The Dynamics of the Age Structure, Dependency and Consumption*

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September 20, 2005

Abstract

This paper considers the evolution of the population age structure and consumption, focusing on the relationship between fertility and economic dependency due to intergenerational transfers. We develop a new continuous-time overlapping generations model that captures these dynamics. Authors such as Bloom, Canning and Sevilla (2001) highlight the short-term consumption benefits of reduced youth dependency after a reduction in fertility. However, the quantitative predictions of our model suggest that the fertility rates exhibited by highly-developed nations are so low that they will ultimately lead to an increase in transfers to the elderly that more than offsets this "demographic dividend". As mortality rates fall among the elderly, an even higher share of income will need to be diverted toward old-age support. We demonstrate that workers respond by further reducing their fertility, leading to an even deeper demographic deficit.

*This paper has benefited from the feedback provided by participants at the Brown University Department of Economics Macro Lunch and the Brown University Population Studies and Training Center Colloquium. The usual disclaimer applies.

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1 Introduction

This paper examines the dynamic interaction of the population age structure, economic dependency, and fertility, paying particular attention to the role of intergenerational financial transfers. Our goal is to model how changes in fertility affect a country's demographic structure and how, via its effect on the dependency burden faced by working age adults, the age structure in turn affects fertility. As will be seen below, none of the pieces that constitute our model is new. However, the particular channel on which we focus is of great potential importance, and has so far not been examined by economists. We also develop a new continuous time overlapping generations model that is appropriate to examining this issue.

The effect of population age structure on economic outcomes is a widely studied topic. For example, Cutler et al. (1990) and Elmendorf and Sheiner (2000) discuss the effect of population aging in the United States on feasible and optimal paths of consumption. Because much of the transfer of resources to the elderly is channeled through governments, population aging will have a particularly dramatic effect on government finances (see Lee and Edwards, 2001). Bloom, Canning and Sevilla (2001) similarly examine how a "demographic dividend" resulting from reductions in fertility - that is, a period of several decades in which the ratio of working age adults to dependent children and elderly is unusually high - affects overall economic growth in developing countries.

In the above literature, the important chain of causality is from the demographic to the economic. That is, the underlying demographic inputs - most notably changes over time in fertility - are either taken as exogenous or related to phenomena (such as declining child mortality) that are outside the economic model being examined. In this paper we close the circle, and concentrate our attention on the interdependence of fertility and population age structure, through the channel of economic dependency. Specifically, we look at how changes in fertility affect population age structure and economic outcomes, and how these feed back to affect fertility.

A moments consideration suggests that the problem that we are interested in is inherently dynamic. Changes in fertility have effects on the population age structure that take generations to play out. Most significantly, while the immediate effect of a decline in fertility is a reduction in a society's dependency burden, and thus an expansion in the feasible level of consumption, the long-run effect of such a decline may be to actually raise dependency and lower consumption. Fertility and age structure are thus linked in a dynamical system. Such a system will have both steady states (in which fertility is constant and the population age structure is stable) and a long lasting dynamic response to external shocks.

Our interest in the mutual dependency of fertility and a society's age structure is primarily motivated by thinking about the future prospects of some of the most developed countries in the world. Countries such as Italy and Japan are now coming to the end of a decades-long period during which low fertility produced a transitorily low level of population dependency. During the period 2010-2030, rapid population aging will drastically impact consumption possibilities (and even more drastically impact government budgets). The effect of this consumption crunch on fertility - and thus on the age structure of the population even further down the road - is an issue that has not yet been addressed by economists and demographers.[1]

The link between population age structure and fertility that we examine is not the only such channel of causation. Most famously, Easterlin (1987) hypothesized that a

[1] Micevska and Zak (2002) examine fertility during income contractions that were even larger than those projected to result from population aging: The Great Depression (in the United States and Germany) and the transition from communism during the 1990s. In both cases, income declines produced significant reductions in total fertility. Micevska and Zak argue that such a phenomenon can be explained by a Malthusian model in which the perceived level of "subsistence" consumption is a function of an individual's own past consumption.

cohort's size was linked to its fertility through its effect on the earnings of young adults relative to those of their parents: members of a large cohort would find themselves with income that was low relative to the standard of living they had grown up with, and would adjust fertility downward to partially restore their standard of living. The mechanism that we examine here shares the feature of Easterlin's model that reductions in living standards trigger compensating reductions in fertility. However, our focus is on a fiscal effect on after tax earnings of income transfers to the elderly, rather than on the effect of cohort size on the pre-tax wage of young workers. For this reason, the key aspect of population age structure on which we focus is the ratio of elderly dependents to working age adults, rather than the ratio of older to younger workers on which Easterlin focuses. In principle the two mechanisms could easily co-exist.

In addition to identifying the feedback from population age structure to fertility via the channel of dependency, a second contribution of this paper is to construct a dynamic model of population age structure. Specifically, we build a continuous time overlapping generations model in which population is divided into three groups (young, working age, and old), and transitions between groups take place in a probabilistic fashion à la Blanchard (1985). Within this model, fertility can be taken as exogenous or made an endogenous function of the population age structure. The model is simple enough to be analyzed graphically, and yet captures the dynamic adjustment of age structure, fertility, and consumption to external shocks such as changes in old-age mortality, retirement age, or preferences regarding children. The model has applications well beyond those pursued here, and also represents a new and convenient way of conceptualizing demographic-economic interactions.

The rest of this paper is structured as follows. In Section 2 we lay out our model of population age structure and develop the basic dynamic equations that will be used in the subsequent analysis. Section 3 analyzes our model under the assumption that the rate of fertility is exogenous. We also show how the model can be used to calculate the rate of fertility that maximizes consumption in steady state and discuss the consumption-maximizing response of fertility to increases in elderly dependency. In section 4 we allow fertility to be an endogenous function of income, analyze the dynamics of the complete system, and compare the actual and consumption-maximizing responses of the economy to an exogenous shock to old-age mortality. Section 5 concludes.

2 A Dynamic Model of the Age Structure

Economists have long recognized the need to incorporate age heterogeneity into macroeconomic analysis. Samuelson (1958) and Diamond (1965) developed simple economic-demographic models in which agents with a finite life span progressed through a small and discrete progression of ages. Overlapping generations (OLG) models of this type have been used extensively in the economic growth literature. OLG growth models typically assume a two or three period lifecourse, which might be suitable for an analysis of fertility and old-age dependency. However, periods within such model represent long spans of time in the real world (e.g. 20-30 years). As a result, the dynamics generated are lumpy, with jumps in rates and stocks occurring only between model periods. This is satisfactory for an understanding of long-term transitions and equilibria. However, such models do not allow changes in fertility and the population structure to be analyzed over smaller time increments.

Adding more periods to a discrete time OLG model might allow for smoother dynamics, but adding more time periods implies adding more age groups. This increase in the state space implies difficulties in aggregation, making it difficult to cleanly analyze macrostructural relationships. Under certain circumstance switching to continuous time may simplify matters. A classic example is Blanchard's (1985) "model of perpetual youth."[2] We develop here a somewhat stylized continuous-time model that borrows from both the traditional OLG framework and that of Blanchard. Given our interest in economic dependency, rather than focus on the lifecycle as a series of ages, we instead model it as a progression through a series of stages of economic life. Most individuals follow a pattern whereby they are first dependent on their parents, work for some amount of time, and then retire. Hence, we divide the population into three groups: A_Y is the stock of young people who have never worked; A_M is the stock of people in the economy who are in their working years; finally, A_O is the stock of people who once worked, but are now retired. The triple (A_Y, A_M, A_O) characterizes what we call the *age structure* of the population.

Each individual i undergoes a monotone progression through the age structure:[3]

$$i \in A_Y \rightarrow i \in A_M \rightarrow i \in A_O \ . \tag{1}$$

We apply the Blanchard idea to this demo-economic progression by assuming constant exit probabilities from each group A_j where $j \in \{Y, M, O\}$. For the young and working, λ_Y and λ_M give the hazard of transition to work and retirement, respectively. Among the elderly, λ_O is the probability of dying. All of the flows are determined by the structural parameters, λ, except the flow of births into A_Y, which is given by $N(t)$. The following

[2] In a more recent paper, Bommier and Lee (2000) develop a model of an economy with a continuous age distribution, and are able to derive a number of interesting aggregate steady state results. However, they make only limited progress in analyzing aggregate dynamics, and even these are made under the assumption that the fertility rate is fixed at its steady-state value.

[3] There are of course cases in which individuals cycle between working and retirement. However, since such individuals represent a relatively small fraction of the population, we rule out such "Michael Jordan" effects in our analysis.

system of equations summarizes this model of the evolution of the age structure:

$$\dot{A}_Y(t) = N(t) - \lambda_Y A_Y(t) \tag{2}$$

$$\dot{A}_M(t) = \lambda_Y A_Y(t) - \lambda_M A_M(t) \tag{3}$$

$$\dot{A}_O(t) = \lambda_M A_M(t) - \lambda_O A_O(t)\ . \tag{4}$$

We will use this system, under various assumptions regarding how fertility is determined, to characterize the evolution of the age structure.[4]

The parameters λ_j give the inverse of the expected (or average) time of remaining in the given group, T_j, which may be used to calibrate the model to empirical data. Since our paper is geared toward describing issues affecting the highly-developed world, we focus our attention to 12 members of the Organization for Economic Cooperation and Development (the "OECD12").[5] Since data on the age of entry into the labor force (T_Y) are difficult to come by, we assume that age 20 is a reasonable number. We compute $T_M = R - 20$, where R is the average age of retirement, which for OECD12 member nations is available from OECD (2004, Table SS8.1).[6] The United Nations (2005, Table 22) provides data on life expectancy in five-year increments. Since the decline in life

[4] In our model we ignore childhood and early adult mortality, assuming that death only occurs among the retired. The model could easily be adapted to incorporate these other forms of mortality by changing the basic equations to:

$$\dot{A}_Y(t) = n(t)\, A_M(t) - (\lambda_Y + \mu_Y)\, A_Y(t) \tag{$\tilde{1}$}$$

$$\dot{A}_M(t) = \lambda_Y A_Y(t) - (\lambda_M + \mu_M)\, A_M(t) \tag{$\tilde{2}$}$$

$$\dot{A}_O(t) = \lambda_M A_M(t) - \mu_O A_O(t)\,. \tag{$\tilde{3}$}$$

With this set-up, λ_j is the transition probability to the next group, and the μ_j is the death rate in the group. Analysis of this model would parallel that in the main text, with qualitatively similar results.

[5] These countries are selected so that they are among the top twenty in both GDP per capita and population among the OECD nations. The members of our OECD12 are: Australia, Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Spain, Sweden, the United Kingdom, and the United States.

[6] The measure of the average age of retirement is based on "observed changes in participation rates over a 5-year period for successive cohorts of workers (by 5-year age groups) aged 40 and over" (OECD 2004, notes accompanying Table SS8.1).

Table 1: Base Data for the OECD12

Country	R	T_M	T_O
Australia	62.0	42.0	21.5
Belgium	57.7	37.7	23.5
Canada	62.2	42.2	21.2
France	59.3	39.3	23.2
Germany	60.6	40.6	20.9
Italy	60.8	40.8	26.0
Japan	67.5	47.5	21.1
Netherlands	60.1	40.1	21.1
Spain	61.5	41.5	21.2
Sweden	62.7	42.7	20.4
United Kingdom	62.1	42.1	19.6
United States	63.9	43.9	18.7
Average	61.7	41.7	21.5

Data Sources: OECD (2004, Table SS8) and United Nations (2005, Tables 7 and 22).

expectancy is roughly linear between ages 55 and 75, we compute life expectancy at retirement (T_O) through linear interpolation of life expectancies at the surrounding five-year increments.[7] Table 1 presents the basic data (R, T_M, T_O) for the OECD12. As can be seen in the table, life expectancy after retirement is substantial. The proportion of life spent as an elderly dependent ranges from 0.22 in the United States to almost 0.3 in Italy.

It is worthwhile noting that our set-up is not, in general, an age-based "first in, first out" (FIFO) motion. Using a FIFO progression implies that the stocks A_j are a direct function of historical fertility rates, which introduces an accounting complexity into the analysis. Historical fertility rates shape the economic age structure in our model, although

[7]Since our model does not account for sex differences, we compute R and T_O as the gender-weighted average of the retirement age and life expectancy values for each sex. Population weights are from United Nations (2005, Table 7).

less directly. In particular, our probabilistic framework smooths over some of the lags that would be generated by a FIFO model. It is also worth noting that our model implies that some individuals spend less time in some groups than others. This makes a certain amount of sense: not all individuals enter the workforce at the same time, and there is a spread in retirement age.[8] Setting up the model in this fashion implies that the A_j's are not age groups in a formal sense, but rather stages of life or stages of economic activity, as discussed earlier. Nonetheless, the average age in A_M is greater than that in A_Y, and likewise for A_O and A_M, and so we may sometimes refer to these stocks as "age groups" in our discussion. In referring to the number of members in A_j, we will use the term "size", and when discussing the average amount of time spent in group j,i.e. T_j we will call this the "width" of A_j.

2.1 Production and Dependency

In order to focus our attention on the dynamics of the age structure, we assume that all productive resources are generated by the labor input of working, which is provided inelastically.[9] The total pool of resources available for consumption is

$$\Omega(t) = W(t) A_M(t) \ ,$$

where $W(t)$ is the prevailing wage at time t. The young and the elderly are supported through a system of transfers from the working. As a result, we focus on the *population*

[8]While some individuals may never leave "childhood" in this framework, the probabilities of such "perpetual youth" events are very low.

[9]Under a small open economy assumption, leaving capital out of the model in this manner is largely innocuous for the purposes of this paper. The only point at which capital will play a role where we make fertility endogenous. Old-age dependency will affect not only current, but past and present fertility decisions due to intertemporal optimization. Even so, analysis of age structure equilibrium and dynamics in a model with savings is qualitatively similar to that in our transfer-based model.

dependency ratios:

$$y(t) = \frac{A_Y(t)}{A_M(t)} \text{ and } o(t) = \frac{A_O(t)}{A_M(t)}. \tag{5}$$

(This also simplifies the analysis of the dynamical system in (2)-(4).) In the remainder of this section we establish various properties of the equations governing the evolution of the old-age and youth dependency ratios. These will serve as the basis for our subsequent analysis of dynamic equilibrium.

2.1.1 Dynamics of the old-age dependency ratio

The equation of motion for the old-age dependency ratio can be derived from (3) and (4) as

$$\dot{o}(t) = \lambda_M - (\lambda_O - \lambda_M) o(t) - \lambda_Y y(t) o(t) . \tag{6}$$

The dynamics of $o(t)$ in (y, o)-space are relatively straightforward, and we present some basic results here that are applicable throughout the paper.

Based on (6), the zero motion locus for $o(t)$ is given by

$$o(t)|_{\dot{o}=0} = \frac{\lambda_M}{(\lambda_O - \lambda_M) + \lambda_Y y(t)} \equiv Z_o(y(t)) . \tag{7}$$

Hence, for a given value of the youth dependency ratio, there is exactly one equilibrium value of o. Considering the graphical properties of the $Z_o(y)$ in (y, o)-space, the vertical intercept is given by $\lambda_M / (\lambda_O - \lambda_M) > 0$. That the intercept is positive follows from the fact that the expected time in retirement is less than the expected time working. Thus, since $T_O < T_M$, we have that $\lambda_O > \lambda_M$. Because $Z_o'(y) < 0$ and $Z_o''(y) > 0$, the zero motion locus for is downward sloping and convex with respect to the origin.

Finally, noting that $d\left[\dot{o}\left(t\right)\right]/d\left[o\left(t\right)\right] < 0$, all equilibrium values of o are stable. That is, for a given value of the youth dependency ratio, the old-age dependency ratio steadily converges to the corresponding point on the zero motion locus.

2.1.2 Fertility and the dynamics of the youth dependency ratio

Analysis of the youth dependency ratio requires depends on the flow of births, which is a function of the stock of fertile persons and the rate of fertility among the fertile. We give here a fairly general dynamic equation for the youth dependency ratio, laying out a basic assumption regarding the stock of fertile of persons. In later sections we specify the rate of fertility among the fertile in greater detail.

From (2) and (3) the equation of motion for the youth dependency ratio can be written

$$\dot{y}\left(t\right) = \frac{N\left(t\right)}{A_M\left(t\right)} - \left(\lambda_Y - \lambda_M\right) y\left(t\right) - \lambda_Y \left[y\left(t\right)\right]^2. \tag{8}$$

For simplicity, we assume that the fertile population is a fixed proportion, ϕ, of the workforce. Since all members of the workforce are homogenous in our model, an alternative interpretation of ϕ is to decompose it as $\phi = T_F/T_M$, where $T_F < T_M$ is the expected length of time spent fertile among each member of M.[10] Letting $n\left(t\right)$ represent the average fertility rate at time t among the fertile, under this assumption (8) reduces to

$$\dot{y}\left(t\right) = \phi n\left(t\right) - \left(\lambda_Y - \lambda_M\right) y\left(t\right) - \lambda_Y \left[y\left(t\right)\right]^2. \tag{9}$$

We will refer to $\phi n\left(t\right)$ as the flow of births per worker.

The equation of motion for the youth dependency ratio (9) does not depend explicitly

[10] An additional set of equations, consistent with those governing the economic age structure, could be specified to allow ϕ to vary over time. However, doing so makes analyzing the model substantially more complicated, while adding very little qualitatively to our analysis.

on $o(t)$. Thus, if fertility is determined in a manner unrelated to the old-age dependency ratio, equilibrium analysis of the relative economic age structure (y, o) is straightforward. This is the case in Sections 3 where we take the fertility rate as given. In Section 4 we allow fertility to be endogenously determined. Since fertility will be a function of after-tax income, old-age dependency will play a role in the dynamics of the youth dependency ratio as a consequence of Social Security transfers.

3 Fertility and Economic Dependency

In this section we analyze our model assuming that the fertility rate is fixed and exogenous. (That is, $n(t) = n$ for all t.) We focus on the relationship between fertility, old-age mortality and a measure of total dependency in the economy. The traditional demographic measure of total dependency is simply the sum of youth dependency, y, and old-age dependency, o. However, as pointed out by Cutler et al. (1990), the consumption requirements of the young and the elderly are not necessarily the same. This implies that an increase in youth dependency will not generally have the same implications for resource availability as an increase in old-age dependency. Consequently, we focus on a measure of needs-weighted economic dependency, $e(t)$. In particular we assume that the children and elderly have consumption needs equivalent to ρ_Y and ρ_O times that of the working, which implies that economic dependency at a given moment is

$$e(t) = \rho_Y y(t) + \rho_O o(t) \ . \tag{10}$$

Given our labor-based production structure, we assume further, as in Weil (1999) that the consumption of the all three groups is indexed to the wage. That is $c_Y(t) = \rho_Y c_M(t)$,

$c_O = \rho_O c_M(t)$, and $c_M(t) = \eta(t) W(t)$. Given the aggregate resource constraint,

$$c_Y(t) A_Y(t) + c_M(t) A_M(t) + c_O(t) A_O(t) = \Omega(t) \ ,$$

the consumption index must satisfy

$$\eta(e(t)) = \frac{1}{1 + e(t)} \ . \tag{11}$$

The index η affects the consumption of all groups proportionately. Increases in youth and elderly dependency reduce the per-capita resources in the economy, but their effects on η are proportional to the relative consumption needs of the young and old. Below we consider the impact of changes in the fertility rate on economic dependency and consumption. We then consider the consequences population aging due to increased life-expectancy among the elderly. Before proceeding we characterize the basic equilibrium of in our model, given consant rate of fertility.

3.1 Equilibrium with Exogenous Fertility

With a fixed fertility rate, the equation of motion for the youth dependency ratio is

$$\dot{y}(t) = \phi n - (\lambda_Y - \lambda_M) y(t) - \lambda_Y [y(t)]^2 \ . \tag{12}$$

Since the dynamic equation does not depend on $o(t)$, (12) can simply be examined for equilibrium values of the youth dependency ratio. Setting $\dot{y}(t)$ equal to zero yields one positive root, denoted by $\bar{y}$. Writing this in terms of the widths of the age groups (T_j), rather than the exit parameters (λ_j),

$$\bar{y} = \sqrt{\left(\left[\frac{1}{2}\left(1 - \frac{T_Y}{T_M}\right)\right]^2 + T_Y \phi n\right)} - \frac{1}{2}\left(1 - \frac{T_Y}{T_M}\right). \tag{13}$$

The equation of motion for y implies that this root is stable. The corresponding equilibrium old-age dependency ratio is given by

$$\bar{o} = Z_o(\bar{y}) = \frac{1}{(T_M/T_O - 1) + (T_M/T_Y)\,\bar{y}}. \tag{14}$$

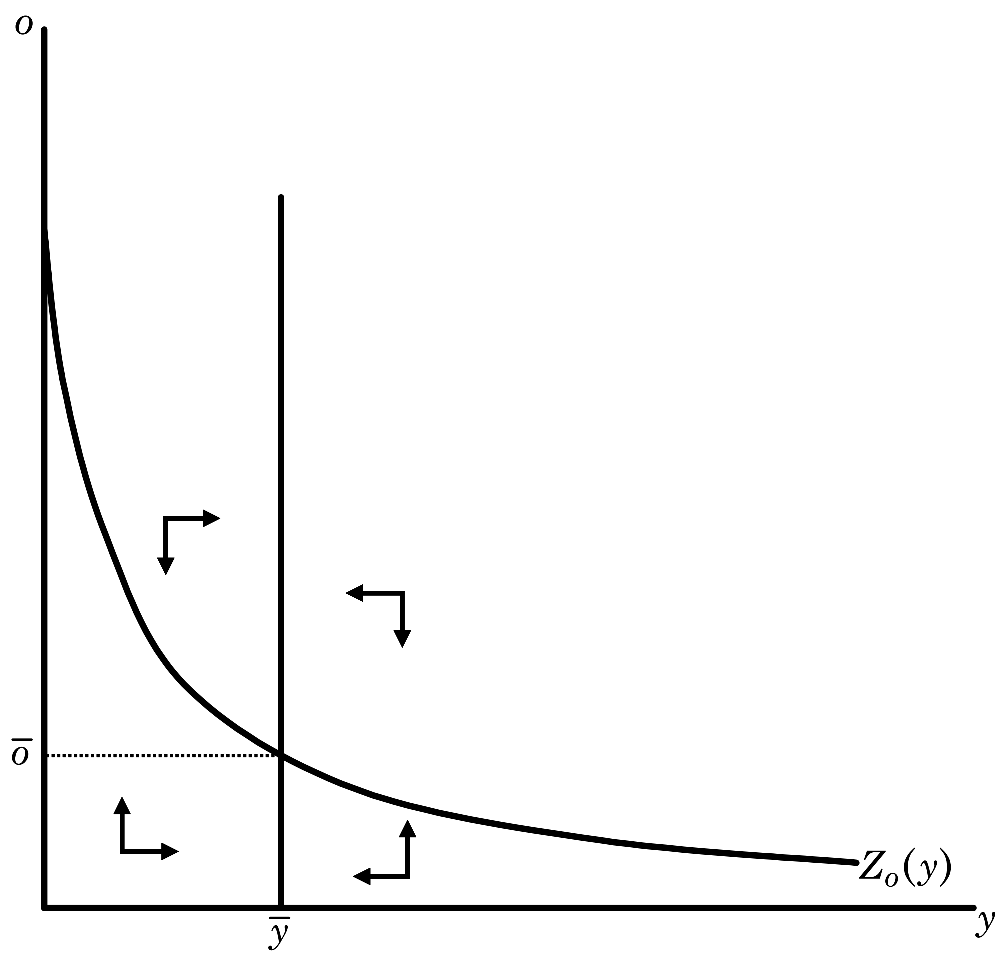
Graphically, Figure 1 portrays the global dynamics of the population dependency ratios. The equilibrium point is globally stable and the population dependency ratios converge monotonically to the point $(\bar{y}, \bar{o})$.

It is convenient to consider the flow of births per worker in terms of the gross reproductive rate (GRR), a common fertility concept used by demographers. Since individuals in our model are the reproductive unit of analysis (or alternatively couples are the economic unit of analysis) the GRR is the number of children an individual can expect to have, assuming that they are subject to the currently-prevailing rate of fertility for the entirety of their childbearing years. Given our assumptions regarding the stock of fertile persons, this implies that $G = T_F n$, where G is the GRR, and that the flow of births per worker is

$$\phi n = \frac{G}{T_M}, \tag{15}$$

Note that when G equals one, the population of workers is just replacing itself. Substituting in (15) for the flow of births, the equilibrium youth dependency ratio can be

Figure 1: Global Dynamics with Exogenous Fertility



rewritten in terms of the GRR:

$$\bar{y}=\sqrt{\left[\frac{1}{2}\left(1-\frac{T_Y}{T_M}\right)\right]^2+\frac{T_Y}{T_M}G}-\frac{1}{2}\left(1-\frac{T_Y}{T_M}\right). \qquad (16)$$

When the GRR is at the replacement rate the equilibrium dependency ratios simplify to $\bar{y}^r = T_Y/T_M$, and $\bar{o}^r = T_O/T_M$. These results are fairly intuitive, and parallel those in basic demographic theory. In a stable population with replacement fertility, the relative sizes of the age groups in equilibrium should simply be the ratios of their widths.

3.2 Declining Fertility and the Consequences for Consumption

Much of the twentieth century was characterized by declines in period fertility rates, with the exception of the post-World War II baby boom. Considering the implications for the population dependency ratios, given a fall in G, equation (16) implies that the equilibrium youth dependency ratio falls. As shown in Figure 2, the decline in $\bar{y}$ maps to a higher equilibrium value of the old-age dependency ratio along the $Z_o(y)$ locus.

Considering the transition to an equlbrium with lower fertility, initially the declines in youth dependency are larger than the increases in old-age dependency, so that economic dependency (e) declines. This leads to an increase in the consumption index η, reflecting the "demographic dividend" discussed by Bloom, Canning and Sevilla (2001). Later in the transition, there comes a turning point where the declines in e caused by falling y are more than offset by the increases in o, in which case η begins to increase as the economy approaches the new equilibrium.[11] Hence, the consumption benefits from the demographic dividend are time limited. The net effect of a fertility decline on equilibrium economic dependency is *a priori* ambiguous. The inset of Figure 2 shows a possible set

[11] The initial decline in e should be evident from an analysis of the phase diagram. While o is still close to the $Z_o(y)$ locus, y is very far from its steady state. Our assertion of a turning point makes intuitive sense, and can be established based on numerical simulations of the model using parameter values describing the OECD12 nations.

Figure 2: The Effects of a Decline in the Fertility Rate

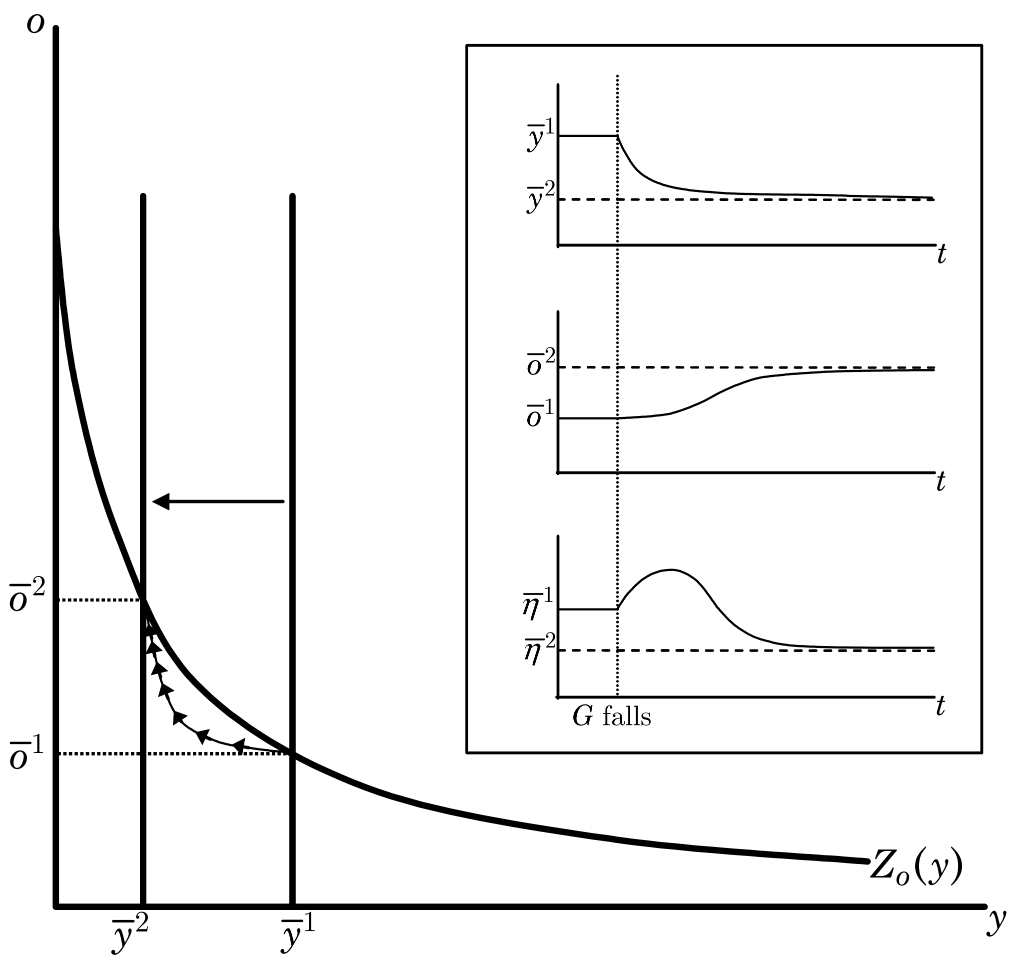
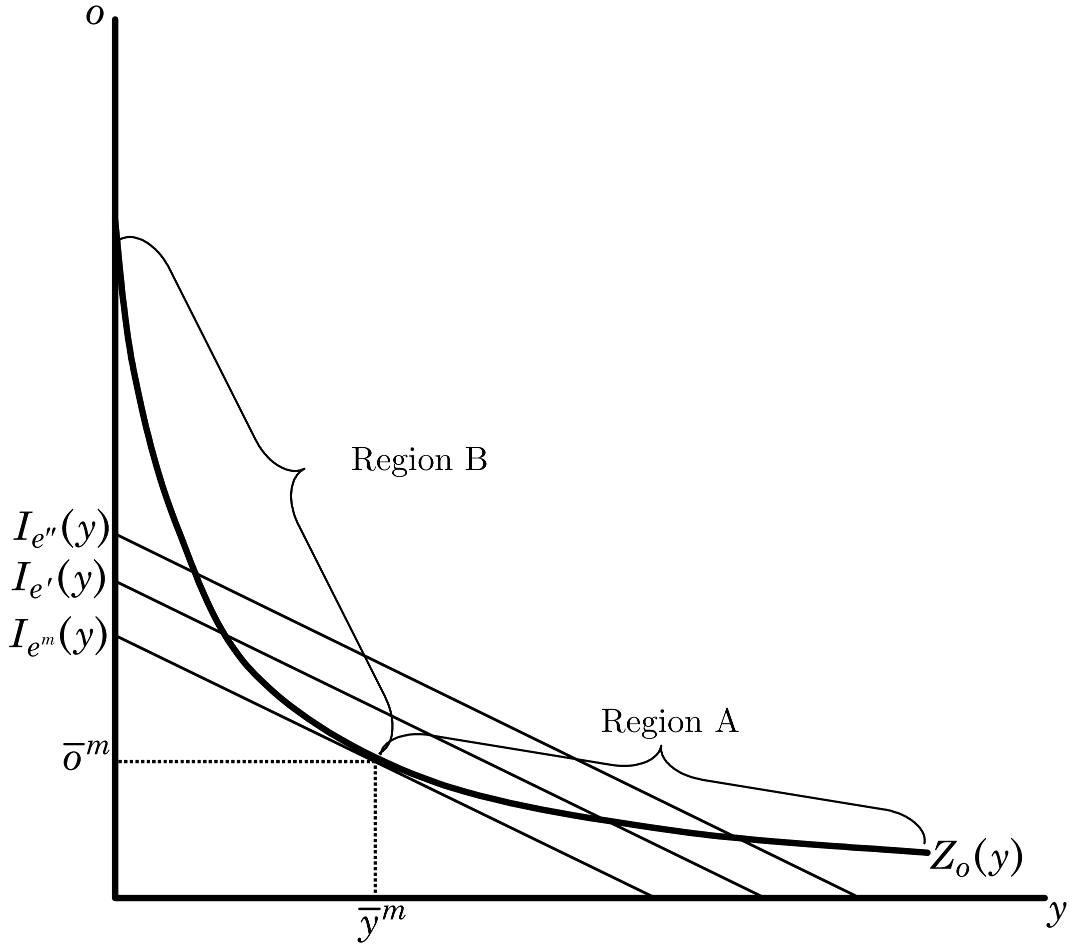


Figure 3: Economic Dependency and the $Z_o(y)$ Locus



of paths for y, o, and η after a decline in the fertility rate in the case where the new equilibrium has a higher level of economic dependency (and a lower level of consumption) than the original equilibrium. As we will see, this likely represents the effects of further fertility declines in most of the highly developed world.

Following Weil (1999), given a certain value of economic dependency, $\tilde{e}$, we can use (10) to graph iso-dependency lines in (y, o) space as

$$o = \frac{\tilde{e}}{\rho_O} - \frac{\rho_Y}{\rho_O} y \equiv I_{\tilde{e}}(y) \tag{17}$$

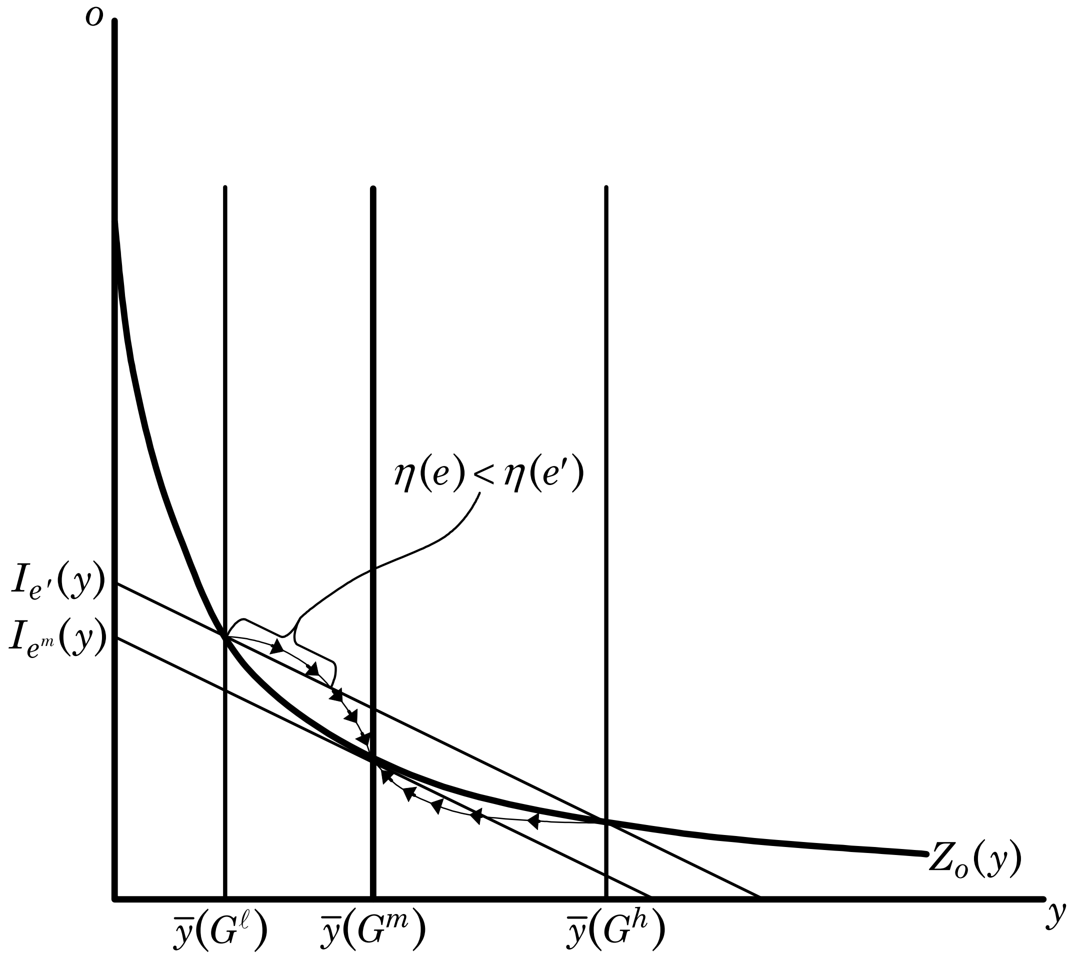
Since changes in fertility trace out equilibrium population dependency ratios along $Z_o(y)$, we may consider the relationship between the iso-dependency lines and this locus. Iso-dependency lines that are closer to the origin are associated with a lower rate of economic dependency and a higher level of the consumption index, η. Because $Z_o(y)$ is convex with respect to the origin, there is a unique equilibrium pair $(\bar{y}^m, \bar{o}^m)$ that is tangent to the $Z_o(y)$ locus. (See Figure 3.) The pair $(\bar{y}^m, \bar{o}^m)$ minimizes economic dependency, which implies that consumption of all members of the economy is maximized. At this point, the slope of the $Z_o(y)$ locus must equal the slope of the iso-dependency line, which based on (14), (16), and (17) implies that

$$\bar{o}\left(G, \frac{T_Y}{T_M}, \frac{T_O}{T_M}\right) = \sqrt{\left(\frac{\rho_Y}{\rho_O}\right)\frac{T_Y}{T_M}}. \tag{18}$$

Given that $\bar{o}$ is a monotonically negative function of G, there is a unique fertility rate that generates this equilibrium, G^m, given values of the other parameters. At equilibria corresponding to higher fertility rates, i.e. those in Region A of the $Z_o(y)$ locus in Figure 3, small declines in G are associated with lower equilibrium economic dependency and higher consumption. The opposite occurs in Region B, which is associated with lower fertility rates.

For a country with a fertility rate $G^h > G^m$ (i.e. in Region A), as the fertility rate declines toward the consumption-maximizing rate, consumption increases at all future times, assuming that the other parameters remain stable. This means that the equilibrium associated with G^h is dynamically inefficient. On the other hand, for a country with a fertility rate below the consumption-maximizing rate the transition to G^m is costly. As shown in Figure 4, in a country with the same initial economic dependency as implied by G^h and a fertility rate of $G^\ell < G^m$, economic dependency must initially increase, implying a reduction in η. It is only later in the transition, as the elderly dependency

Figure 4: Transition to Consumption-Maximizing Equilibrium from Above and Below



ratio declines, that the consumption benefits from the increase in fertility are realized.

Given a set of widths of the age groups and relative consumption needs, ascertaining whether an economy is in Region A amounts to checking whether the observed fertility rate (G^a) is at or above the consumption-maximizing fertility rate. We undertake this calculation for the OECD12 nations. Based on measures of education, health and private consumption expenditures, Elmendorf and Scheiner (2000) estimate that $\rho_Y = 0.62$ and $\rho_O = 1.37$ in the United States.[12] It seems reasonable that ρ_Y and ρ_O should follow

[12] These update the estimates of the relative consumption needs given by Cutler et al. (1990).

a similar pattern in the other highly-developed nations. Taking the values of ρ_Y and ρ_O from Elmendorf and Scheiner, and using the base data on age widths from Table 1, we compute G^m for the OECD12 nations based on equation (18). In the first column of Table 2, we report the actual gross reproductive rate and in the second we give our calculation of G^m.[13] In eleven out of the twelve nations, the current GRR is less than the consumption-maximizing fertility rate, and the difference is quite substantial. It is only in the United States that the currently-observed fertility rate exceeds G^m.[14] Hence, based on present demographic data, the equilibria for most highly-developed economies fall in Region B of the $Z_o(y)$ locus. This suggests that recent declines in fertility will ultimately be associated with a higher equilibrium rate of economic dependency and a lower level of consumption. Conversely, equilibrium consumption would increase in most countries through increases in the fertility rate.

In order to determine the equilbrium benefits from moving to the consumption-maximizing rate of fertility, we use equations (14) and (16) to calculate the steady-state population dependency ratios for the OECD12 consistent with the age widths in Table 1 and the values of G^a and G^m reported in Table 2. Weighting these by the relative consumption needs gives us the equilibrium level of economic dependency associated with the actual fertility rate and with the consumption-maximizing fertility rate. We denote these as $\bar{e}^a$ and $\bar{e}^m$, respectively. Finally, equation (11) allows us to compute the per-capita consumption index η associated with these two values. The third column of Table 2 reports the percentage increase in $\eta(\bar{e}^*)$ relative to $\eta(\bar{e}^a)$. The average potential steady-state consumption gain in the OECD12 is approximately 1.6%. Countries with fertility rates furthest away from their respective values of G^m obviously have the

[13] Data on the observed gross reproductive rates are from Population Reference Bureau (2004).

[14] That the consumption-maximizing rate is so low in the U.S. is largely a function of the relatively short life expectancy after retirement. Similarly, G^m is the highest in Italy where life expectancy after retirement is the longest. We discuss the relationship between G^m and life expectancy more explicitly below.

Table 2: Actual and Consumption-Maximizing Gross Reproductive Rates and Potential Consumption Gains for the OECD12

Country	G^a	G^m	Potential gain in η (%)
Australia	0.85	1.32	0.76
Belgium	0.80	1.78	3.50
Canada	0.75	1.26	0.99
France	0.95	1.66	1.62
Germany	0.65	1.28	1.69
Italy	0.65	1.98	7.20
Japan	0.65	1.06	0.67
Netherlands	0.90	1.32	0.61
Spain	0.65	1.28	1.69
Sweden	0.85	1.12	0.26
United Kingdom	0.85	1.01	0.09
United States	1.00	0.79	0.14
Average	0.80	1.32	1.60

Notes: G^a is from Population Reference Bureau (2004); G^m and the percentage gain in the consumption index, η, are calculated as described in the text.

most to gain. For example, if the fertility rate in Italy were to rise to its consumption-maximizing rate, the steady-state per-capita consumption index would rise by over 7%. However, since most of the countries under consideration have fertility rates below their consumption-maximizing rate, the transition to G^m would entail a transitory period of higher economic dependency and lower consumption.

A number of explanations have been proposed for the currently low levels of fertility in the most-developed nations, which we have so far taken as given.[15] Nonetheless the future path of fertility in these "lowest-low fertility" countries is a matter of some debate among demographers, and there is limited theory to guide predictions. At the same time, it is almost a certainty that old-age mortality will decline in the future. In Section 3.3, we explore the consequences of falling mortality among the elderly in terms of economic dependency and consumption, and in Section 4 we consider a mechanism whereby increased life expectancy itself may affect the path of fertility.

3.3 The Effects of Falling Old-Age Mortality

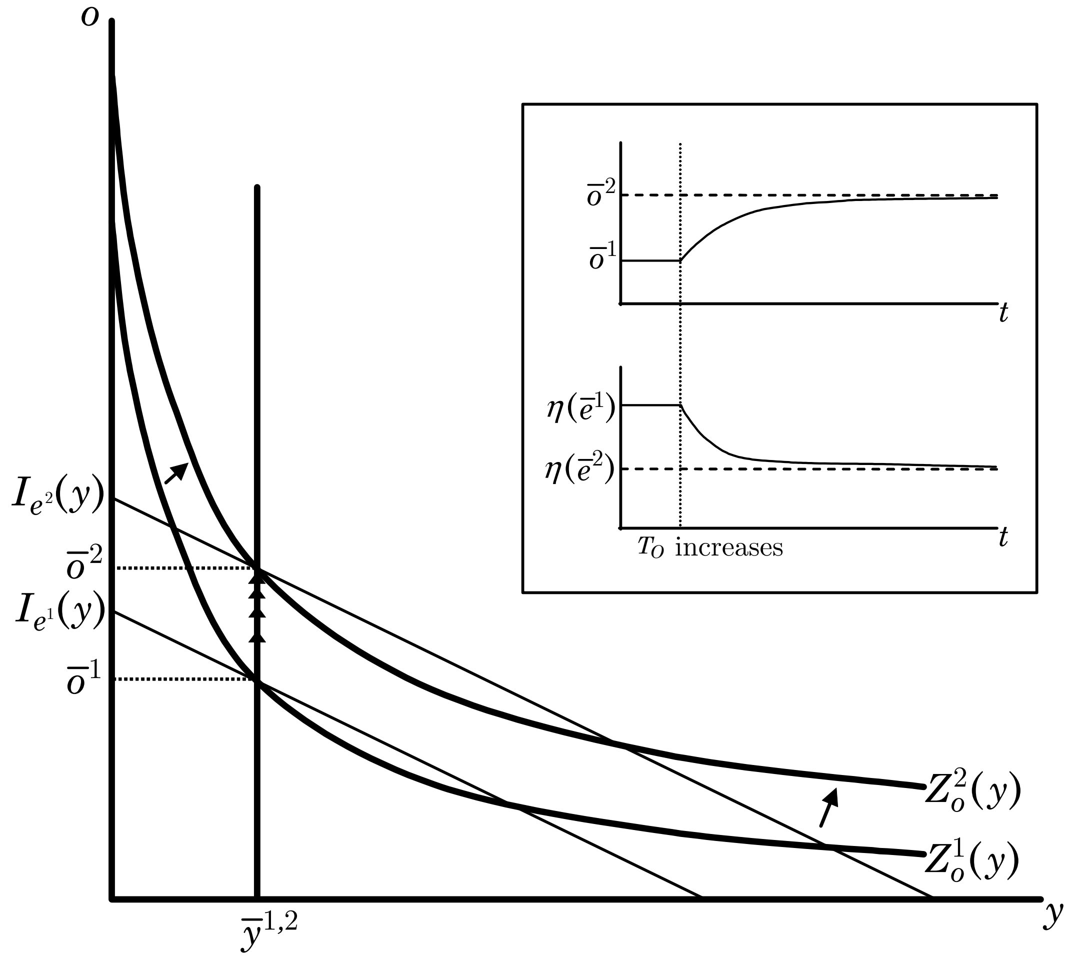
Old-age mortality has fallen substantially over the past fifty years, and is expected to decline further in the future. In the United States, for example, life expectancy at age 65 has risen by 3.25 years since 1955 and is expected to rise by a comparable amount over the next 50 years.[16] Other highly-developed countries have seen comparable or greater reductions in old-age mortality over the past half-century[17] and can be expected to follow a similar trend in the future. These gains in life expectancy will clearly be associated an increase in elderly dependency. Assuming a constant rate of fertility and constant

[15] See, e.g. Kohler, Billari and Ortega (2002).

[16] Here we use the simple, rather than gender-weighted average of sex-specific life-expectancies. The data on period life expectancies and come from the Board of Trustees, Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds (2005, Table V.A3). For the projected gain in life expectancy, we use the "intermediate" variant. Other independent mortality projections (e.g. Lee and Carter 1992) predict even higher values of life expectancy.

[17] See, e.g., Munnell, Hatch and Lee (2004).

Figure 5: The Effects of a Decrease in Old-age Mortality on Dependency and Consumption with Fertility Held Constant



values of the other parameters, Figure 5 demonstrates the effects of an increase in life expectancy among the elderly in the context of our model. As life expectancy rises from T_O^1 to T_O^2, the $Z_o(y)$ locus shifts upward, leaving youth dependency unchanged. The old-age dependency ratio increases monotonically toward the new equilibrium, while the period index of consumption declines due to the higher burden of elderly dependency. Reductions in old-age mortality also affect the optimal level of fertility in the economy.

Again, assuming that none of the other parameters change, the consumption-maximizing fertility rate reduces to a function of the life expectancy of the elderly. As life expectancy rises, so does G^m because the total cost of support per elderly dependent rises. An increase in fertility would cause the equilibrium youth share of the population to rise, but the effect of this on consumption is more than offset by the concomitant decrease in the elderly share. For countries that initially have a fertility rate $G^h > G^m$, the consumption-maximizing rate draws closer to their existing fertility rate. As seen in Figure 6, not only does the best-attainable level of consumption fall, so does the potential equilibrium gain from reducing the fertility rate.[18] For countries with a fertility rate $G^\ell < G^m$, the consumption-maximizing fertility rate moves further away as old-age mortality falls. Although the best-attainable level of consumption falls, the potential equilibrium gains from a higher fertility rate are greater.

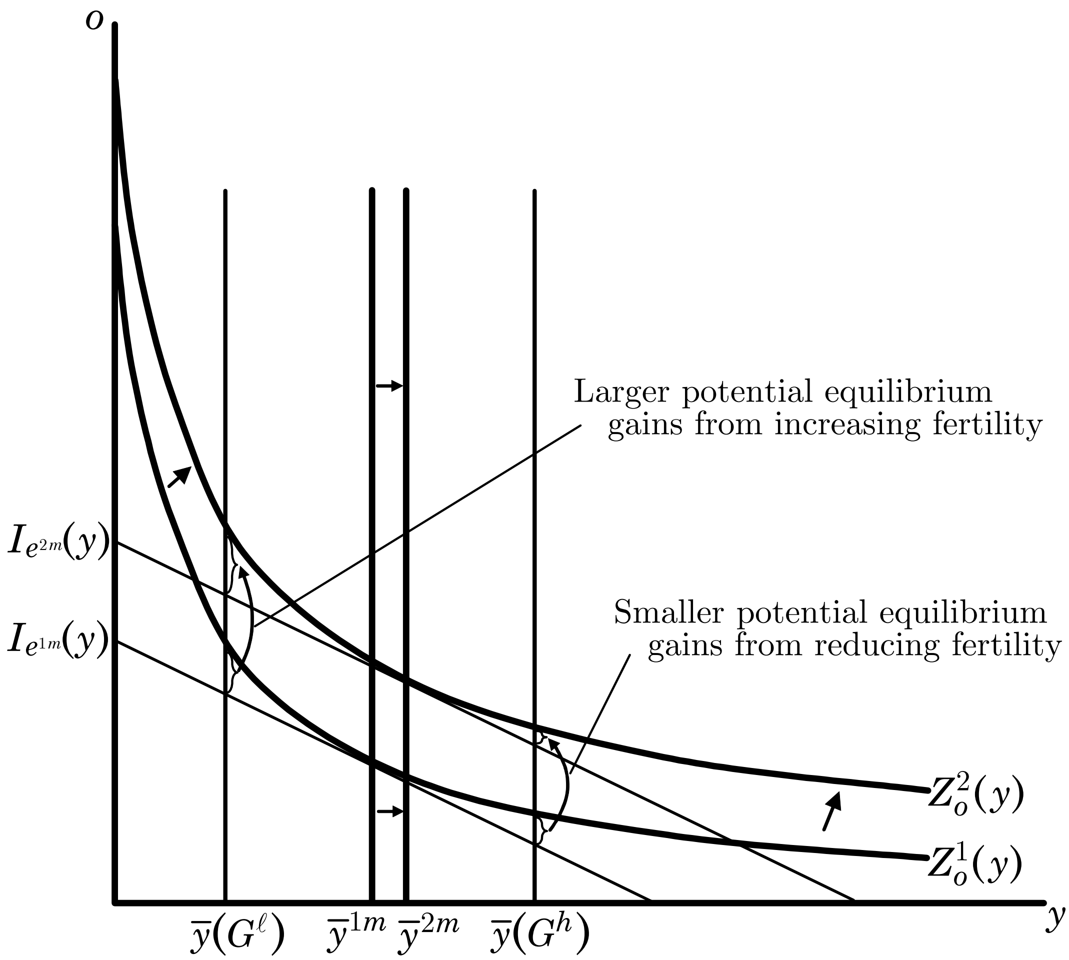
Repeating the numerical calculation from Section 3 for all OECD12 under the assumption that T_O increases by 3.25 years and actual fertility rates remain unchanged results in an average potential increase in η of 4.0%. Further, the United States joins the eleven other nations in having a fertility rate below the consumption-maximizing rate. As we will see below, the burden of old-age dependency itself may cause fertility rates to decline (rather than rise), leading to long-run outcomes that are even less efficient from a consumption standpoint.

4 Old-Age Dependency and Fertility

In our analysis of economic dependency and consumption thus far, we have taken the fertility rate as exogenous. This ignores the vast economics literature that regards fertility, for the most part, as the result of a decision making by potential parents who weigh the costs and benefits of having children. Hence, in addition to differences in consumption needs, there is a more profound asymmetry between youth dependency and

[18] This can be established analytically since the $Z_o(y)$ locus gets steeper as T_O increases.

Figure 6: The Effects of a Decrease in Old-age Mortality on the Consumption-Maximizing Equilibrium



elderly dependency. The economic burden of youth dependency is partially the result of the fertility choices of the working. On the other hand, adults have no choice over the number of elderly persons in the economy, but are typically required to support them through public pension systems. In a pay-as-you-go social security system, rising elderly dependency reduces the per-capita resources available to the working. Given standard economic models of child-bearing decisions, this potentially provides a partial explanation for declines in the fertility rate over the last half-century. Drucker (1990, p.7) takes a stronger stand on this issue, arguing that the primary cause of low and declining fertility in the developed world is that "its younger people are no longer able to bear the increasing burden of supporting a growing population of older, nonworking people."

Boldrin et al. (2005) provide some empirical evidence in support of this idea, employing a panel sample from 1960 to the present for the following countries: Austria, Belgium, Denmark, Finland, France, Ireland, Norway and Spain. Controlling for a measure of the generosity of the social security system (as well as infant morality), they find a strong and negative relationship between fertility and the share of the population 65 and over (p_{65}). In the eight countries in their sample the average increase in p_{65} between 1960 and 2000 was approximately 4.7 percentage points.[19] Applying their regression coefficient to this difference implies a predicted decline of 0.15 in the GRR, which was over a quarter of the average actual reduction in the GRR in the sample. Thus, increases in elderly dependency potentially explain a good deal of the recent declines in fertility.

We develop here an extension to our basic model that allows this sort of dynamic interaction between old-age dependency, fertility and the age structure. We retain the production structure described above, where workers provide labor inelastically in return for a wage of $W(t)$. Wages are subject to a per-worker tax τ, which is used to fund public

[19] Our data on the share of the population age 65+ (p_{65}) and fertility rates drawn from United Nations (2000 and 2004).

transfers to the old and to the young. We assume that the elderly are supported entirely through public transfers in the form of a pension system, while the young receive both public support and direct transfers from their parents. Although somewhat stylized, this distinction mirrors the substantial difference in the pattern of transfers to the old and young observed in reality. According to Mason et al. (2005), private transfers account for over 61% of the transfer-based consumption of persons under the age of 20 in the United States, but only 11% of the total transfers to persons over the age of 65.

The pension system replaces after-tax wages at a rate β, and we assume that per-capita public transfers to the young are equal to a fixed proportion, π, of the net transfers to the elderly. The tax rate at time t is determined by the aggregate resource equation

$$A_Y \alpha W(t)(1-\tau(t)) + A_M(t) W(t)(1-\tau(t)) + A_O \beta W(t)(1-\tau(t)) = \Omega(t) \ , \qquad (19)$$

where $\alpha = \pi\beta$. Solving (19) for the balanced-budget tax rate gives

$$\tau(t) = \frac{\alpha y(t) + \beta o(t)}{1 + \alpha y(t) + \beta o(t)} \, .$$

Higher rates of youth and old-age dependency both increase the tax rate. As a result, increases in both forms of dependency decrease the take-home wages of the workers and elderly, as well as reduce the public transfers to the young. Given values of α and β, we can solve for the tax-rate minimizing gross reproductive rate, G_τ^m, which maximizes the transfer-based consumption of all groups. Similar to the analysis in Section 3, G_τ^m is the implicit solution to

$$\bar{o}\left(G_\tau^m, \frac{T_Y}{T_M}, \frac{T_O}{T_M}\right) = \sqrt{\frac{\alpha}{\beta}\frac{T_Y}{T_M}} \, . \qquad (20)$$

Because of the large share of the consumption of children that is privately funded, (α/β)

will be substantially less than (ρ_Y/ρ_O). Since $\bar{o}$ is a negative function of G, this will imply that the tax-rate minimizing rate of fertility is substantially higher than that which maximizes consumption in the framework above (where all transfers were essentially publicly provided). We do not attempt to quantify G_τ^m, focusing instead on the divergence between the privately chosen rate of fertility and the tax-minimizing rate.

Because of the labor-based production structure in the economy, at each moment in time the fertile population of workers allocate their after-tax income to their own consumption and to bearing children.[20] Further, the price of consumption will be equal to the pre-tax wage rate. We assume that childrearing requires χ units of goods and services per period, which must be purchased on the market.[21] As a consequence, the cost of children will also be indexed to W. The nature of our model of the age structure makes it difficult to link children to their parents, so we assume that childrearing expenses are paid up front into a trust fund. With a constant rate of wage growth, g, to ensure that children always receive χ units of goods and services, an actuarially-neutral trust fund will set

$$p_n = \xi W(t) ,$$

where $\xi = \chi T_Y / (1 - gT_Y)$.[22]

For simplicity we assume a log utility function, which implies fertile workers face the

[20] Incorporating saving into this model will yield similar predictions as the one we present here. Changes in wages due to persistent changes in old-age dependency will represent changes in permanent income and should similarly affect consumption and fertility choices in the steady state. The main difference will be in the transitional dynamics; in a model with saving, consumption (and fertility) smoothing may lead to slightly different paths and speeds of approach toward steady state.

[21] The consumption requirement of children could be indexed to that of the parents, and we could also include a time cost of children. Incorporating these would yield qualitatively similar results to the model we present, so we ignore them for ease of exposition.

[22] For the path of trust fund payments to remain bounded, we require that $g < 1/T_Y$. Twenty years is a reasonable upper limit on the average length of youth dependency. This implies that g must be less than 5% over the long run, which is quite plausible.

following optimization problem

$$\max_{c(t),n(t)} \ln\left[c\left(t\right)\right] + \theta \ln\left[n\left(t\right)\right] \tag{21}$$

subject to

$$W\left(t\right)c\left(t\right) + \xi W\left(t\right)n\left(t\right) = w\left(t\right), \tag{22}$$

where $c\left(t\right)$ is the consumption, $n\left(t\right)$ is the annual flow of births per fertile worker, and $w\left(t\right) = W\left(t\right)/\left(1+\alpha y\left(t\right)+\beta o\left(t\right)\right)$ is the take-home wage of the working. Solving for the flow of births

$$\tilde{n}\left(t\right) = \psi\frac{w\left(t\right)}{W\left(t\right)} = \frac{\psi}{1+\alpha y\left(t\right)+\beta o\left(t\right)}, \tag{23}$$

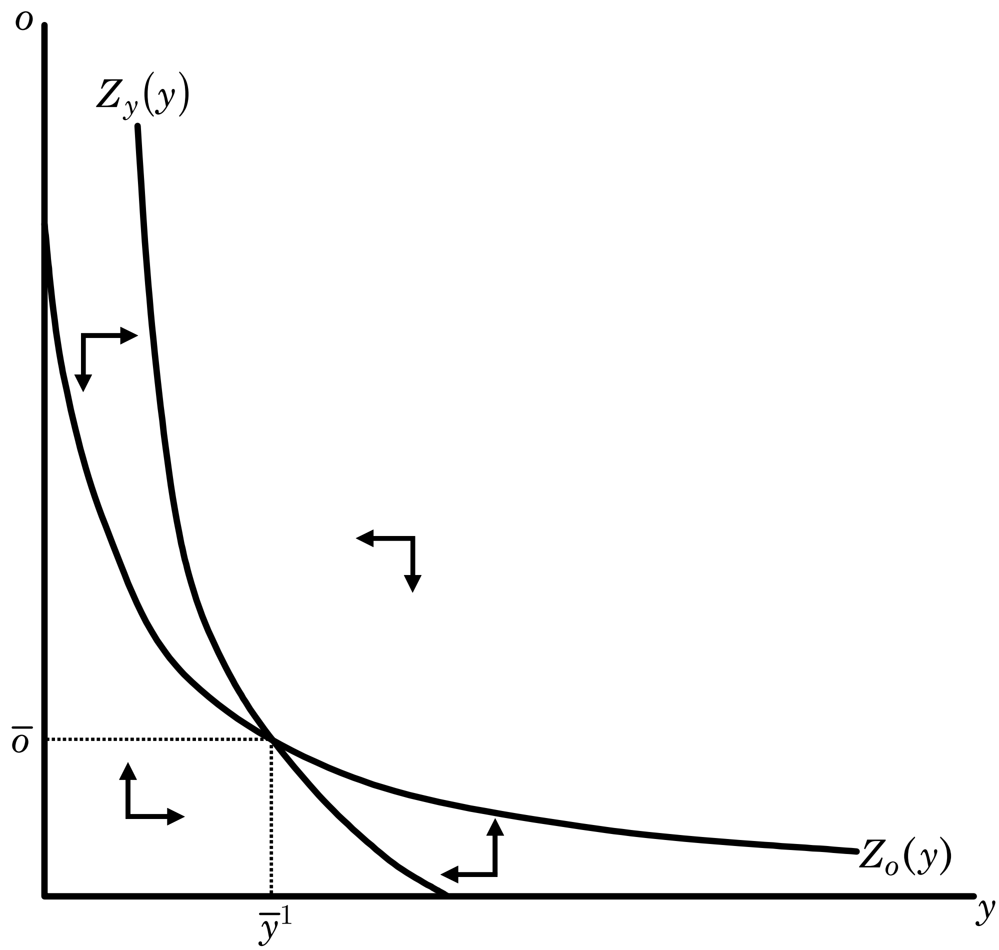
where $\psi = \theta/(\xi(1+\theta))$. The flow of births per fertile worker satisfies some usual properties; fertility is a positive function of the relative preference for children (θ) and a negative function of childrearing costs. Finally, due to a standard income effect, increases in o (as well as y) reduce fertility. As a result, the dynamics of the youth dependency ratio will also be affected by the old-age dependency ratio.

Putting the flow of births per fertile worker, $\tilde{n}$, into the equation of motion of the youth dependency ratio (equation (9)), the graph of the zero-motion locus of y in (y, o)-space is

$$o\big|_{\dot{y}=0} = \frac{1}{\beta}\left[\frac{\phi\psi T_Y}{\left(1-T_Y/T_M\right)y+y^2} - \left(1+\alpha y\right)\right] \equiv Z_y\left(y\right).$$

This locus represents equilibrium values of y that are stable (because $d\dot{y}/dy < 0$). Further, it is downward sloping and convex with respect to the origin, asymptotes to infinity

Figure 7: Global Dynamics with Endogenous Fertility



as y approaches zero, and crosses the axis for large enough values of y. Finally, there is a single crossing in the positive orthant between the $Z_y(y)$ and $Z_o(y)$ loci, which implies that a globally stable equilibrium in the population dependency ratios, depicted in Figure 7.[23]

We depict the transition associated with a fall in mortality in this model of endogenous fertility in Figure 8. As discussed above, lower rates of old-age mortality translate directly into increases in the rate of elderly dependency due to longer life expectancy

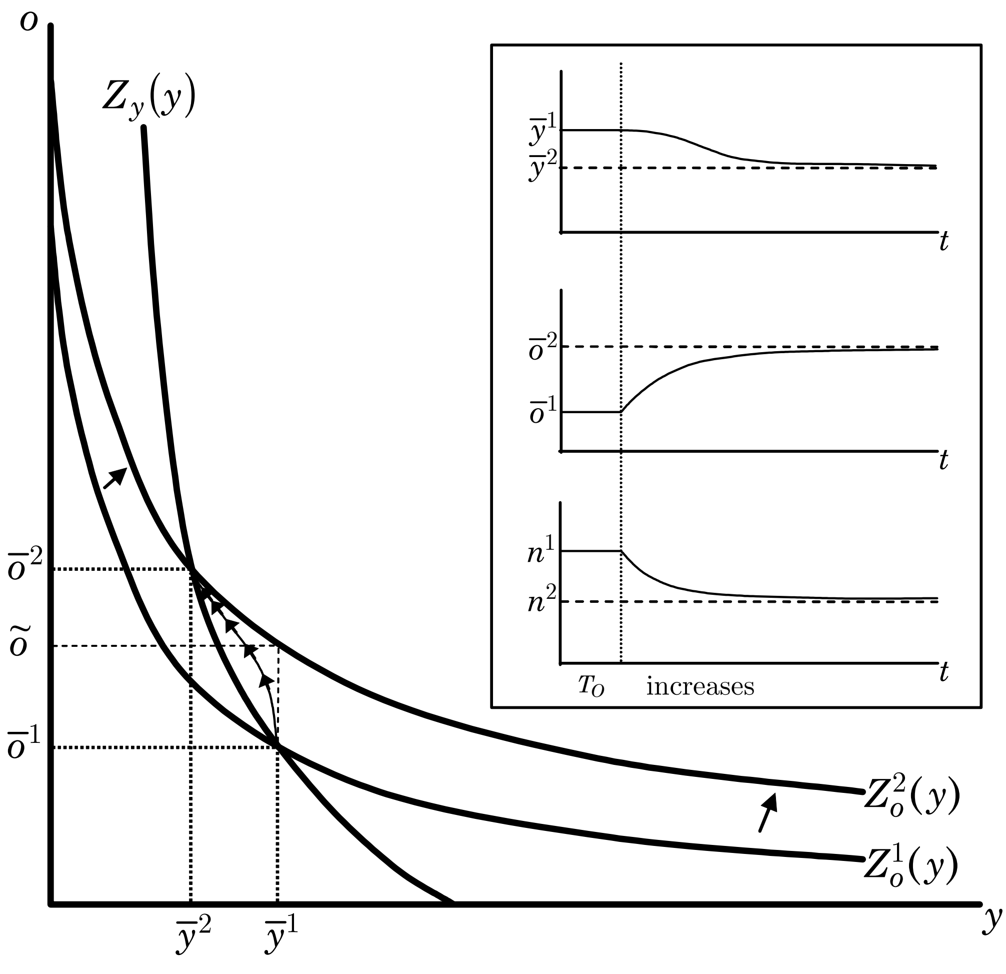
[23]A proof of the single crossing between the loci is available upon request.

after retirement. If fertility rate were to remain unchanged, old-age dependency would rise from $\bar{o}^1$ to $\tilde{o}$, and the youth dependency ratio would remain constant at $\bar{y}^1$. However, increases in old-age dependency drive up the social security tax rate in our model, given the government's balanced budget constraint. Since workers will have less disposable income, the rate of childbirth among the fertile will fall. We have already shown in Section 3 that declines in fertility lead, *ceteris paribus*, to increases in old-age dependency. As a result, there is a multiplier effect on declines in old-age mortality, whereby increases in of old-age dependency reduce fertility, which further increase old-age dependency. However, as the fertility rate falls, so does the youth dependency ratio. The consequent reductions in public expenditures on the young acts as a brake on the feed-back cycle between rising old-age dependency and falling fertility. Nontheless, the net result is a transition to an equilibrium with an old-age dependency ratio $\bar{o}^2$ that is even higher than $\tilde{o}$. Further the ratio of young to working falls from $\bar{y}^1$ to $\bar{y}^2$ due to the declines in fertility induced by the rise in elderly dependency.

As life expectancy among the elderly rises, the tax-minimizing fertility rate ($n_\tau^m \equiv G_\tau^m/\phi$) will increase due to the mechanism discussed in Section 3.3. However, the response of the fertile workers to an increase in life expectancy is to reduce their fertility. As a result, in the highly developed countries where $\tilde{n}$ is initially less than n_τ^m, the privately-optimal fertility response of workers to a decrease in old-age mortality results in a reduction of workers' take-home pay in the steady state. It also lowers the steady-state transfer-based consumption of the young and the elderly. Further, it leads to a reduction in their utility relative to the fertility rate, $\hat{n}$, chosen by a social planner whose goal is to maximize the equilbrium utility of the fertile workers.

To see this last point, consider the optimization problem of the social planner who takes into account the both the utility that fertile workers get from fertility and consumption as well as the effects of fertility decions on the dependency ratios. Substituting in

Figure 8: The Effects of a Decrease in Old-age Mortality on Dependency and Fertility



the budget constraint (22) into the objective function (21), the social planner's problem can be written

$$\max_{\hat{n}} \ln\left[\frac{1}{1+\alpha y(\hat{n})+\beta(\hat{n})}-\xi\hat{n}\right]+\theta\ln[\hat{n}]\ . \tag{24}$$

The first order condition for (24) can be written

$$\hat{n}=\tilde{n}(\hat{n})+\frac{1}{\xi(1+\theta)}\frac{[-\alpha y'(\hat{n})-\beta o'(\hat{n})]\,\hat{n}}{(1+\alpha y(\hat{n})+\beta o(\hat{n}))^2}\,, \tag{25}$$

where $\tilde{n}(\hat{n})$ is the fertility rate that workers would chose, conditional on the equilibrium $(o(\hat{n}), y(\hat{n}))$. Based on the equilibrium equations (13) and (14) term in brackets on the right-hand side of (25) is equal to

$$[-\alpha y'(\hat{n})-\beta o'(\hat{n})]=\left[-\alpha+\beta[o(\hat{n})]^2(T_M/T_Y)\right]y'(\hat{n})\ .$$

Since $y'(\cdot)$ is negative, the social planner's chosen fertility rate, $\hat{n}$, is higher than the worker's private choice based on $\hat{n}$ when

$$\beta[o(\hat{n})]^2(T_M/T_Y)-\alpha<0\,,$$

and a lower fertility rate otherwise. Consequently, $\hat{n}$ equals $\tilde{n}(\hat{n})$ only when (20) holds, which implies that $\hat{n}=n_\tau^m$. That is, optimizing the fertile workers' equilibrium utility is equivalent to setting the fertility rate equal to that which minimizes the tax rate. Thus, as fertile workers in highly-developed nations reduce their fertility in response to declining old-age mortality, this will eventually lead to reductions in the well-being of future workers as well as to decreases the pool of resources available to all of the other individuals in the economy.

5 Conclusion

In this paper we have analyzed the dynamic evolution of a country's population age structure and fertility rate. Our particular concern was with the feedback from population age structure to fertility via the channel of old age dependency. In a country with a high level of old-age dependency, working age individuals will see a large fraction of their labor income redistributed to the elderly. In response, working age individuals will lower fertility.

The dynamic aspects of the problem that we study are particularly important because the short and long-run effects of changes in fertility on dependency are so different. In the short run, a reduction in fertility, by reducing the number of children relative to working age adults, unambiguously lowers a society's dependency burden. In the long run, reductions in fertility raise a country's of old age dependency ratio, potentially undoing the reduction in youth dependency. Our calculations indicate that the countries in the word with the lowest levels of fertility have already passed the point where reductions in fertility raise rather than lower the long-run dependency burden.

To conduct our analysis we constructed a new continuous-time overlapping generations model that divides the population into three age groups: dependent young, working age, and dependent elderly. Individuals in each age group face constant hazards of transitioning into the next group (or into death, in the case of the elderly). The model allows us to examine the dynamic evolution of age structure of the population and the consumption dependency burden in response to changes in fertility (in the case where fertility is exogenous) or the joint evolution of fertility, age structure, and dependency to exogenous shocks such as old age mortality.

Using our model, we show that, for countries that are already below the level of fertility that maximizes consumption in the steady state, the actual and optimal responses of fertility to a shock to old-age survival have opposite signs. In response to greater old age survival, such countries will effectively dig themselves into a deeper demographic hole by cutting fertility in order to maintain consumption in the short run.

The model that we present is somewhat stylized, in the interests of analytic tractability and ease of exposition. It could be extended along two major dimensions in order to improve its realism and forecast accuracy. First, it would be useful to expand the number of age groups beyond the three basic ones we use. Moving beyond three age groups would mean that the dynamics of the model could not be examined analytically, however, and we would be forced to take a computational approach along the lines of Auerbach and Kotlikoff (1987). Second, the model could also be extended to incorporate intertemporal optimization on the part of households, which would entail shifting consumption between periods in response to anticipated changes in demographics. Such intertemporal shifting, either through investment in physical capital or purchase of foreign assets, could allow agents to save for their own old age, or the country as a whole to save for the period when there will be a high level of old-age dependency. Although this would have little effect on the steady state of the model, it would significantly influence the model's dynamics. As discussed in Cutler et al. (1990) and Elmendorf and Sheiner (2000), the ability of the economy as a whole to insulate itself against demographic shocks by building extra capital is limited by both depreciation and capital's declining marginal product. Attempts to smooth consumption in the face of demographic shocks may also result in a run-up of price of capital assets when everyone is trying to save and a corresponding asset meltdown when everyone is trying to dis-save.[24]

We suspect that the extensions discussed above would not alter the fundamental predictions of our model, which are relatively grim. Demographers have been wrestling for decades with explaining below-replacement fertility in some of the world's richest

[24]See Lim and Weil (2003).

countries. Low fertility is viewed as a troubling outcome in and of itself. The failure of a country's citizens to replace themselves is cited as evidence of some sort of social illness. Ironically, this hand-wringing has taken place during a period when the economic consequences of low fertility were positive. Our results suggest that over the next several decades, as the effect of low fertility on consumption possibilities turns from positive to negative, there will be a further reduction in fertility, which will, in the long run, produce further reductions in consumption.

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