on a log-log scale generates a slope of less than 1.5. In all the other cases the scaling parameter are well outside the 2-3 range.

These results suggest that when the amount of error in the tail of the degree distribution is taken into account, the power law models do not fit the empirical data well. Further, there is little evidence that the sexual network degree distribution is scale free. Almost all of the observed scaling parameters are well out side the scale free range.

The social process models generally fit the empirical data better, in particular the Negative Binomial. It is not at all surprising that the Negative Binomial model is so often the best fitting because the process that generates the distribution is so intuitive. The Negative Binomial is simply the probability of some number of failures before success is attained; so we can imagine people selecting partners until they find the right one and enter a long term monogamous relationship.

As a caveat, we do not believe that any of the 11 models tested here are particularly good fits to the data or should be interpreted as the sole underlying mechanism that generates sexual network degree distributions. We simply use them as a tool to demonstrate that there are a host of possible models that fit the empirical data as well or better than the much talked about power law models. Partnership formation is a heterogeneous process that should be modeled in such a way as to include at least some of the complexities of the social world. Further research is required to identify sources of heterogeneity in the acquisition and selection of sexual partners.

Given the results of our analyses we conclude that power law models do not describe the degree distribution of sexual networks, these networks are not scale free and there is an epidemic threshold. The public health implications of these results are clear, broad population based intervention strategies like education programs, condom education and distribution and antiretroviral therapies can be effective in reducing the spread of HIV/AIDS and other incurable STI and bring the reproductive rate below Ro.

TABLE 1. An Overview of the Five Population Based Surveys Used in this Study

Survey	Years	Age	Sex	Interview method	Question	Data adjustments
BRFS	1996-2000	18+	M/F	Telephone	During the past twelve months, with how many people have you had sexual intercourse?	No adjustments. Responses topcoded at 76+. 3 respondents reported 76+ partners.
GSS	1988-91, 93, 94, 96, 98, 2000	18+	M/F	Self-administered Questionnaire	How many sex partners have you had in the last 12 months?	Responses were categorical for values greater then 4 so all responses greater then 4 were coded as 5. These data were only used in the truncated analysis.
NHSLS	1992	18-59	M/F	Face to Face Interview	Thinking back over the past 12 months, how many people, including men and women, have you had sexual activity with, even if only one time?	An additional question was asked on the SAQ portion of the questionnaire which is also used as is a variable constructed by the original researchers.
NSM	1991	19-41	M	Face to Face Interview	Since January 1990, how many different women have you had vaginal intercourse with? Since January 1990, how many different partners have you had anal sex with?	Constructed from the greater of vaginal sex partners in 1990 and anal sex partners in 1990
NSW	1991	19-38	F	Face to Face Interview	With how many different men did you have vaginal intercourse since January 1990? With how many different men did you have vaginal intercourse since January 1990?	Constructed as the greater of vaginal sex partners in 1990 or anal sex partners in 1990

Figure 1a. Histograms of the degree distributions of sexual partners in the last year from the three NHSLS variable. (for categorical variables the midpoint is ploted)

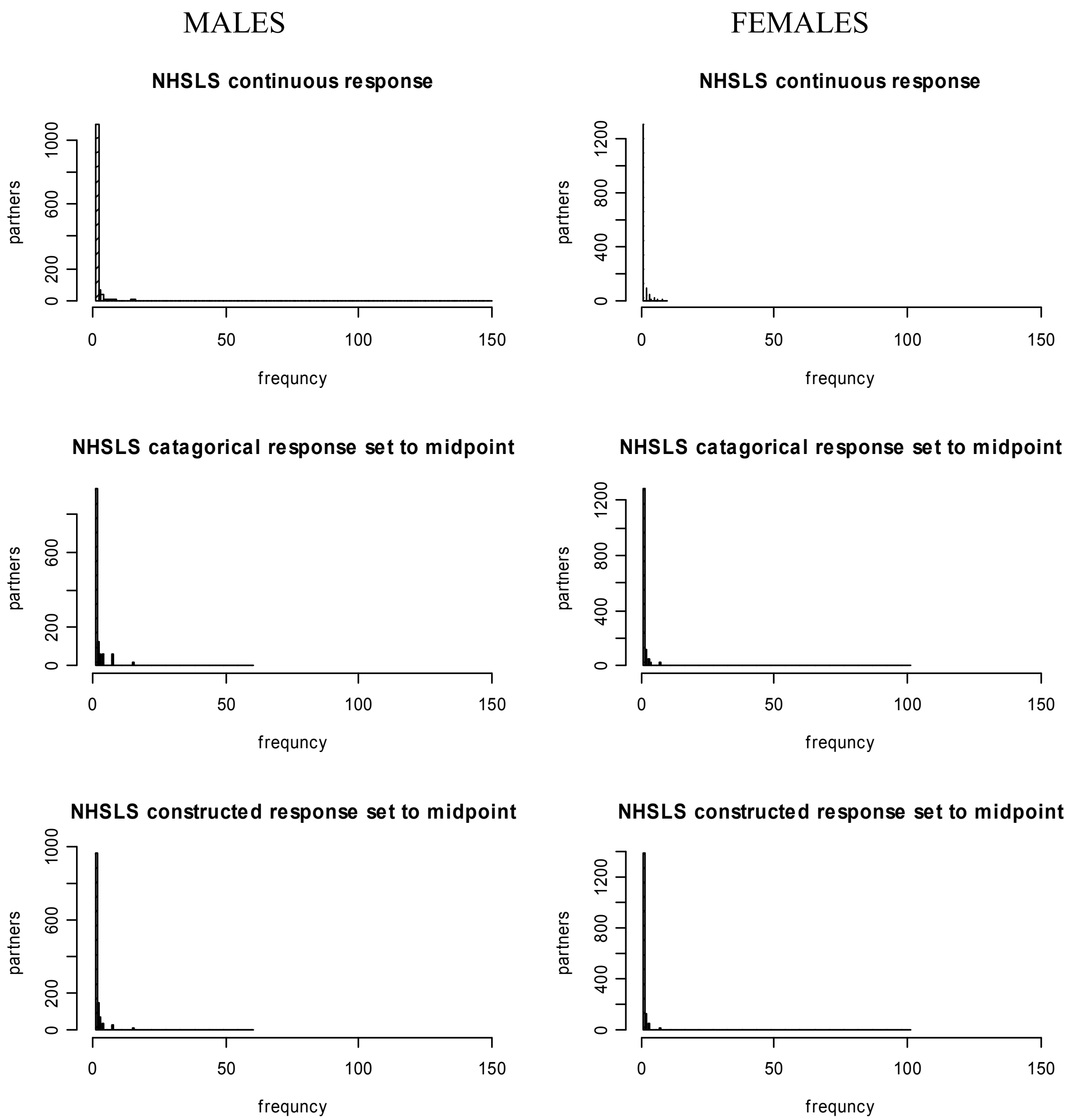

Figure 1b **Histograms of the degree distributions of sexual partners in the last year from the NSM, NSW, GSS and BRFS. (for categorical variables the midpoint is ploted)**

MALES FEMALES

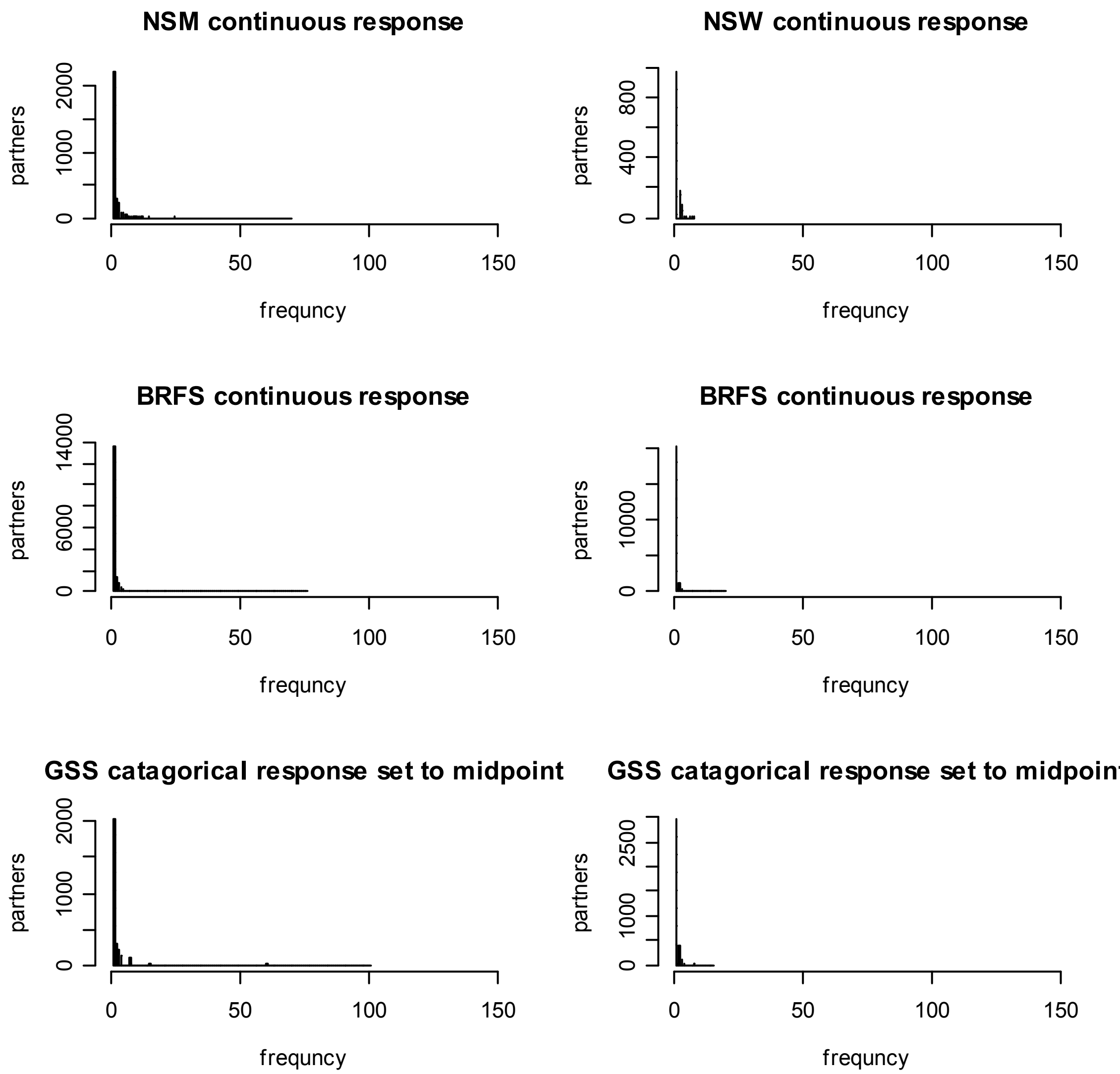

Table 2. The four best fitting models to each of the six distributions in order of rank by BIC.

Best fit models when the data are cutoff at 1

	Rank of fit	NHSLS P12	NHSLSNPS	NHSLSPARTN	GSS	BRFS	NSM_NSW
Males	1	NB	DP	NB	NB	NBY	NBY
	2	NBY	waring	NBY	NBY	NB	NB
	3	waring	DPE	waring	waring	waring	waring
	4	DP	NBY	DP	DP	DP	DPE
Females	1	NBY	NB	waring	waring	waring	NB
	2	waring	NBY	DP	NBY	NBY	DPE
	3	DP	waring	NBY	NB	DP	NBY
	4	NB	DP	DPE	DP	DPE	waring

Best fit models when the data are cutoff at 2

	Rank of Fit	NHSLS P12	NHSLSNPS	NHSLSPARTN	GSS	BRFS	NSM_NSW
Males	1	NB	yule	NB	NB	PLN	PLN
	2	PLN	DP	Geo	Geo	waring	waring
	3	waring	waring	yule	DPE	NBY	NBY
	4	NBY	Gyule	waring	waring	DPE	DPE
Females	1	yule	Geo	DP	yule	DP	Geo
	2	waring	NB	waring	PLN	DPE	PLN
	3	DP	DPE	yule	waring	waring	waring
	4	DPE	waring	DPE	Gyule	NBY	DPE

Best fit models when the data are cutoff at 3

	Rank of Fit	NHSLS P12	NHSLSNPS	NHSLSPARTN	GSS	BRFS	NSM_NSW
Males	1	Geo	DP	NB	NB	PLN	PLN
	2	NB	yule	Geo	Geo	yule	yule
	3	PLN	waring	DP	DP	NBY	DPE
	4	DPE	DPE	yule	waring	waring	NBY
Females	1	DP	Geo	DP	yule	DP	NB
	2	yule	DPE	waring	PLN	DPE	DP
	3	waring	NB	yule	DP	waring	yule
	4	DPE	waring	NB	waring	NBY	NBY

Table 3 The log-likelihood, AIC and BIC of the four best fitting models to each of the six distributions.

a) Model Fit to the Degree Distribution of the US Population Age 20-39 (Data: NHSLS, 1 open ended question)

	MALES				FEMALES			
Cutoff	Model	log-lik	AICC	BIC	Model	log-lik	AICC	BIC
1	DP	-1715.987	3435.983	3446.468	NB	-1545.366	3096.746	3113.128
	waring	-1715.549	3437.116	3452.838	NBY	-1550.975	3109.973	3131.811
	DPE	-1715.92	3437.856	3453.578	waring	-1554.284	3114.582	3130.964
	NBY	-1715.163	3438.355	3459.312	DP	-1559.419	3122.844	3133.767
2	yule	-1715.018	3436.054	3451.776	Geo	-1543.688	3093.391	3109.772
	DP	-1715.877	3437.771	3453.493	NB	-1543.539	3095.1	3116.937
	waring	-1715.005	3438.039	3458.996	DPE	-1543.602	3095.227	3117.064
	Gyule	-1715.019	3438.068	3459.024	waring	-1543.707	3095.436	3117.274
3	DP	-1714.561	3437.152	3458.108	Geo	-1543.481	3094.985	3116.822
	yule	-1714.969	3437.967	3458.924	DPE	-1543.421	3096.877	3124.168
	waring	-1714.546	3439.136	3465.325	NB	-1543.48	3096.995	3124.286
	DPE	-1714.559	3439.161	3465.35	waring	-1543.598	3097.231	3124.522

b) Model Fit to the Degree Distribution of the US Population Age 20-39 (Data: NHSLS, 1 Categorical question)

	Males				Females			
Cutoff	Model	log-lik	AICC	BIC	Model	log-lik	AICC	BIC
1	NB	-1601.786	3209.589	3225.376	waring	-1611.399	3228.812	3245.356
	NBY	-1633.177	3274.383	3295.428	DP	-1612.932	3229.87	3240.902
	waring	-1634.494	3275.004	3290.792	NBY	-1613.074	3234.17	3256.225
	DP	-1635.814	3275.636	3286.164	DPE	-1622.227	3250.466	3267.01
2	NB	-1607.555	3223.138	3244.183	DP	-1611.197	3228.408	3244.952
	Geo	-1609.555	3225.126	3240.914	waring	-1611.274	3230.57	3252.625
	yule	-1632.959	3271.936	3287.724	yule	-1613.175	3232.363	3248.908
	waring	-1632.209	3272.446	3293.491	DPE	-1616.177	3240.376	3262.431
3	NB	-1609.536	3229.114	3255.413	DP	-1611.081	3230.184	3252.239
	Geo	-1610.962	3229.953	3250.998	waring	-1610.632	3231.296	3258.859
	DP	-1631.144	3270.315	3291.36	yule	-1612.76	3233.541	3255.596
	yule	-1631.343	3270.713	3291.758	NB	-1612.981	3235.994	3263.558

c) Model Fit to the Degree Distribution of the US Population Age 20-39 (Data: NHSLS, Constructed by Laumann et al)

	MALES				Females			
Cutoff		**log-lik**	**AICC**	**BIC**		**log-lik**	**AICC**	**BIC**
1	NB	-1653.337	3312.691	3328.418	NBY	-1619.94	3247.904	3269.746
	NBY	-1681.071	3370.17	3391.133	waring	-1621.08	3248.173	3264.558
	waring	-1694.547	3395.111	3410.837	DP	-1624.739	3253.485	3264.411
	DP	-1696.551	3397.111	3407.598	NB	-1639.533	3285.079	3301.464
2	NB	-1656.211	3320.451	3341.414	yule	-1619.419	3244.851	3261.236
	PLN	-1679.715	3367.459	3388.421	waring	-1619.351	3246.726	3268.568
	waring	-1679.828	3367.685	3388.648	DP	-1620.724	3247.461	3263.846
	NBY	-1680.455	3370.954	3397.15	DPE	-1626.015	3260.053	3281.895
3	Geo	-1658.05	3324.129	3345.092	DP	-1618.399	3244.82	3266.662
	NB	-1658.05	3326.143	3352.339	yule	-1618.901	3245.825	3267.667
	PLN	-1676.911	3363.864	3390.06	waring	-1618.462	3246.958	3274.255
	DPE	-1679.492	3369.026	3395.222	DPE	-1622.411	3254.856	3282.153

d) Model Fit to the Degree Distribution of the US Population Age 20-39 (Data: GSS)

Cutoff	MALES				FEMALES			
1	**Model**	**log-lik**	**AICC**	**BIC**	**Model**	**log-lik**	**AICC**	**BIC**
	NB	-3700.485	7406.978	7425.088	waring	-3363.522	6733.051	6751.822
	NBY	-3834.577	7677.168	7701.311	NBY	-3362.892	6733.794	6758.82
	waring	-3838.177	7682.363	7700.472	NB	-3368.355	6742.717	6761.488
	DP	-3841.573	7687.15	7699.225	DP	-3369.718	6743.44	6755.955
2	NB	-3743.639	7495.29	7519.434	yule	-3362.946	6731.897	6750.669
	Geo	-3753.134	7512.275	7530.385	PLN	-3362.147	6732.304	6757.331
	DPE	-3822.577	7653.168	7677.312	waring	-3362.896	6733.802	6758.829
	waring	-3825.482	7658.977	7683.121	Gyule	-3362.946	6733.903	6758.929
3	NB	-3736.468	7482.956	7513.133	yule	-3362.896	6733.803	6758.83
	Geo	-3765.825	7539.664	7563.807	PLN	-3361.943	6733.901	6765.181
	DP	-3808.55	7625.112	7649.256	DP	-3363.488	6734.986	6760.013
	waring	-3808.211	7626.441	7656.618	waring	-3362.894	6735.803	6767.084

e) Model Fit to the Degree Distribution of the US Population Age 20-39 (Data: NSM/NSW)

CUTOFF	MALES				FEMALES			
1	Models	log-lik	AICC	BIC	Models	log-lik	AICC	BIC
	NBY	-4496.015	9000.041	9024.46	NB	-1930.883	3867.78	3884.025
	NB	-4510.353	9026.714	9045.03	DPE	-1931.376	3868.767	3885.013
	waring	-4513.352	9032.712	9051.028	NBY	-1931.387	3870.798	3892.454
	DPE	-4516.545	9039.097	9057.413	waring	-1932.637	3871.287	3887.533
2	PLN	-4473.554	8955.12	8979.539	Geo	-1931.005	3868.025	3884.271
	waring	-4482.717	8973.447	8997.866	PLN	-1930.17	3868.364	3890.019
	NBY	-4491.088	8992.195	9022.715	waring	-1930.374	3868.772	3890.428
	DPE	-4492.825	8993.662	9018.081	DPE	-1930.637	3869.298	3890.954
3	PLN	-4473.349	8956.716	8987.236	NB	-1925.295	3860.626	3887.69
	yule	-4475.92	8959.853	8984.272	DP	-1927.516	3863.056	3884.712
	DPE	-4475.275	8960.569	8991.089	yule	-1928.033	3864.09	3885.746
	NBY	-4474.88	8961.784	8998.405	NBY	-1926.209	3864.468	3896.937

f) Model Fit to the Degree Distribution of the US Population Age 20-39 (Data: BRFS)

CUTOFF	MALES				FEMALES			
1	Model	log-lik	AICC	BIC	Model	log-lik	AICC	BIC
	NBY	-20425.91	40859.81	40891.23	waring	-17764.76	35535.51	35559.93
	NB	-20461.64	40929.28	40952.84	NBY	-17766.75	35541.5	35574.06
	waring	-20473.56	40953.11	40976.67	DP	-17774.93	35553.86	35570.14
	DP	-20528.99	41061.99	41077.69	DPE	-17774.93	35555.86	35580.28
2	PLN	-20366.77	40741.54	40772.96	DP	-17764.93	35535.87	35560.29
	waring	-20399.06	40806.12	40837.53	DPE	-17764.55	35537.1	35569.66
	NBY	-20411.92	40833.83	40873.1	waring	-17764.69	35537.38	35569.94
	DPE	-20418.74	40845.47	40876.89	NBY	-17766.36	35542.73	35583.43
3	PLN	-20366.76	40743.52	40782.79	DP	-17763.34	35534.69	35567.25
	yule	-20391.66	40791.32	40822.73	DPE	-17763.34	35536.69	35577.39
	NBY	-20390.5	40793.01	40840.13	waring	-17763.36	35536.72	35577.42
	waring	-20391.59	40793.17	40832.44	NBY	-17763.39	35538.79	35587.63

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