

Nonparametric Estimation of Disability-Free Life Expectancy Using Period Life Table and Cross-Sectional Disability Survey*

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Abstract

A rapidly aging population, such as the United States today, is characterized by the increased prevalence of chronic impairment, especially pronounced among the elderly. An important question is whether additional years of life are spent in poor health. Robust estimation of disability-free life expectancy (DFLE) is essential for addressing this question. Thirty years after publication, Sullivan's method still remains the most widely used method for estimating DFLE when large-scale longitudinal data are not available. Sullivan's method simply partitions the total number of person-years lived in a given age interval by the proportion disabled in that interval. We prove that in doing so Sullivan's method imposes a previously unnoticed assumption. To improve upon Sullivan's method, we relax this assumption and derive the nonparametric bounds of DFLE. A bootstrap method is used to compute the balanced confidence intervals for the bounds. It is also possible to improve these bounds by incorporating additional assumptions that are theoretically credible. We identify such assumptions and show that under these conditions Sullivan's method is likely to underestimate the DFLE. Finally, we apply the proposed methodology to estimate DFLE for the 1999 United States population using the data from the period life table, the National Health Interview Survey, and the National Nursing Home Survey. We find important race, sex, and education differentials in DFLE and proportion of remaining life spent without disability.

Key Words: Bounds, Demography, Disability-adjusted Life Expectancy, Morbidity, Mortality, Sullivan's Method.

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1 Introduction

A rapidly aging population, such as the United States today, is characterized by the increased prevalence of chronic impairment, especially pronounced among the elderly. An important question is whether additional years of life are spent in poor health. Robust estimation of disability-free life expectancy (DFLE) is essential for addressing this question because DFLE represents the expected number of years of *disability-free* life a member of the life table cohort would experience if current age-specific rates of mortality and disability prevailed throughout the cohort's lifetime.

In his seminal paper, Sullivan (1971) developed a method for combining mortality and morbidity rates into a single summary measure of a population's health status. Over thirty years after publication, Sullivan's method remains the most widely used method for estimating DFLE when large-scale longitudinal data are not available. The key idea of the method is to combine the period life table with age-specific disability prevalence from survey data to estimate DFLE. In particular, Sullivan's method simply partitions the total number of person-years lived in a given age interval, which is obtained from the life table, into the disabled and disability-free life expectancy based on the proportion disabled in that interval, which is measured from the disability survey.

Sullivan's method is of prime methodological importance in the ongoing exploration of morbidity (e.g., Crimmins *et al.*, 1989, 1997). It has been used extensively to estimate DFLE in various populations (e.g., Iburg *et al.*, 2001) as well as differences in DFLE by socioeconomic status (e.g., Molla *et al.*, 2004; Sihvonen *et al.*, 1998), educational levels (e.g., Minicuci, 2004), occupational groups (e.g., Bronnum-Hansen, 2000), and between time periods (e.g., Graham *et al.*, 2004; Crimmins *et al.*, 1989; Bronnum-Hansen *et al.*, 2004). The method has also been used to estimate the burden of disease from chronic conditions such as diabetes (e.g., Manuel and Schultz, 2004) and the contribution of specific diseases to educational disparities in DFLE (e.g., Nusselder *et al.*, 2005). Nusselder and Looman (2004) analyzed the contribution of various causes of death and disability

to differences in health expectancy among populations and over time. Murray and Lopez (1996) compared the disability-adjusted life expectancy, which is a quantity closely related to DFLE, across world regions using Sullivan’s method as part of the Global Burden of Disease Study. In addition to academic researchers, a number of governments and international health organizations employ Sullivan’s method. The United States (U.S.) National Center for Health Statistics uses Sullivan’s method to compute health expectancy as part of the Healthy People 2010 Study (Molla *et al.*, 2003). The Australian Institute of Health and Welfare also used Sullivan’s method in its 1996 Burden of Disease study (Mathers *et al.*, 2001). The World Health Organization also used the method to estimate disability-adjusted life expectancy for 191 member states (Musgrove *et al.*, 2000).

Robust estimation of DFLE is also vital to the theoretical understanding of morbidity. The existing research on DFLE of various populations and time periods has often reached contradictory conclusions regarding the competing nature of mortality and morbidity. For example, Gruenberg (1977) and Kramer (1980) argue that the decline in mortality rates only reflects a decline in the fatality rate of chronic diseases rather than a decline in their incidence. Greater life expectancy will result in more severe chronic diseases. Fries (1980), on the other hand, argues for the compression of morbidity. If the onset of the chronic condition can be postponed and adult life expectancy is relatively constant, morbidity will be compressed into a shorter period of time. Moreover, Manton (1982) offers an alternative theory of dynamic equilibrium in which the decline in mortality leads to an increase in the prevalence of milder chronic diseases. Therefore, the accurate estimation of DFLE is essential for the empirical evaluation of these competing theories.

In this paper, we identify the previously unnoticed assumption of Sullivan’s method and propose an alternative nonparametric method that avoids such an assumption. In Section 2, we prove that Sullivan’s method imposes an assumption about the conditional probability of death and

the average number of person-years lived among those who will die within each age interval. In Section 3, we relax this assumption and derive nonparametric bounds for DFLE. We use bootstrap to compute the balanced confidence intervals for the nonparametric bounds. We also show how to improve these bounds by incorporating additional assumptions that are theoretically credible. In particular, the age-specific conditional probability of death is often significantly higher for the disabled than for the disability-free. Another plausible assumption is that the frailer are more likely to die earlier within a given age interval. We show that under these assumptions, Sullivan’s method is likely to underestimate the DFLE. In Section 4, we apply our proposed methodology to estimate DFLE for the 1999 U.S. population using the data from the period life table, the National Health Interview Survey, and the National Nursing Home Survey. We find important race, sex, and education differentials in DFLE and proportion of remaining life spent without disability. In Section 5, we present conclusions.

2 Sullivan’s Method Revisited

Sullivan’s method utilizes the mortality data from a period life table and the disability prevalence data from a cross-sectional survey. Of course, when longitudinal data are available, DFLE is best modeled by a multi-state life table, which follows a specific cohort and provides the detailed information about each individual’s transitions between the disability-free and disabled status and from each status to death over time (e.g., Rogers *et al.*, 1989; Robine *et al.*, 1995). Yet, multi-state life table methodology is often difficult to implement because of considerable data requirements (Mathers and Robine, 1993). Sullivan’s method continues to be widely used to estimate DFLE because of the inherent difficulty in collecting large-scale longitudinal data. In contrast, the life table mortality data and various large-scale cross-sectional disability surveys are publicly available. In this section, we first introduce the standard notation of the period life table used in the field of demography (e.g., Chiang, 1984; Preston *et al.*, 2001) and define the estimand, DFLE. We then

derive the key assumption of Sullivan’s method.

2.1 Period Life Table and Life Expectancy

A main purpose of period life tables is to calculate the life expectancy of a hypothetical cohort that experiences the same cross-sectional mortality rates during a particular period. In general, the construction of period life tables assumes the stationarity of the population during that period. Let x represent the age at the start of an age group interval. Period life tables assume discrete time, and are typically measured by years. In this paper, we first consider *unabridged* period life tables where the length of age group interval is one year. As discussed in Section 3.4, the estimation of DFLE using *abridged* period life tables where the interval length is two years or longer is possible but requires stronger assumptions.

Period life tables are created by first observing the mid-year population, P_x , and the total number of deaths, D_x , for each interval starting at age $x = 0, 1, 2, \dots, \omega$, where ω is the oldest age observed. The observed mortality rate M_x for each interval is calculated as the ratio of D_x and P_x , i.e., $M_x = D_x/P_x$. Given the stationarity assumption and the fact that P_x and D_x are directly obtained from the Census data, the mortality rate of the hypothetical cohort m_x is assumed to equal the observed mortality rate of the population M_x without sampling variability. Then, it can be shown that the conditional probability of death q_x is equal to $m_x/[1 + (1 - a_x)m_x]$ where a_x ($0 < a_x < 1$) is the average person-years lived in the interval $[x, x + 1)$ among those who are alive at age x but die in the interval. Although period life tables are based on discrete time, the values of a_x are often obtained from the complete life tables and used in subsequent calculations as a known quantity (e.g., Preston *et al.*, 2001; Molla *et al.*, 2001).

Given q_x and the known value of l_0 , which represents the total number alive at age 0, we can derive the remainder of the life table. First, the number of survivors at age x is sequentially defined as $l_{x+1} = (1 - q_x)l_x$ for each $x = 0, 1, \dots, \omega$. If l_0 is set to 1, then l_x represents the survival

Age	l_x	q_x	a_x	L_x	e_x
20	0.986	0.001	0.506	0.986	55.693
25	0.982	0.001	0.500	0.981	50.943
30	0.977	0.001	0.495	0.976	46.179
35	0.971	0.001	0.500	0.970	41.437
40	0.963	0.002	0.500	0.962	36.754
45	0.952	0.003	0.500	0.950	32.160
50	0.935	0.004	0.500	0.933	27.686
55	0.911	0.007	0.499	0.908	23.345
60	0.875	0.011	0.501	0.870	19.213
65	0.820	0.016	0.500	0.813	15.310
70	0.743	0.025	0.500	0.733	11.629
75	0.638	0.038	0.500	0.626	8.112
80	0.505	0.059	0.500	0.490	4.565
85	0.345	1.000	0.500	0.328	0.500

Table 1: 1999 U.S. Period Life Table and Life Expectancy for Selected Ages. The period life table is created from the conditional probability of death, q_x , and the average person-years lived in the age interval by those dying in the interval, a_x . l_x is the proportion of survivors at age x , whereas L_x represents the total number of person-years lived within the age interval $[x, x+1)$ for those who were alive at age x . The final column gives the life expectancy e_x at each age.

probability at age x . Next, the total number of person-years lived within the age interval $[x, x+1)$ among those who are alive at age x is given by,

$$L_x \equiv (1 - q_x)l_x + d_x a_x, \quad (1)$$

$$= l_x[1 + q_x(a_x - 1)], \quad (2)$$

where d_x represents the number of death in the age interval $[x, x+1)$, i.e., $d_x \equiv l_x - l_{x+1} = l_x q_x$.

Each of those who do not die within the age interval $[x, x+1)$ contributes one person-year and each of those who die within the interval contributes a_x person-years on average. Finally, by adding the total number of person-years for the subsequent age intervals, the life expectancy at age x is defined as,

$$e_x \equiv \frac{1}{l_x} \sum_{i=x}^{\omega} L_i. \quad (3)$$

Table 1 shows the 1999 U.S. unabridged period life table for selected ages (Arias, 2002). The radix, l_0 , is set at 1.0 (not shown) so that l_x represents the survival probability. At age 25 years,

98.2% of the hypothetical life table cohort remains. From age 25 to ω , the remaining 98.2% of the cohort will live $\sum_{i=25}^{\omega} L_i = 50.0$ person-years. Hence, a 25 year-old member of the hypothetical cohort will live, on average, $e_x = 50.9$ years given he or she experiences the prevailing period age-specific conditional probabilities of death. See Section 4 for a comprehensive analysis of the 1999 U.S. population.

2.2 Defining Disability-Free Life Expectancy (DFLE)

In a manner completely analogous to the derivation of the overall life table developed in Section 2.1, a disability-free life table may be constructed from the age-specific disability prevalence at age x , denoted by π_x , and the period life table. For any age x , the total number of disability-free persons starting the age interval is defined as,

$$l_x^{DF} \equiv (1 - \pi_x)l_x. \quad (4)$$

Analogous to equation 2, the person-years lived disability-free in the interval $[x, x + 1)$ is given by,

$$L_x^{DF} \equiv (1 - q_x^{DF})l_x^{DF} + d_x^{DF} a_x^{DF}, \quad (5)$$

$$= (1 - \pi_x)l_x [1 + q_x^{DF}(a_x^{DF} - 1)], \quad (6)$$

where d_x^{DF} represents the number of deaths among the disability-free in the given age interval, i.e., $d_x^{DF} = l_x^{DF} q_x^{DF}$. Each disability-free survivor contributes one person-year while those who die contribute a_x^{DF} person-years. While $l_{x+1} = (1 - q_x)l_x$ holds for the overall population, the equation does not necessarily hold for the disability-free population, i.e., $l_{x+1}^{DF} \neq (1 - q_x^{DF})l_x^{DF}$. This is because some previously healthy people transition to the disabled state at the beginning of the next age interval and vice versa. It is assumed that such transitions do not occur during age intervals. While a_x and q_x are best measured separately for the disabled and disability-free populations, this is often not the case. The life table gives q_x for each x but does not give q_x^{DF} . Similarly, in most cases a_x^{DF} is not directly observed and only a_x is observed.

The estimand DFLE can now be defined as,

$$e_x^{DF} \equiv \frac{1}{l_x} \sum_{i=x}^{\omega} L_i^{DF}. \quad (7)$$

DFLE can be interpreted as the expected number of years of disability-free life a member of the life table cohort would experience if current age-specific rates of mortality and disability prevailed throughout the cohort's lifetime. Finally, the same argument can be repeated to construct the life table for the disabled population, which is of less interest to researchers than the disability-free population. We use the superscript D to denote the same quantities for the disabled population (e.g., q_x^D and a_x^D).

2.3 Sullivan's Method

Sullivan's estimator of DFLE is found by partitioning the person-years lived in the age interval into the proportion with and without disability based on the disability prevalence. Within each age interval, the number of person-years lived is simply multiplied by the fraction of the disability-free at the beginning of the interval. Specifically, Sullivan's estimator of e_x^{DF} is defined by,

$$\widehat{e_x^{DF}} \equiv \frac{1}{l_x} \sum_{i=x}^{\omega} (1 - \pi_i) L_i \quad (8)$$

where π_i is ordinarily estimated using the sample average of the disability prevalence within each age group i of the cross-sectional survey data.

A common way to obtain the variance of Sullivan's estimator is to assume that the total number of the disabled within each age interval follows an independent binomial process (e.g., Mathers, 1991; Montpellier, 1997; Molla *et al.*, 2001). Given this assumption, if π_i is being estimated using the sample average, then the variance of Sullivan's estimator is equal to,

$$\text{Var} \left(\widehat{e_x^{DF}} \right) = \frac{1}{l_x^2} \sum_{i=x}^{\omega} \frac{\pi_i (1 - \pi_i) L_i^2}{N_i}, \quad (9)$$

where N_i represents the total number of survey respondents within the age group $[x, x + i)$. This

variance is then estimated by substituting the sample average of the disability prevalence for each i obtained from the disability survey data.

2.4 The Key Assumption of Sullivan's Method

The following proposition shows that Sullivan's method relies on a key assumption concerning the relationship between the conditional probability of death and the average person-years lived within a given interval among those who die in the interval.

PROPOSITION 1 *Sullivan's method consistently estimates the disability-free life expectancy at every age if and only if $q_x(a_x - 1) = q_x^D(a_x^D - 1) = q_x^{DF}(a_x^{DF} - 1)$ for all $x = 0, 1, 2, \dots, \omega$.*

The proof is given in Appendix. The proposition implies that in order for the Sullivan's estimator to be consistent for every age group, the equality $q_x(a_x - 1) = q_x^D(a_x^D - 1) = q_x^{DF}(a_x^{DF} - 1)$ must hold for all x . First, if $q_x^{DF} = q_x^D$ and $a_x^{DF} = a_x^D$, then the condition is satisfied. This amounts to the assumption that the disabled die at the same rate as the disability-free population throughout the cohort's lifetime. Second, although by definition a_x , a_x^{DF} , and a_x^D are strictly less than 1 and empirically they are never equal to 1, it is important to note a limiting case $a_x = a_x^{DF} = a_x^D = 1$ where the equality, $q_x(a_x - 1) = q_x^D(a_x^D - 1) = q_x^{DF}(a_x^{DF} - 1)$, is satisfied regardless of the value of q_x , q_x^{DF} , and q_x^D . In this case, everyone dies only at the beginning of age interval, and the mortality rate m_x equals the conditional probability of death q_x . Then, it follows that $L_x = l_x$ and $L_x^{DF} = (1 - \pi_x)L_x$, which imply the consistency of Sullivan's estimator. Finally, it can be readily seen that in continuous time, Sullivan's estimator directly corresponds to DFLE, via the identity $\int_x^\infty l_t^{DF} dt = \int_x^\infty (1 - \pi_t) l_t dt$, which follows directly from equation 4.

3 Methodology

In this section, we propose a nonparametric method that improves upon Sullivan's method. Our goal is to estimate DFLE without making the assumption identified in the previous section. To do

this, we apply the method of bounds. This method has been employed in many areas of statistics, including causal inference (e.g., Manski, 1990; Balke and Pearl, 1997; Robins, 1989), ecological inference (e.g., Duncan and Davis, 1953), retrospective sampling (e.g., King and Zeng, 2002), and missing data (e.g., Horowitz and Manski, 1998), where due to the limitation of data availability, strong and often controversial assumptions are invoked. In such situations, the method of bounds is particularly attractive because it illuminates how informative the data *alone* are about quantities of interest, thereby separating the issue of identification from that of estimation (see Manski, 2003, for a comprehensive monograph).

3.1 Nonparametric Bounds of DFLE

Proposition 1 in Section 2.4 shows that Sullivan’s estimator *point identifies* e_x^{DF} by assuming $q_x^{DF}(a_x^{DF} - 1) = q_x^D(a_x^D - 1) = q_x(a_x - 1)$. Here, we avoid this assumption and derive the nonparametric bounds of DFLE. The following proposition gives the upper and lower bounds of DFLE. These bounds are sharp, i.e., they cannot be made narrower without imposing an additional assumption.

PROPOSITION 2 *For each age group $x = 1, 2, \dots, \omega$, the sharp upper and lower bounds of DFLE are given by,*

$$B_x^U = \frac{1}{l_x} \sum_{i=x}^{\omega} (1 - \pi_i) l_i \min \left[1, \frac{q_i(a_i - 1) + 1}{1 - \pi_i} \right], \quad (10)$$

$$B_x^L = \frac{1}{l_x} \sum_{i=x}^{\omega} (1 - \pi_i) l_i \max \left[0, \frac{q_i(a_i - 1)}{1 - \pi_i} + 1 \right], \quad (11)$$

respectively.

The proof is given in Appendix. Note that given an age group i if $\pi_i \geq q_i(1 - a_i)$, which is likely because q_i tends to be much smaller than π_i , then $\min \left[1, \frac{q_i(a_i - 1) + 1}{1 - \pi_i} \right] = 1$. Similarly, if $\pi_i \geq 1 - q_i(1 - a_i)$, which is unlikely for the same reason, $\max \left[0, \frac{q_i(a_i - 1)}{1 - \pi_i} + 1 \right] = 0$.

The lower and upper bounds, B^U and B^L , can then be consistently estimated by substituting the consistent estimate of π_x from the disability survey data, since the only unknown quantity in the expressions of the bounds is the prevalence of disability. The consistent estimate of π_x can be easily obtained from the survey data, for example, by computing the weighted average of the disability indicator variable within each age group. An alternative and perhaps more appropriate estimation strategy, when estimating the disability prevalence of a hypothetical cohort from a cross-sectional dataset, is to model the disability prevalence as a smooth function of age and use a nonparametric regression.

The length of the bounds $B_x^U - B_x^L$ depends on the observed values of a_i , q_i , and π_i for $i = x, x+1, \dots, \omega$ as well as l_x . Here, we consider how the quantity directly related to the informativeness of the bounds of DFLE, $(1 - \pi_i) \left\{ \min \left[1, \frac{q_i(a_i-1)+1}{1-\pi_i} \right] - \max \left[0, \frac{q_i(a_i-1)}{1-\pi_i} + 1 \right] \right\}$, varies as a function of q_i and π_i for a particular age group i (a_i is set to 0.5). The left panel of Figure 1 presents this quantity as a function of q_i and π_i using a level plot where darker shades represent smaller values of this quantity, thereby implying narrower and more informative bounds of DFLE. The plot shows that the bounds tend to be informative when q_x and π_x are small. The plot also presents the actual mortality and estimated disability prevalence for ages 25 to 85 of the 1999 U.S. population as solid circles using the data from the period life table (see Table 1), the National Health Interview Survey (NHIS), and the National Nursing Home Survey (NNHS) (See Section 4 for a comprehensive description of these data and the definition of disability we use). The data are quite informative about DFLE because q_x and π_x are generally small in this population, suggesting the method of bounds may be useful for estimating DFLE.

3.2 Monotonicity Assumptions

It is straightforward to incorporate additional inequality (or monotonicity) assumptions into the method of bounds while still avoiding the strong equality assumption of Sullivan's method, i.e.,

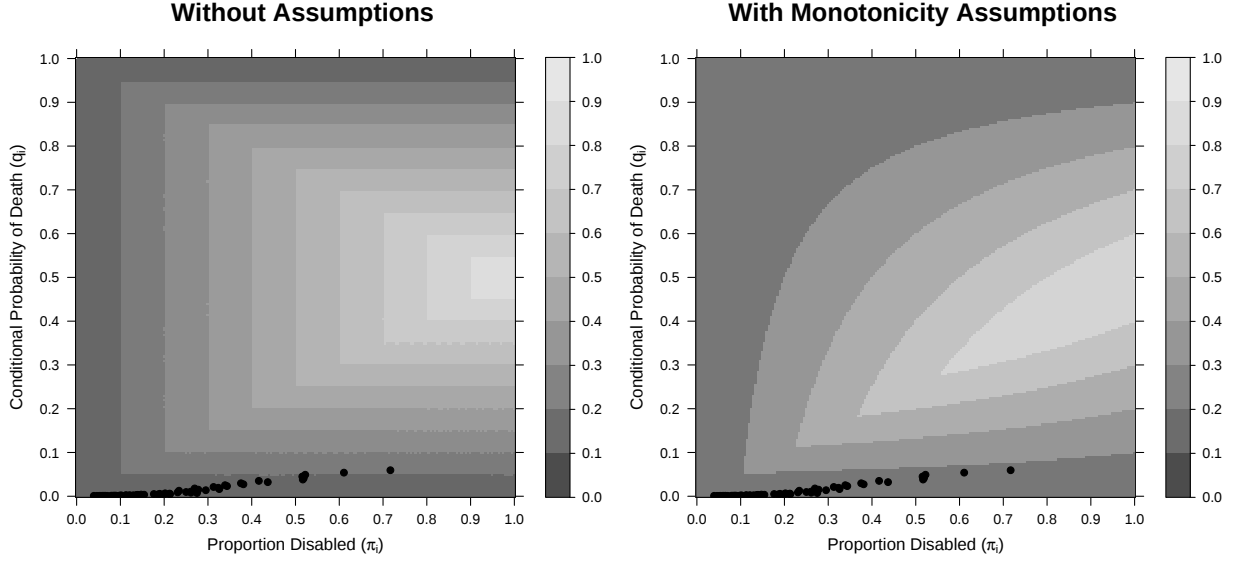


Figure 1: The Informativeness of the Nonparametric Bounds for DFLE as a Function of Disability Prevalence, π_i , and the Conditional Probability of Death, q_i , Without Assumptions and With Monotonicity Assumptions. The value of a_i is set to 0.5 years. The plots show the quantity directly related to the length of the bounds of DFLE, $(1 - \pi_i) \left\{ \min \left[1, \frac{q_i(a_i - 1) + 1}{1 - \pi_i} \right] - \max \left[0, \frac{q_i(a_i - 1)}{1 - \pi_i} + 1 \right] \right\}$, as a function of q_i and π_i . Darker shades represent smaller values of this quantity, thereby implying narrower and more informative bounds of DFLE. The sold circles represent the actual mortality and estimated disability prevalence for ages 25 to 85 of the 1999 U.S. population using the data from the period life table, the National Health Interview Survey, and the National Nursing Home Survey.

$q_x^{DF}(a_x^{DF} - 1) = q_x^D(a_x^D - 1) = q_x(a_x - 1)$. In particular, we consider the assumptions $q_x^D \geq q_x^{DF}$ and $a_x^D \leq a_x^{DF}$ for all x . This case is theoretically most likely among the four cases that exist in the monotonic relationships between q_x^{DF} and q_x^D and between a_x^{DF} and a_x^D . The conditional probability of death is assumed to be higher for the disabled population than the disability-free population in each age group, i.e., $q_x^{DF} \leq q_x^D$. It is theoretically plausible because the disabled are, *ceteris paribus*, more likely to die. One can also assume that the life expectancy among those who will die in the interval is less for the disabled population within each age interval, i.e., $a_x^{DF} \geq a_x^D$. Among those who will die in an interval, the disabled may do so sooner than the disability-free because higher morbidity often corresponds to a higher hazard rate.

The following proposition shows the implications of the monotonicity assumptions specified

above about the nonparametric bounds of DFLE.

PROPOSITION 3 *For a given age group x , under the assumption that $q_i^D \geq q_i^{DF}$ and $a_i^D \leq a_i^{DF}$ for all $i = x, x+1, \dots, \omega$, the upper bound of e_x^{DF} remains same as equation 10, whereas the lower bound B_x^L equals $\frac{1}{l_x} \sum_{i=x}^{\omega} (1 - \pi_i) L_i$.*

The proof is given in Appendix. The assumptions imply that the resulting lower bound for e_x^{DF} becomes greater, yielding more informative bounds than before. The new lower bound is equal to Sullivan's estimate of e_x^{DF} , implying that if these assumptions are correct, Sullivan's estimate is likely to be biased downwards. Figure 1 illustrates the fact that with the monotonicity assumptions (the right panel of the figure) the bounds are narrower and hence more informative than without assumptions (the left panel). Figure also shows that even under the monotonicity assumptions, the bounds are informative when q_i and π_i are small as it is the case in the absence of such assumptions.

3.3 Confidence Intervals

Since π_x is estimated from the disability survey data, the resulting nonparametric bounds have sampling variability. In general, there are several types of $(1 - \alpha)\%$ confidence intervals that can be constructed for nonparametric bounds (e.g., Cheng and Small, 2005). First, we consider a simple approach based on the Bonferroni's inequality. Let Y_{ij} represent the disability indicator variable for the j th individual of the i th age group in the cross-sectional survey where $j = 1, \dots, N_i$. We obtain the consistent (and unbiased) estimates of the nonparametric bounds, $\hat{B}_x^L$ and $\hat{B}_x^U$, by using, for example, the sample averages $\hat{\pi}_i = \sum_{j=1}^{N_i} Y_{ij}/N_i$ to estimate π_i for $i = x, \dots, \omega$ and substituting them into equations 10 and 11. To obtain the $(1 - \alpha)\%$ confidence interval for the resulting bounds, we apply the Bonferroni's inequality, $\Pr([B_x^L, B_x^U] \subset [\hat{B}_{x\alpha}^L, \hat{B}_{x\alpha}^U]) \geq \Pr(B_x^L \geq \hat{B}_{x\alpha}^L) + \Pr(B_x^U \leq \hat{B}_{x\alpha}^U) - 1 = 1 - \alpha$ and find the upper and lower confidence bounds, denoted by $\hat{B}_{x\alpha}^U$ and $\hat{B}_{x\alpha}^L$, such that $\Pr(B_x^L \geq \hat{B}_{x\alpha}^L) = \Pr(B_x^U \leq \hat{B}_{x\alpha}^U) = 1 - \alpha/2$. Such values of $\hat{B}_{x\alpha}^U$ and $\hat{B}_{x\alpha}^L$

are obtained by consistently estimating the variance of the upper and lower bounds via,

$$\text{Var}(\hat{B}_x^U) = \frac{1}{l_x^2} \sum_{i=x}^{\omega} \sigma_i^U l_i^2, \quad \text{and} \quad \text{Var}(\hat{B}_x^L) = \frac{1}{l_x^2} \sum_{i=x}^{\omega} \sigma_i^L l_i^2, \quad (12)$$

where σ_i^U is equal to $\text{Var}(\hat{\pi}_i)$ if $q_i(1 - a_i) < \pi_i$ and is equal to 0 otherwise. Similarly, σ_i^L is equal to $\text{Var}(\hat{\pi}_i)$ if $q_i(a_i - 1) + 1 < \pi_i$ and is equal to 0 otherwise. For example, if sample averages are used to estimate π_i , then $\text{Var}(\hat{\pi}_i) = \sum_{j=1}^{N_i} (Y_{ij} - \hat{\pi}_i)^2 / [N_i(N_i - 1)]$. Despite the ease of computing, the drawback of this approach is that the coverage probability of the resulting confidence intervals is often greater than $1 - \alpha$, yielding wider confidence intervals than necessary.

To overcome this problem, we apply a general bootstrap method developed by Beran (1988) to compute the balanced simultaneous confidence intervals with the exact coverage probability. We can estimate π_x using sample averages as before. Other estimators of π_x can also be used in conjunction with this bootstrap method. For example, we can nonparametrically estimate π_x as a smooth function of age from the disability survey data since we know theoretically that the disability prevalence does not change rapidly from one age group to another. Unlike longitudinal data, cross-sectional surveys often exhibit a high fluctuation in disability prevalence across age groups. Hence, the smoothing technique may be appropriate for identifying the underlying functional form of disability prevalence over time. Moreover, such an approach is desirable when age of survey respondents is measured in months or days instead of years as done in the National Health Interview Survey as well as when one wishes to efficiently incorporate variables other than age to predict the disability prevalence (See Section 4).

We choose $\tilde{c}_{x\alpha}^L = \hat{F}_{xL}^{-1}[\hat{F}_x^{-1}(1 - \alpha)]$ and $\tilde{c}_{x\alpha}^U = \hat{F}_{xU}^{-1}[\hat{F}_x^{-1}(1 - \alpha)]$, where $\hat{F}_{xL}$ and $\hat{F}_{xU}$ are the empirical distribution functions of $\tilde{B}_x^L - \hat{B}_x^L$ and $\hat{B}_x^U - \tilde{B}_x^U$, and $\hat{F}_x$ is the empirical distribution function of $\max\{\hat{F}_{xL}(\tilde{B}_x^L - \hat{B}_x^L), \hat{F}_{xU}(\hat{B}_x^U - \tilde{B}_x^U)\}$. The resulting confidence interval, $[\hat{B}_x^L - \tilde{c}_{x\alpha}^L, \hat{B}_x^U + \tilde{c}_{x\alpha}^U]$ asymptotically covers the true bounds by the fixed probability $1 - \alpha$. Moreover, these confidence intervals are balanced in a sense that they treat upper and lower bounds fairly, i.e., $\Pr(\hat{B}_x^L - \tilde{c}_{x\alpha}^L \leq$

$B_x^L) = \Pr(\widehat{B}_x^U + \widehat{c}_{x\alpha}^U \geq B_x^U)$ hold asymptotically. In contrast, the bootstrap bounds proposed by Horowitz and Manski (2000) are not balanced.

3.4 Estimation of DFLE Using Abridged Period Life Table

Although our discussion so far is based on unabridged period life tables, in many situations only abridged period life tables, in which the width of age interval is $n > 1$ years, are available to applied researchers. Indeed, many of the studies cited in Section 1 as well as the original article by Sullivan (1971) apply Sullivan's method to abridged life tables. DFLE can be defined similarly for abridged life tables. Analogous to equation 2, the total person-years lived in the given age interval $[x, x+n)$ among those alive at age x is given by ${}_nL_x = l_x [n + {}_nq_x({}_na_x - n)]$ where the prescript n indicates the interval width of abridged life tables. ${}_nL_x^{DF}$ can also be defined in an completely analogous manner (see equation 5), and is equal to, ${}_nL_x^{DF} = (1 - \pi_x) l_x [n + {}_nq_x^{DF}({}_na_x^{DF} - n)]$. Then, DFLE under abridged life tables is defined as $e_x^{DF} \equiv \frac{1}{l_x} \sum_{i=0}^{(\omega-x)/n} {}_nL_{x+in}^{DF}$ where ω now represents the beginning age of the last age interval.

Given this setup, Proposition 1 directly applies to the case of abridged life tables with the condition ${}_nq_x^{DF}({}_na_x^{DF} - n) = {}_nq_x^D({}_na_x^D - n) = {}_nq_x({}_na_x - n)$. The sharp nonparametric bounds of e_x^{DF} given an abridged life table can also be derived in a completely analogous manner (see Proposition 2), and they are given by,

$$B_x^U = \frac{1}{l_x} \sum_{i=0}^{(\omega-x)/n} (1 - \pi_{x+in}) l_{x+in} \min \left[n, \frac{{}_nq_{x+in}({}_na_{x+in} - 1) + n}{1 - \pi_{x+in}} \right], \quad (13)$$

$$B_x^L = \frac{1}{l_x} \sum_{i=0}^{(\omega-x)/n} (1 - \pi_{x+in}) l_{x+in} \max \left[0, \frac{{}_nq_{x+in}({}_na_{x+in} - n)}{1 - \pi_{x+in}} + n \right]. \quad (14)$$

The resulting bounds are often much wider than those of unabridged life tables because the information is lost in the process of aggregation. It is also easy to verify that under the monotonicity assumptions considered in Section 3.2, i.e., ${}_na_x^{DF} \geq {}_na_x^D$ and ${}_nq_x^{DF} \leq {}_nq_x^D$, the lower bound equals Sullivan's estimator $\frac{1}{l_x} \sum_{i=1}^{(\omega-x)/n} (1 - \pi_{x+in}) l_{x+in}$ (see Proposition 3).

The estimation of these bounds and calculation of their confidence intervals can be done in the same manner as the case of unabridged life tables. In practice, applied researchers often apply Sullivan’s method by approximating π_x with the disability prevalence of the overall age group interval $[x, x + n)$, i.e., ${}_n\pi_x$, rather than estimating it at the beginning of the interval (e.g., Molla *et al.*, 2001). Although this is done to increase the precision of the sample mean estimator of the disability prevalence by using a larger sample size, the approximation may be poor if the interval is wide.

We emphasize that whether one uses Sullivan’s method or the proposed nonparametric method, the estimation of DFLE under abridged life tables relies upon the assumption that there is no transition between disability and disability-free status within the age intervals. Such an assumption may not be realistic when the width of age interval is large. Moreover, the validity of the assumption cannot be verified from the observed data, and hence the direction of the resulting bias can never be known to researchers. To some extent, this is due to the nature of any discrete data as well as the aggregation of age intervals. Even with unabridged life tables or longitudinal data, the statistical framework assumes that the transition between disability and disability-free status only occurs at the beginning of age intervals. Nevertheless, this problem is severe with wider age intervals.

4 An Empirical Analysis of the 1999 U.S. Population

In this section, we apply the proposed nonparametric method to the 1999 U.S. population. We first estimate DFLE and the proportion of remaining life spent disability-free for the overall population. Next we disaggregate the 1999 U.S. population by age, sex, education, and race to examine socioeconomic differentials of DFLE.

4.1 DFLE of the Overall Population

In this section, we apply the proposed method to estimate DFLE using the 1999 U.S. period life table (Arias, 2002), which is summarized in Table 1, as well as the disability prevalence data from the 1999 National Health Interview Survey (NHIS) and 1999 National Nursing Home Survey (NNHS), which are briefly mentioned in Section 3.1. The NHIS is a multi-purpose health survey conducted by the National Center for Health Statistics and is the principal source of information on the health of the civilian, noninstitutionalized population of the United States (The number of observations is 97,059). The NNHS is a survey of nursing homes and related care facilities in the United States conducted by the National Center for Health Statistics (The number of observations is 8,215). The use of the two surveys gives a complete picture of disability prevalence among the noninstitutionalized and institutionalized populations. In all the analyses presented below, the survey weights are incorporated so that respondents from the two surveys are appropriately weighted according to their population sizes.

The third and fourth columns of Table 2 show the proportion disabled and life expectancy for selected ages from the two survey datasets. Following the literature, a respondent was considered disabled if he or she responded affirmatively to either of the following questions: “Do you need help with activities of daily living?” and “Because of a physical, mental, or emotional problem, do you need the help of other persons in handling routine needs ...?” where “...” represents various independent activities of daily living (e.g., Molla *et al.*, 2004; Crimmins and Saito, 2001; Crimmins *et al.*, 1997). While activities of daily living include bathing and showering, dressing, eating, getting in/out of bed or chair, using the toilet, and getting around in home, the independent activities of daily living include household chores, doing necessary business, and shopping. We observe that the proportion disabled increases roughly monotonically with age (e.g., 5% at age 20, 19% at age 55, and 58% at age 85). Life expectancy on the other hand decreases monotonically

Age	Proportion Disabled ($\hat{\pi}_x$)	Life Expectancy (e_x)	No Smoothing				Smoothing			
			Estimated DFLE	95% C.I.			Estimated DFLE	95% C.I.		
				Lower	Upper	Width		Lower	Upper	Width
20	0.05	55.69	46.32	46.20	46.43	0.23	46.30	46.12	46.49	0.37
25	0.04	50.94	41.81	41.70	41.92	0.22	41.76	41.58	41.94	0.36
30	0.07	46.18	37.24	37.13	37.35	0.22	37.23	37.05	37.41	0.36
35	0.08	41.44	32.79	32.68	32.90	0.22	32.77	32.59	32.94	0.35
40	0.10	36.75	28.41	28.31	28.52	0.21	28.41	28.24	28.59	0.35
45	0.14	32.16	24.22	24.12	24.32	0.21	24.22	24.05	24.39	0.34
50	0.14	27.69	20.24	20.14	20.34	0.20	20.23	20.07	20.40	0.33
55	0.19	23.34	16.48	16.38	16.58	0.19	16.49	16.33	16.64	0.32
60	0.21	19.21	13.08	12.99	13.17	0.18	13.04	12.90	13.19	0.29
65	0.24	15.31	10.03	9.94	10.11	0.17	9.91	9.78	10.04	0.27
70	0.29	11.63	7.17	7.10	7.24	0.15	7.08	6.96	7.20	0.24
75	0.31	8.11	4.60	4.53	4.67	0.14	4.54	4.44	4.64	0.20
80	0.49	4.56	2.19	2.14	2.24	0.09	2.29	2.22	2.36	0.15
85	0.58	0.50	0.21	0.18	0.24	0.05	0.22	0.21	0.23	0.02

Table 2: Estimated DFLE based on Sullivan’s Method for Selected Ages. The first two columns of the table show the proportion disabled in the sample of the two surveys and the life expectancy of the 1999 U.S. population. The table also presents the point estimates of DFLE and their 95% confidence intervals using Sullivan’s Method. Two sets of the results are presented; one using sample weighted averages (“No Smoothing”) and the other using the estimates from the binomial Generalized Additive Model with the logistic link (“Smoothing”) to estimate the disability prevalence.

with age (e.g., approximately 56 years at age 20, 23 years at age 55, and 0.5 years at age 85).

We first estimate DFLE using Sullivan’s method. The fourth column of Table 2 presents the point estimate of DFLE, while the next three columns show the associated 95% confidence intervals and their width (under the heading “No Smoothing”). When computing these estimates, the disability prevalence π_x is estimated using sample averages and the confidence intervals are based on the expression of the variance in equation 9. For example, the 95% confidence interval of Sullivan’s estimator is $[46.20, 46.43]$ years for age 20, while that for age 85 is approximately $[66, 88]$ days.

We also estimate π_x as a smooth function of age using the binomial General Additive Model (GAM) with the logistic link (Hastie and Tibshirani, 1990). Such a strategy may be useful to effectively uncover the structural relationship between the disability prevalence and age. It is also

Age	No Assumption						Monotonicity Assumptions					
	Bounds of DFLE			Balanced 95% C.I.			Bounds of DFLE			Balanced 95% C.I.		
	Lower	Upper	Width	Lower	Upper	Width	Lower	Upper	Width	Lower	Upper	Width
20	46.12	46.60	0.48	45.94	46.88	0.94	46.30	46.60	0.29	46.13	46.88	0.76
25	41.57	42.05	0.48	41.40	42.33	0.93	41.76	42.05	0.29	41.58	42.33	0.75
30	37.04	37.52	0.48	36.87	37.80	0.93	37.23	37.52	0.29	37.06	37.81	0.75
35	32.58	33.06	0.48	32.41	33.33	0.93	32.77	33.06	0.29	32.60	33.34	0.74
40	28.23	28.70	0.48	28.06	28.98	0.92	28.41	28.70	0.29	28.25	28.98	0.74
45	24.03	24.51	0.48	23.86	24.78	0.92	24.22	24.51	0.29	24.05	24.78	0.73
50	20.04	20.52	0.48	19.87	20.79	0.92	20.23	20.52	0.28	20.07	20.80	0.73
55	16.29	16.77	0.48	16.13	17.04	0.91	16.49	16.77	0.28	16.32	17.04	0.72
60	12.84	13.32	0.48	12.68	13.60	0.92	13.04	13.32	0.28	12.88	13.61	0.73
65	9.71	10.18	0.47	9.55	10.47	0.92	9.91	10.18	0.27	9.75	10.47	0.72
70	6.86	7.33	0.47	6.70	7.63	0.93	7.08	7.33	0.26	6.91	7.63	0.72
75	4.32	4.78	0.47	4.18	5.09	0.90	4.54	4.78	0.24	4.41	5.09	0.68
80	2.06	2.52	0.46	1.99	2.77	0.78	2.29	2.52	0.23	2.22	2.77	0.56
85	0.00	0.44	0.44	0.00	0.44	0.44	0.22	0.44	0.22	0.20	0.44	0.24

Table 3: The Nonparametric Bounds of DFLE Without Assumption and With Monotonicity Assumptions. The table shows the estimated bounds and 95% balanced confidence intervals for selected ages. The disability prevalence was estimated as a smooth function of age using the binomial Generalized Additive Model with the logistic link.

more appropriate in our case because the age of respondents at the time of interview is measured in quarters rather than years in NHIS. For implementation, we use an R package `gam` written by Tibshirani to fit the model and compute the 95% bootstrap confidence interval for each age. The final four columns of Table 2 (under the heading “Smoothing”) presents the results based on GAM for selected ages. When compared with the results based on the sample averages, both the point estimates and the 95% confidence intervals are quite different for older age groups. For example, for ages 75, 80, and 85, the 95% confidence intervals from the two sets of the estimates do not overlap with each other. The 95% confidence intervals based on GAM are also shorter for older age groups.

Next, we use the methodology discussed in Section 3 to estimate the nonparametric bounds of DFLE and compute the 95% balanced confidence intervals based on 10,000 bootstrap replications. We estimate π_x using the GAM as before. The columns under the heading “No Assumption” of

Table 3 presents the estimated nonparametric bounds and their 95% confidence intervals for selected age groups. The comparison with the estimates based on Sullivan’s method in Table 2 (the columns under the heading “Smoothing”) shows that the resulting 95% confidence intervals based on the method of bounds are significantly wider especially for older age groups. For example, in the group of 80 years old, the 95% confidence interval based on the nonparametric bounds is $[1.99, 2.77]$ years with the width of approximately 9 months, whereas Sullivan’s method yields $[2.22, 2.36]$ with the width of roughly two months.

We also estimate the nonparametric bounds of DFLE with the monotonicity assumptions discussed in Section 3.2. The results are presented in the columns under the heading “Monotonicity Assumptions” of Table 3. As expected, the width of the bounds narrows significantly when the monotonicity assumptions are invoked. For example, the 95% confidence interval of DFLE at age 60 is $[12.88, 13.61]$ with the monotonicity assumptions, whereas it is $[12.68, 13.60]$ years without assumption. The latter confidence interval is more than 26% wider than the former. Furthermore, as shown in Proposition 3, the lower bound under the monotonicity assumptions equals exactly the point estimate of DFLE based on Sullivan’s method, indicating that Sullivan’s method is likely to underestimate DFLE if the monotonicity assumptions are correct.

The proportion of remaining life spent disability-free, e_x^{DF}/e_x , is another important quantity of interest, and its nonparametric bounds can be computed using the bounds of DFLE. Figure 2 shows the bounds of the proportion and their 95% balanced confidence intervals with no assumption (left panel) and with the monotonicity assumptions (right panel). We observe that both the upper and lower bounds of the proportion decrease gradually over age except that the upper bound sharply increases at the last age groups. The width of the bounds also increases monotonically with age and sharply goes up for older age groups. As shown in the left panel of Figure 2, between 0.82 and 0.84 of remaining life is spent disability-free at age 20, whereas the proportion decreases to between 0.62

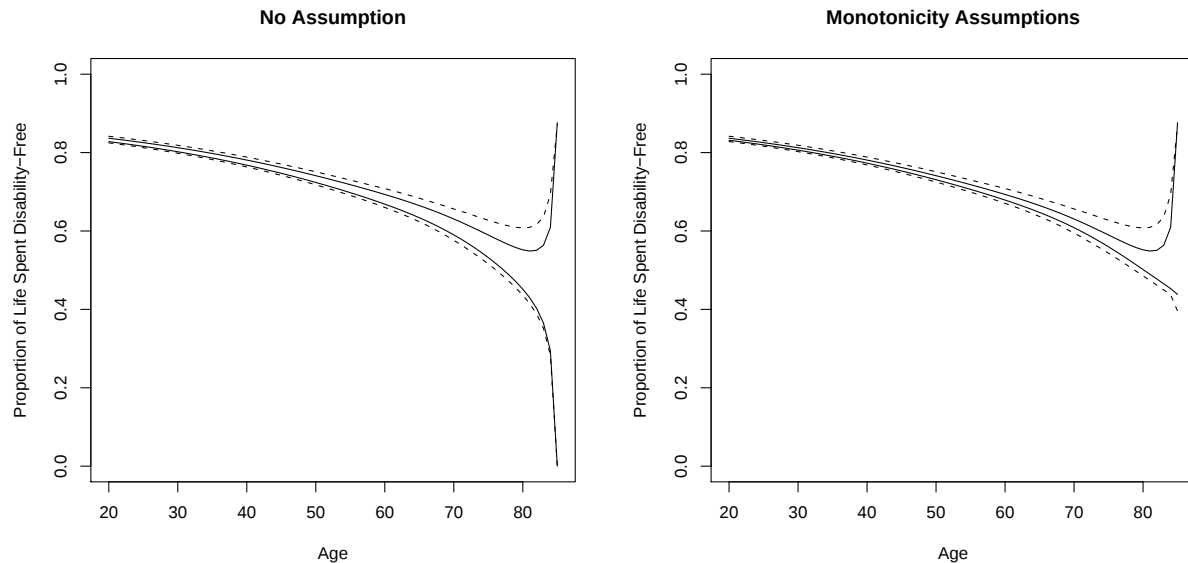


Figure 2: The Nonparametric Bounds for the Proportion of Remaining Life Spent Disability-Free. The proportion of remaining life is the ratio of disability-free life expectancy and overall life expectancy (i.e., e_x^{DF}/e_x). The left and right panels of the figure show the proportion with no assumption and with the monotonicity assumptions, respectively. The bounds (dashed line) and 95% confidence interval (solid line) are presented. The upper and lower bounds of the proportion decreases monotonically with age up until the last age groups.

and 0.68 at age 65 and 0.44 and 0.61 at age 80. Moreover, the 95% balanced confidence intervals of the proportion widen over age. The right panel of Figure 2 displays the proportion when the monotonicity assumptions are invoked. The confidence interval of the proportion widens less over age. The estimated proportion is between 0.83 and 0.84 at age 20, 0.64 and 0.68 at age 65, and 0.49 and 0.61 at age 80.

4.2 DFLE by Sex, Race, and Education

Previous research has found important differences in mortality with respect to race (e.g., Sorlie *et al.*, 1995), sex (e.g., Bird and Rieker, 1999), and education (e.g., Rogot *et al.*, 1992). Pappas *et al.* (1993) argue the disparity in mortality rates have increased over time between men and women, whites and non-whites, and by education level. Molla *et al.* (2004) found similar differences in morbidity by sex and education. Finally, Crimmins and Saito (2001) argue that sex, race, and

education differentials are increasing over time.

We estimate life expectancy and DFLE for the 1999 U.S. population separately by single year of age, sex, race, and educational attainment. Sex, race, and educational attainment are important factors of socioeconomic status (Sorlie *et al.*, 1995). We begin our analysis at age 30, the age at which education is considered complete for most adults (Crimmins and Saito, 2001). Following Molla *et al.* (2004) and Crimmins and Saito (2001), educational attainment was broken into three categories: 0 – 8 years (low), 9 – 12 years (medium), and 13 or more years of schooling (high). For race, we use two categories: white and non-white. We then determine mortality information by age, sex, race, and education for 1999 using the vital statistics obtained by the National Center for Health Statistics. The population estimates for these groups were obtained from the 1999 Current Population Survey. Based on death and population information, we calculate group specific conditional probabilities of death. Finally, life expectancy for each group was calculated using the q_x values specific to each group as well as the overall U.S. period life table a_x values, which are assumed to be applicable to all groups.

An individual was considered disabled using the same definition of disability in Section 4.1. For the noninstitutionalized population (i.e., community residents), disability status by factor was estimated by the NHIS. For the institutionalized population (e.g., nursing home and long-term care residents), disability status by factor was estimated by the NNHS. Similar to Molla *et al.* (2004), all institutionalized persons age 65 years and higher were considered disabled. The NNHS does not include the educational attainment of respondents. This information was obtained from the institutionalized residents surveyed in the 1993 **(note: use 1999 when we receive the medicare data)** Medicare Current Beneficiary Survey (MCBS). The distribution of the institutionalized population by education for a given age, race, and sex was estimated from the MCBS Access to Care Module.

We use the methodology proposed in Section 3 to estimate the nonparametric bounds of DFLE with the monotonicity assumptions by sex, education, and race for each age from 30 to 80 years. We model the disability prevalence for each category by using the binomial GAM with the logistic link where along with the smooth function of age, the main effects for sex, race, and education levels are estimated. The smoothing approach is more appropriate than computing sample averages within each group because some groups have very few respondents. Although it is possible to add interaction effects, we find that they do not significantly improve the model fit.

Figure 3 presents the estimated upper and lower bounds of the differences in DFLE across different groups by using one of the four middle educated groups as the reference group within a given row and comparing them with the other groups. For example, the first row, second column plot (i.e., plot (1,2)) compares the estimated DFLE of high, medium, and low educated non-white males with middle educated white males. The DFLE of highly educated non-white males is between 2.73 and 3.36 years higher at age 30 and 0.74 and 1.51 years higher at age 65 than for middle educated white males at these ages. Middle educated non-white men experience significantly less years of DFLE than equally educated white men (3.98 to 4.59 years less at age 30 and 1.17 to 1.71 years less at age 65).

Within race, high and medium educated women experience more years of disability-free life than medium educated men (see plots (1,3) and (2,4)). Within sex, high and medium educated whites experience more years of disability-free life than middle educated non-whites. Of significant note is that even low educated whites experience nearly equivalent years of disability-free life (see plots (2,1) and (4,3)). The interaction of sex and race also reveal important differences. Plot (2,3) compares white women of all education levels against medium educated non-white men. High and medium educated white women experience significantly more years of DFLE for than this reference group for all ages. Low educated white women also experience more years of DFLE

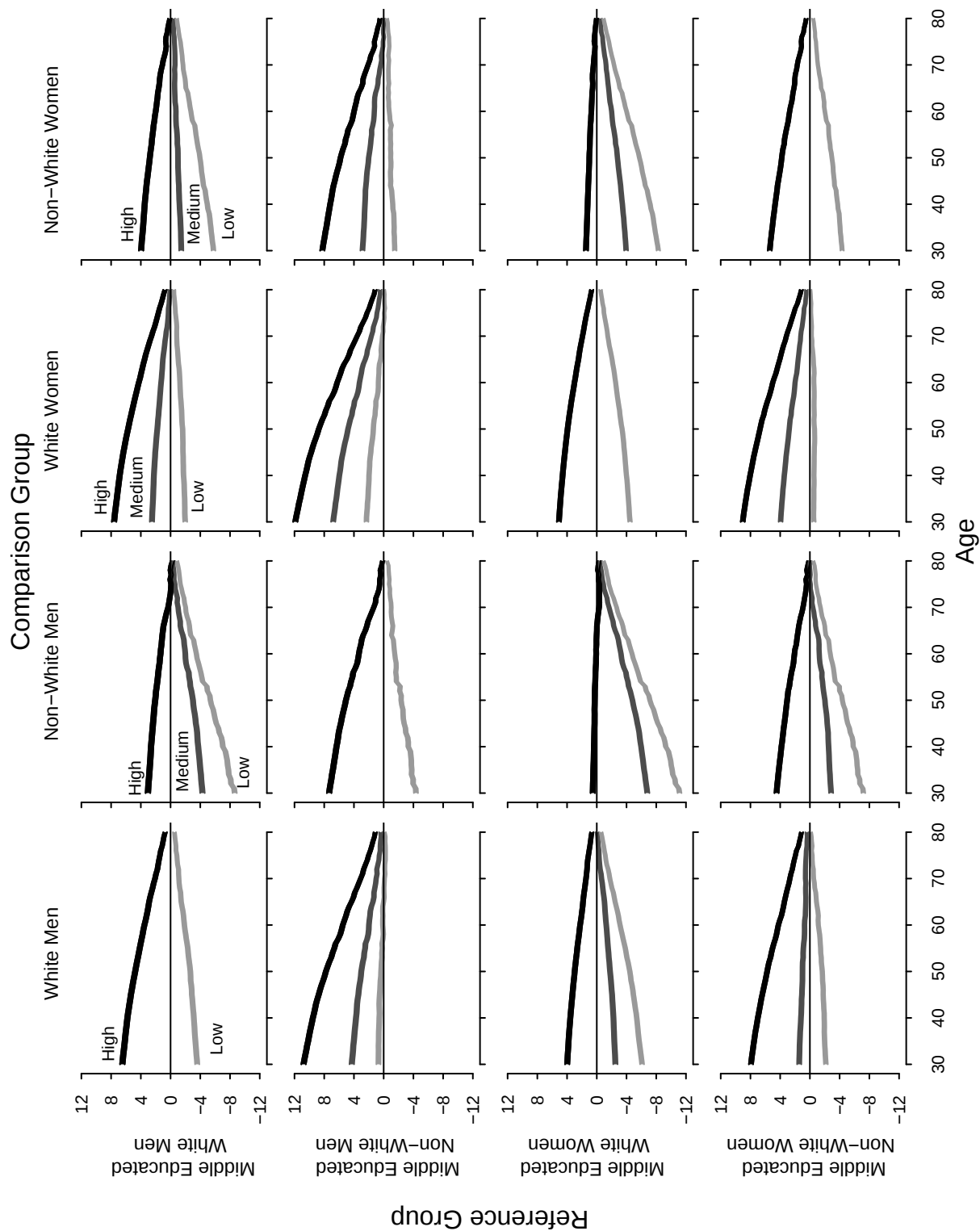


Figure 3: The Nonparametric Bounds of Differences in DFLE by Sex, Race, and Education. The figure shows the difference in the nonparametric bounds of DFLE for high, medium, and low education of the comparison group relative to the medium education of the reference group.

between ages 30 and 70, after which their morbidity experience is approximately equal despite lower educational attainment. The same is not true for white men and non-white women (see plot (4,1)). Low educated white men do not experience more years of DFLE compared to medium educated non-white women.

Figure 4 presents the estimated upper and lower bounds for the differences in the proportion of remaining life spent disability-free (i.e., e_x^{DF}/e_x) among different groups for ages from 30 to 80 years in the exactly same way as in Figure 3. The main diagonal compares each group to itself where the middle educated are taken as the reference category. For example, plot (1,1) shows that among white males at age 30, the proportion of life disability-free is between 0.76 and 0.78 for the highly educated, while only between 0.68 and 0.71 for the middle educated and 0.57 and 0.59 for the low educated. The proportion of remaining life increases for the high educated and decreases for the low educated compared to the middle educated for all four race-sex groups over age (see the plots in the main diagonal). Within race, equally educated men experience a higher proportion of remaining life free of disability until approximately age 70 (see plots (3,1) and (4,2)). Within sex, the differences in the proportions are persistent; equally educated whites experience a higher proportion of disability-free life than non-whites for all ages (see plots (2,1) and (4,3)).

Two comparisons that demonstrate the interaction of sex and race are plots (4,1) and (1,4). In the former plot, white men of all education levels are compared to middle educated non-white women. The proportion of remaining life spent disability-free is approximately equal between low educated white men and middle educated non-white women, despite their difference in the education level. Non-white women are the only group where the highly educated do not experience significantly higher proportion of remaining life without disability. Similarly, the latter plot shows that the proportion for highly educated non-white women is approximately equal to middle educated white men, also despite their difference in the education level. These results indicate that

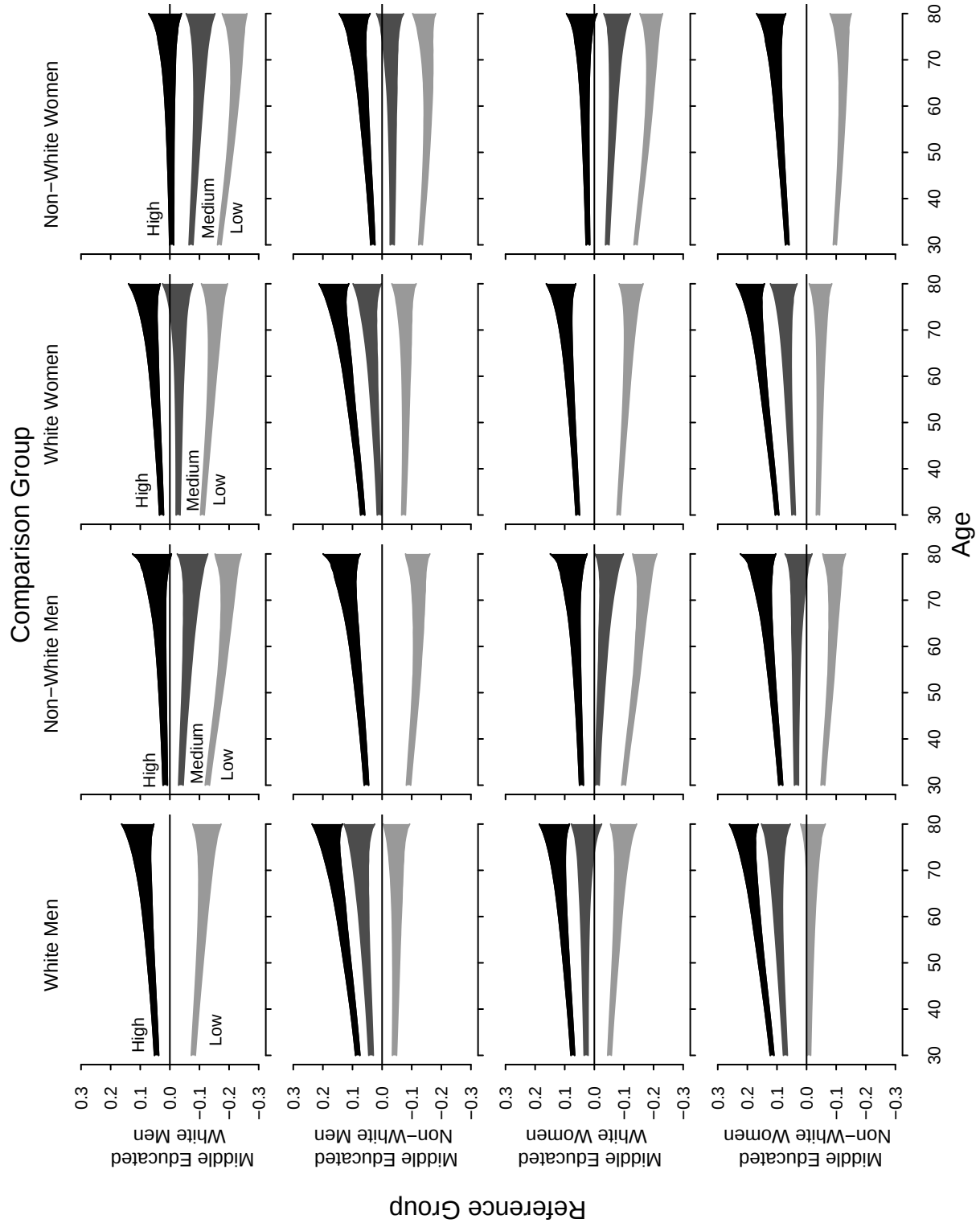


Figure 4: The Nonparametric Bounds for Differences in Proportion of Life Spent Disability-Free across Different Groups. The figure shows the difference in the nonparametric bounds of the proportion of life spent disability-free for high, medium, and low education of the comparison group relative to the medium education of the reference group.

the effect of race and sex may be especially pronounced among non-white women of all ages.

Important sex, race, and sex-race interaction differences exist in DFLE and proportion of remaining life spent disability-free. While medium educated women experience more years of DFLE than medium educated men of the same race, they experience less proportion of remaining life without disability (see plots (1,3) and (2,4) of Figures 3 and 4). However, this pattern does not hold when sex is held constant. For men and women, medium educated whites experience both more years of DFLE and a greater proportion of remaining life without disability than medium educated non-white counterparts (see plots (2,1) and (4,3) of Figures 3 and 4).

5 Concluding Remarks

Robust estimation of DFLE is vital to testing the competing theories of morbidity and mortality. Such estimates help to answer the important question of whether additional years of life are spent in poor health. Over the last 30 years, Sullivan’s method has been the most widely used method for estimating DFLE. Academic and government researchers use this method because of the relative ease of obtaining mortality data from a period life table and disability data from a cross-sectional survey. Yet, we show that Sullivan’s method relies on the strong assumption to point identify DFLE. In this paper, we develop an alternative nonparametric method that avoids such an assumption. We derive the bounds of DFLE and use the bootstrap procedure to construct accurate and balanced confidence intervals. We also showed that under plausible monotonicity assumptions, Sullivan’s method is likely to underestimate DFLE. Our analysis of the 1999 U.S. population demonstrates that the proposed nonparametric method provides a valid estimate of DFLE. The resulting bounds are informative even without any assumption, but they yield inferences different from Sullivan’s method especially for older age groups. The methodology developed in this paper is relatively easy to implement and provides a robust estimate of DFLE using period life table and cross-sectional disability survey data.

Appendix: Proofs of the Propositions

Proof of Proposition 1. Sullivan's estimator consistently estimates the disability-free life expectancy if and only if the following equation holds, $e_x^{DF} = \frac{1}{l_x} \sum_{i=x}^{\omega} (1 - \pi_i) L_i$. By comparing equation 2 with equation 5, it is immediate that Sullivan's estimator is equal to e_x^{DF} for all x if and only if $q_x^{DF}(a_x^{DF} - 1) = q_x(a_x - 1)$. Now, note that the following equalities hold,

$$q_x = \pi_x q_x^D + (1 - \pi_x) q_x^{DF}, \quad (15)$$

$$q_x a_x = \pi_x q_x^D a_x^D + (1 - \pi_x) q_x^{DF} a_x^{DF}, \quad (16)$$

for all $x = 1, 2, \dots, \omega$. From equation 15 and $q_x^{DF}(a_x^{DF} - 1) = q_x(a_x - 1)$, we obtain $q_x a_x = q_x^{DF}(a_x^{DF} - 1) + \pi_x q_x^D + (1 - \pi_x) q_x^{DF}$. Substituting this into equation 16 and rearranging yield $q_x^{DF}(a_x^{DF} - 1) = q_x^D(a_x^D - 1)$. ■

Proof of Proposition 2. By substituting equation 5 into equation 7, we obtain $e_x^{DF} = \frac{1}{l_x} \sum_{i=x}^{\omega} (1 - \pi_i) l_i [1 + q_x^{DF}(a_i^{DF} - 1)]$, where l_x is observed for each age group, and π_x can be consistently and nonparametrically estimated from the cross-sectional disability survey. Therefore, we seek to bound $q_i^{DF}(a_i^{DF} - 1)$ for each $i = x, x + 1, \dots, \omega$ in order to bound e_x^{DF} . Using equations 15 and 16, we write a_i^D as a function of q_i, a_i, π_i , and q_i^{DF} . Since a_i^D is bounded below by 0 and above by 1, after rearranging we obtain the following inequality conditions if $q_i \neq (1 - \pi_i) q_i^{DF}$,

$$q_i^{DF}(a_i^{DF} - 1) > \frac{q_i(a_i - 1)}{1 - \pi_i}. \quad (17)$$

$$q_i^{DF} a_i^{DF} < \frac{q_i a_i}{1 - \pi_i}. \quad (18)$$

Using inequality 17 and the fact that q_i^{DF} and a_i^{DF} are bounded below by 0 and above by 1, the greatest lower bound of $q_i^{DF}(a_i^{DF} - 1)$ equals $\max \left[-1, \frac{q_i(a_i - 1)}{1 - \pi_i} \right]$. To derive the least upper bound, subtracting q_i^{DF} from both sides of inequality 18 yields $q_i^{DF}(a_i^{DF} - 1) < \frac{q_i a_i}{1 - \pi_i} - q_i^{DF}$. Then, using equation 15 and the fact that both q_i^{DF} and q_i^D are bounded below by 0 and above by

1 imply $\max\left(0, \frac{q_i - \pi_i}{1 - \pi_i}\right) \leq q_i^{DF} \leq \min\left(1, \frac{q_i}{1 - \pi_i}\right)$, the least upper bound of $q_i^{DF}(a_i^{DF} - 1)$ equals $\min\left[0, \frac{q_i(a_i - 1) + \pi_i}{1 - \pi_i}\right]$. Finally, when $q_i = (1 - \pi_i)q_i^{DF}$, equations 15 and 16 imply $a_i^{DF} = a_i$ and hence $q_i^{DF}(a_i^{DF} - 1) = q_i(a_i - 1)/(1 - \pi_i)$, which is always within the derived bounds. ■

Proof of Proposition 3 If $q_i^{DF} \leq q_i^D$, then equation 15 implies $\max\left(0, \frac{q_i - \pi_i}{1 - \pi_i}\right) \leq q_i^{DF} \leq q_i$. Moreover, if $a_x^{DF} \geq a_x^D$, then inequality 17 becomes the new inequality $a_x \leq a_x^{DF}$. It follows immediately that under these monotonicity assumptions the lower bound for e_x^{DF} is attained when $a_x^{DF} = a_x$ and $q_x^{DF} = q_x$, while the upper bound stays the same as before. ■

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