Life Expectancies at Older Ages in Destabilized Populations: A Methodological Investigation with Application to Some Developed and Less Developed Countries

Subrata Lahiri
Professor and Head
Department of Public Health and Mortality Studies
International Institute for Population Studies
Deonar, Mumbai-400 088

Abstract:

This paper presents a technique for estimating the ratios e(x+10)/e(x) over ages from two enumerations in a closed population following **Generalized Population Model**, which, in turn, are used to estimate life expectancies at older ages between 65 and 80. The proposed technique makes use of the values of e(75) and (80) estimated through an earlier method, proposed by the author, based on the assumption of local stability of the of the sub-population aged 75 & above. To test the validity of the procedure, the present technique has been applied to different quality of age-data starting with the age-data of Japan (1965-70) having reasonably good age-reporting followed by those of Korea (1990-95) and China (1982-90) with moderate error in age-reporting, and finally with those of India (1981-91 & 1991-2001) that are heavily distorted due to age-misreporting.

Introduction:

Among the methodological investigations carried out in estimating life expectancies at older ages beyond age 70, the techniques developed by Horiuchi and Coale (1980), Mitra (1984) and Lahiri (1990) are worth noting. While developing a short-cut technique for estimating completeness of death registration in a destabilized population, Preston and Lahiri (1991) incidentally proposed a formula for life expectance at older ages. It may be noted that all the above techniques are based on the assumption of sectional stability at older ages (age 65 and above). Because of considerable increase in public health awareness and remarkable development in medical sciences during the later half of the twentieth century, particularly during the last three decades, the overall longevity including that at older ages has also increased considerably worldwide including in many less developed countries. Consequently the age-specific growth rates at older ages have also increased considerably over time in various developed and less developed countries, hence the assumption of approximate stability may not be tenable in such countries. Thus, an attempt has been made in this paper to develop a technique for estimating life expectancies at older ages in a destable population following **Generalized Population Model (GPM)** of age-structure applicable to any population (Bennett and Horiuchi, 1981; and Preston and Coale, 1982).

Methodology:

The Generalized Population Model

According to a destabilized or generalized population model which is applicable to any population, the function **N(x; t)** describing the age-structure of any population at time **t** is given by the following equation (Bennett and Horiuchi, 1981; Preston and Coale, 1982):

$$\mathbf{N(x; t) = B(t) \exp\left[- \int_0^x r(y; t)\, dy \right] p(x; t)} \text{ ....(1)}$$

where, **B(t)** : Number of births at time **t**;

r(y; t): Instantaneous rate of growth of persons aged **y** at time t;

p(x;t): Probability of surviving from birth to exact age **x** according to the stationary population associated with the destabilized population at time t.

It is worth mentioning here that an equation, very similar to that of the equation (1), was originally introduced by Bennett and Horiuchi (1981) while developing a procedure for estimating death registration completeness in any closed population. It appears that this was then not identified as the generalization of the stable equation model. However, Preston and Coale (1982) tackled this problem more systematically and derived the age-structure of any population (closed or open) by using the fundamental concept of instantaneous force of mortality at age **x** at time **t** ($\mu_x(t)$ and instantaneous growth rate of population aged **x** at time **t** [$r_x(t)$]. They have also discussed its utility in the estimation of various demographic parameters.

Estimation of the Ratio – e(x+10)/e(x) under GPM using the Age-data at any Two points of Time (not necessarily multiple of 5 years apart)

In life table terminology **p(x;t) = l(x; t)/l(0; t)**, where **l(x; t)** denotes the number of survivors at exact age **x** out of the initial birth cohort **l(0; t)** in the stationary population.

Taking **l(0; t) = B(t)**, one can easily find the following expression for l(x; t) from the equation (1), for the convenience and simplicity the 'argument' **t** will be omitted henceforth :

$$\mathbf{l(y) = N(y) * \exp\left[\int_0^y r(u) du \right]} \text{ .... (2)}$$

Now, by definition **T(y)**, the person-years lived beyond age **y**, can be written as:

$$\mathbf{T(y) = N(y+) * \exp\left[\int_0^{C_{y+}} r(u)du\right]}$$ ....**(3),** where **N(y+)** represents the number of persons aged **y** and above. The equation (3) can be obtained by integrating both sides of (2) in the age-range (y, ω), where **ω** being the maximum age attainable by a person in the population under study, and according to the first mean value theorem of integral calculus, there exists a point (age) $\mathbf{C_{y+}}$ lying between the ages **y** and **ω** such that the identify (3) holds true. Remembering that **e(x) = T(x)/l(x),** it can be shown by using the equations (2) and (3) for **y = x & x+10** that the ratios of the form **e(x+10)/e(x)** can be obtained through the following formula:

$$\frac{e(x+10)}{e(x)} = \exp\left[\int_{C_{x+}}^{C_{(x+10)+}} r(u)du - \int_{x}^{x+10} r(u)du\right] \times \frac{b_x}{b_{x+10}} \quad \text{.......... (4),}$$

The two points[1] (ages) -- $\mathbf{C_{x+}}$ and $\mathbf{C_{(x+10)+}}$ in the open intervals **(x, ω)** and **(x+10, ω) --** are such that the identity (3) holds true for y = x & x+10. The quantities $\mathbf{b_x[= N(x)/N(x+)]}$ and $\mathbf{b_{x+10}[= N(x+10)/N(x+10)+]}$ in the above equation, stands for ***birthday rates*** or the ***rates of arrival of persons*** at exact ages **'x'** & **'x+10'** respectively. The values of $\mathbf{b_x}$'s can be estimated through the formula adopted by Preston and Lahiri (1991), which will be discussed later on.

Discrete Approximation of the Ratio – e(x+10)/e(x)

To obtain an approximation of the ratio **e(x+10)/e(x)** in discrete form so as apply it to a discrete set of age-data, it is necessary to evaluate the integrals in the R.H.S. of the formula (4). Assuming that the growth curve ($\mathbf{\bar{r}_x}$) follows a second-degree polynomial[2] in the whole range of integration ($\mathbf{C_{x+}, C_{(x+10)+}}$) which is split into two sub-intervals $\mathbf{S_1}$ = ($\mathbf{C_{x+}, C_{(x+5)+}}$) and $\mathbf{S_1}$= ($\mathbf{C_{(x+5)+}, C_{(x+10)+}}$), and noting that $\mathbf{{}_5r_y = \frac{1}{5}\int_y^{y+5} r(u)du}$, **for y = x & x + 5**, an approximate discrete version of the expression in the R.H.S of equation (4) can be obtained through the following equation after evaluating the ***first integral*** through the Simpson one-third rule of

---

[1] Though the exact values of $\mathbf{C_{x+}}$ and $\mathbf{C_{(x+10)+}}$ are not known, it can be shown that the two points (ages), mentioned above, are sufficiently close to the mean ages of persons aged 'x & above', and 'x+10 & above' respectively (Lahiri, 1983).

[2] Empirical investigations with the age-data at two points of time of various countries (developed and developing countries) indicate that the growth curve ($\mathbf{\bar{r}_x}$) at ages 45 and above resembles well to a second-degree polynomial (for details, see Lahiri and Menezes, 2003).

numerical integration, and evaluating the second integral by splitting the whole domain of integration (x, x+10) into two sub-intervals of equal size -- (x, x+5) and (x+5, x+10). It is worth noting that the application of the Simpson one-third rule of numerical integration is theoretically justified only when the whole range of integration is divided into even number of sub-intervals of exactly of equal width in addition to the condition that the growth curve ($\bar{\mathbf{r}}_x$) follows a second-degree polynomial in the whole range of integration ($\mathbf{C}_{x+}, \mathbf{C}_{(x+10)+}$). The point $\mathbf{C}_{(x+5)+}$, being the mean-age of persons aged '**x+5** & above', may not necessarily coincide exactly with the mid-point of the interval ($\mathbf{C}_{x+}, \mathbf{C}_{(x+10)+}$), where $\mathbf{C}_{x+}$ and $\mathbf{C}_{(x+10)+}$ represent the mean-ages of persons aged '**x** & above' and '**x+10** & above' respectively. Thus, taking '**<u>k</u>**' as the mid-point of the interval ($\mathbf{C}_{x+}, \mathbf{C}_{(x+10)+}$), the two new sub-intervals, viz., ($\mathbf{C}_{x+}$, **k**) and (**k** , $\mathbf{C}_{(x+10)+}$) in which the whole interval ($\mathbf{C}_{x+}, \mathbf{C}_{(x+10)+}$) is divided, are of exactly equal width. Thus, the application of the Simpson one-third rule of numerical integration provides the following equation:

$$\frac{\mathbf{e(x+10)}}{\mathbf{e(x)}} = \mathbf{exp}\left[\frac{\mathbf{h}^{*}_{x+5}}{3}\left(\bar{\mathbf{r}}(\mathbf{C}_{x+}) + 4 * \bar{\mathbf{r}}_k + \bar{\mathbf{r}}(\mathbf{C}_{(x+10)+})\right) - 5 * ({}_5\mathbf{r}_x + {}_5\mathbf{r}_{x+5})\right] * \frac{\mathbf{b}_x}{\mathbf{b}_{x+10}} \quad ..(5)$$

It can be shown analytically that the statistics $\bar{\mathbf{r}}(\mathbf{C}_{x+})$ and $\bar{\mathbf{r}}(\mathbf{C}_{(x+10)+})$, representing the average annual exponential growth rates at exact ages $\mathbf{C}_{x+}$ and $\mathbf{C}_{(x+10)+}$ during the intercensal period, are very close to $\mathbf{r}_{x+}$ and $\mathbf{r}_{(x+10)+}$, the average annual exponential growth rates of persons aged '**<u>x</u> & above**' and '**<u>x</u>+10 & above**' respectively during the intercensal period (Lahiri, 1983). Hence,

$$\frac{\mathbf{e(x+10)}}{\mathbf{e(x)}} \approx \mathbf{exp}\left[\frac{\mathbf{h}^{*}_{x+5}}{3}\left(\mathbf{r}_{x+} + 4 * \bar{\mathbf{r}}_k + \mathbf{r}_{(x+10)+}\right) - 5 * ({}_5\mathbf{r}_x + {}_5\mathbf{r}_{x+5})\right] * \frac{\mathbf{b}_x}{\mathbf{b}_{x+10}} \quad \ldots\ldots(5.1)$$

The notation $\mathbf{h}^{*}_{x+5}$ stands for the average of the widths of the sub-intervals $\mathbf{S_1} \equiv (\mathbf{C}_{x+}, \mathbf{C}_{(x+5)+})$ **and** $\mathbf{S_2} \equiv (\mathbf{C}_{(x+5)+}, \mathbf{C}_{(x+10)+})$. In other words, $\mathbf{h}^{*}_{x+5}$ is nothing but half of the distance between $\mathbf{C}_{x+}$ and $\mathbf{C}_{(x+10)+}$. The quantities ${}_5\mathbf{r}_x$**'s** and $\mathbf{r}_{x+}$**'s** in the equation (5.1) denote the exponential growth rates of persons in the age-groups **(x, x+4)** and **'<u>x</u> & above'** respectively, and $\bar{\mathbf{r}}_k$ represents the exponential growth rate at exact age **k**, the mid-point of the interval ($\mathbf{C}_{x+}, \mathbf{C}_{(x+10)+}$). Since the exact value of $\bar{\mathbf{r}}_k$ is not known, its magnitude may be obtained as a weighted average of either $\hat{\mathbf{r}}_{x+}$ **and** $\hat{\mathbf{r}}_{(x+5)+}$ **or** $\hat{\mathbf{r}}_{(x+5)+}$ **and** $\hat{\mathbf{r}}_{(x+10)+}$ depending upon whether the age **k**, the mid-point of the interval $(C_{x+}, C_{(x+10)+})$, belongs to the sub-interval $S_1 \equiv (C_{x+}, C_{(x+5)+})$ **or** $S_2 \equiv (C_{(x+5)+}, C_{(x+10)+})$ respectively. The statistic $\hat{r}_{a+}$ (for a= x, x+5, & x+10) stands for the estimated value of the exponential rate of growth of persons ages **'a & above'** which can be obtained through the following formula:

$$_5\hat{r}_a = \frac{1}{m} . \ln [\ _5P_a(z+m)\ /\ _5P_a(z)] \quad .... (5.1),$$

The quantities $_5P_a(z)$ and $_5P_a(z+m)$ **in (5.1)** represent enumerated number of persons in the age-group (a, a+4) at time **z** and **z+m** respectively, **m** being the intercensal interval (not necessarily multiple of 5). The value of $\bar{r}_k$ in the formula (5.1) can be obtained through either of the following approximations (Lahiri 2004):

$$\hat{\bar{r}}_k = \frac{1}{h_{x+2.5}}[(C_{(x+5)+} - k).\ \hat{r}_{x+} + (k - C_{x+}).\ \hat{r}_{(x+5)+}] \quad ...(5.2)$$

$$\text{if k belongs to the sub-interval } S_1\text{; or}$$

$$\hat{\bar{r}}_k = \frac{1}{h_{x+7.5}}[(C_{(x+10)+} - k).\ \hat{r}_{(x+5)+} + (k - C_{(x+5)+}).\ \hat{r}_{(x+10)+}] \quad ...(5.3)$$

$$\text{if k belongs to the sub-interval } S_2.$$

The notations $h_{x+2.5}$ and $h_{x+7.5}$, used in (5.2) and (5.3), are the widths of the sub-intervals S_1 **and** S_2 respectively**.** One can easily verify from (5.2) or (5.3) that $\hat{\bar{r}}_k$ will be exactly equal to $\hat{r}_{(x+5)+}$ if the sub-intervals S_1 and S_2 are exactly of equal width, that is, when **k** coincides with $C_{(x+5)+}$, the mean age of persons aged 'x+**5 & above**'.

Life Expectancy at Older Ages Under the Assumption of Local Stability:

It may be noted that at some advanced ages, say beyond 75, the mortality situation and its rate of growth are unlikely to be changed significantly in normal situation within period of ten-years or less. Thus for all practical purposes we may safely assume that the population aged **'75 & above'** is approximately stable. Under the assumption of local stability of population at older ages beyond age 70, the life expectancies at some older ages, viz., 75 and 80 can be estimated quite precisely through the following equation (proposed by Lahiri, 1991):

$$e_x^0 \approx \frac{\exp\left[r_{x+} * \left\{A_{P_{x+}} + \frac{r_{x+}}{2} * \sigma^2(P_{x+})\right\}\right]}{b_x} \text{------------} (6)$$

where, the notation $\mathbf{A}_{P_{x+}}$ represents the average mean number of years lived beyond age x by those surviving at ages '**x & above**' during the intercensal period and $\sigma^2(\mathbf{P}_{x+})$ denotes the variance of the age-distribution of persons surviving at ages '**x & above**', and the other notations used in the formula (6) have already been mentioned earlier. Owing to the non-availability of sufficiently reliable age-data of persons at very advance ages, Preston and Lahiri (1990) proposed a procedure for estimating $\mathbf{A}_{P_{x+}}$ under the assumption of approximate stability at some older ages (sectional stability), and because of non-availability of detailed age-data at very old ages, Lahiri (1990) also proposed a set of standard values of $\sigma^2(\mathbf{P}_{x+})$ at quinquennial ages between 65 and 80 based on the average variances of the age-distributions for males and females at ages 65 and above according to the Coale-Demeny "West" model stable populations. A sensitivity analysis carried out by Lahiri (1990) indicates that the estimates of longevity at older ages, particularly at ages 70, 75 and 80, are sufficiently robust enough against the standard values of $\sigma^2(\mathbf{P}_{x+})$ as suggested by Lahiri (1990).

The quantity $\mathbf{b}_x \left(= \overline{\mathbf{N}}(\mathbf{x}) / \overline{\mathbf{N}}(\mathbf{x+})\right)$, the 'birth-day rate' at age x can be estimated through the following formula as adopted by Preston and Lahiri (1991) which can be developed under the assumption of exponential change of persons, and constant death rate aged **y** in the age interval (x-5, x+5)**.** An analytical derivation of the following formula may be found elsewhere (see, Lahiri, Rao, and Srinivasan, 2005)

$$\hat{\mathbf{b}}_x \cong \frac{1}{\hat{\overline{\mathbf{N}}}(\mathbf{x+})}\left[\frac{{}_5\hat{\overline{\mathbf{N}}}_{x-5} * {}_5\hat{\overline{\mathbf{N}}}_x \ln({}_5\hat{\overline{\mathbf{N}}}_{x-5} / {}_5\hat{\overline{\mathbf{N}}}_x)}{5({}_5\hat{\overline{\mathbf{N}}}_{x-5} - {}_5\hat{\overline{\mathbf{N}}}_x)}\right] \text{---------}(7)$$

The symbol ${}_5\overline{\mathrm{N}}_x$ stands for the average person years lived in the age group (x, x+5) during the intercensal period, and it can be estimated through the following formula (Preston and Bennet, 1983).

$${}_5\hat{\overline{\mathbf{N}}}_x = \frac{{}_5\mathbf{P}_x(\mathbf{t}+\mathbf{n}) - {}_5\mathbf{P}_x(\mathbf{t})}{\mathbf{n} \times {}_5\hat{\mathbf{r}}_x} \text{------------}(8),$$

$$\textbf{where } {}_5\mathbf{r}_x = \frac{1}{\mathbf{n}}\ln\left[\frac{{}_5\mathbf{P}_x(\mathbf{t}+\mathbf{n})}{{}_5\mathbf{P}_x(\mathbf{t})}\right] \text{-----------}(9)$$

where 'n' stands for any intercensal interval, not necessarily multiple of 5.

The Data Used and the Application:

The proposed technique requires enumerated age-data at two points of time, not necessarily multiple of 5 years apart, of a closed population. To test the validity of the procedure, the present technique has been applied to different quality of age-data starting with the age-data of Japan (1965-70) having reasonably good age-reporting followed by those of Korea (1990-95) and China (1982-90) with moderate error in age-reporting, and finally with those of India (1981-91 & 1991-2001) that are heavily distorted due to age-misreporting.

For all the aforementioned countries other than India the values of e(x) for ages 75 and 80 were obtained first through the formula (6), applicable in a stable population, under the assumption of local stability of the sub-population aged 75 and above. Now, knowing the ratios **R(x) = e(x+10)/e(x)** estimated through the formula (5.1), the other values of e(x), that is, at ages 70, 65, 60 and 55 were obtained successively through the repeated application the ratios **R(x)** in a reverse order. In other words, e(75) multiplied by quantity **1/R(65)** leads to e(65) which when multiplied by **1/R(55)** gives an estimate of e(55). Similarly, the value of e(80) estimated through the formula (6) gives the values of e(70) and e(60) successively. Since at very advanced ages 75 and above, the age misreporting errors are likely be relatively high, particularly in Korea and China, one may make use of the e(x) values at ages 65 and 70, estimated through the formula (6) under the assumption of local stability, to obtain the values of e(55) & e(60), and e(75) & e(80) through the use of the corresponding **R(x)** ratios in reverse and forward order respectively. The average of the two sets of values for e(x) at various quinquennial ages starting between 55 and 80, estimated through the procedure mentioned above may be taken as the final estimates of e(x) at older ages between 55 and 80 for Japan, Korea and China. In case of India where the quality of the age-data is extremely poor at very old ages, viz., 75 and above, we make use of the second procedure, mentioned above, which is based on the e(x) values at relatively younger ages 65 and 70.

The ratios **e(x+10)/e(x)** along with the **e(x)** values for males and females at quinquennial ages between 55 and 80 for various countries starting with Japan (1965-1970), followed by Korea (1990-1995), China (1982-1990) and India (1981-1991 & 1991-2001) are presented in the columns (9) and (10) of the Tables 1 to 10 respectively. It is encouraging to note that the indirect estimates of the e(x) at older ages for the countries obtained through the successive application of the formulas (6) and (5.1) are very close to those of the official estimates obtained through the life table techniques based on age-specific death rates.

References:

Bennett, N.G., and S. Horiuchi (1981) **Estimating Completeness of Death Registration in a Closed Population**, ***Population Index*,** 47(2), 207-221.

Bennett, N.G., and S. Horiuchi (1984) **Mortality Estimates from Registered Deaths in Less Developed Countries,** ***Demography***, 21(4), pp. 217-233.

Horiuchi, S. and A. J. Coale (1982) **A Simple Equation for Estimating the Expectation of Life at Old Ages,** ***Population Studies***, Vol. 36, pp.317-326.

Lahiri, Subrata (1983): **Life Table Construction from Population Age-distributions Suffering from Response Biases in Age-report:A new Technique with Application to Indian Census Age-returns**, Indian Statistical Institute, Calcutta,unpublished Ph.D. Dissertation.

Lahiri, Subrata (1990) **Some New Approaches to the Estimation of Life Expectancies at Older Ages**, In ***Dynamics of Population and Family Welfare, 1989***, (eds. by Srinivasan and K.B. Pathak), pp.315-341.

Lahiri, Subrata (2003) **Some New Demographic Equations in Survival Analysis under Generalized Population Model: Applications to Swedish and Indian Census Age-data for Estimating Adult Mortality,** Submitted to PAA 2004 Annual Meeting, Boston, Massachusetts, April 1-3, 2004.

Lahiri, Subrata and Lysander Menezes (2003) **Estimation of Adult Mortality from Two Enumerations of A Destabilized Population Subject to Response Biases in Age-Reporting,** The revised and updated version of the paper was included as one of the part of the research project report (unpublished) entitled, "**A Study of Mortality in India and Some Selected Developed and Developing Countries: Perspectives, Contrast and Challenges" which was prepared under the *Fulbright New Century Scholars Award* to the first author during 2001-2002 and presented by him in the "*Conference on Global Health*" held at the Airlie Center, Warrenton, Virginia, USA during November 1-5, 2002.**

Lahiri, Subrata, Arni S. R. Srinivasa Rao, and S. Srinivasan (2005) **Role of Age-specific Growth Rates on Population Ageing in Some Developed and Developing Countries – A Comparative Study**, ***Demography-India***, 34(1): 63-83

Mitra, S. (1084) **Estimating the Expectation of Life at Old Ages, *Population Studies***, Vol. 38, pp. 313-319.

Preston, S.H., and A.J. Coale (1982) **Age structure, growth, attrition, and accession: A new synthesis**, ***Population Index***, 48(2): 217-259.

Preston, S.H., and N.G., Bennett (1983) **A census-based method for estimating adult mortality, *Population Studies***, 37(1):91-104.

Preston, Samuel H., and Subrata Lahiri (1991) **A short-cut method for estimating death registration completeness in destabilized populations**, ***Mathematical Population Studies (U.S.A.)***, Vol. 3, No.1, pp.39-51.

Table 1

Estimation of $\frac{e(x+10)}{e(x)}$ for Japan Males during 1965-1970 at age fifty-five and above

Age	Population 1965	Population 1970	$_5r_a$	r_{a+}	$_5N_x$	C_{a+}	r_k	e(x+10)/e(x)	e_x^0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	2172903	2157091	-0.00146	0.01655	2164987	62.19			
55	1930469	2042055	0.01124	0.02242	1985739	65.34	0.02256	0.6610	19.13
60	1625089	1755397	0.01543	0.02719	1689406	68.69	0.02737	0.6327	15.61
65	1218867	1399180	0.02759	0.03391	1306951	72.22	0.03404	0.5786	12.65
70	788994	961641	0.03958	0.03889	872472	75.94	0.03888	0.5657	9.88
75	451871	531898	0.03261	0.03813	490798	79.77	0.03813		7.32
80	186946	241356	0.05109	0.04734	212994	83.56			5.59
85+	73855	89100	0.03753	0.03753	81239				

Table 2

Estimation of $\frac{e(x+10)}{e(x)}$ for Japan Females during 1965-1970 at age fifty-five and above

Age	Population 1965	Population 1970	$_5r_a$	r_{a+}	$_5N_x$	C_{a+}	r_k	e(x+10)/e(x)	e_x^0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	2485095	2648360	0.01273	0.02639	2565862	62.84			
55	2071540	2382691	0.02799	0.03086	2223488	66.23	0.03087	0.6663	23.00
60	1719370	1970485	0.02726	0.03200	1842076	69.68	0.03201	0.6328	18.96
65	1343444	1584699	0.03303	0.03431	1460753	73.18	0.03432	0.5665	15.33
70	955567	1172155	0.04086	0.03511	1060176	76.77	0.03510	0.5415	12.00
75	644043	736258	0.02676	0.03026	689122	80.38	0.03032		8.68
80	341170	408191	0.03587	0.03453	373679	83.91			6.50
85+	176068	206511	0.03190	0.03190	190885				

Table 3

Estimation of $\frac{e(x+10)}{e(x)}$ for Korea Males during 1990-1995 at age fifty-five and above

Age	Population 1990	Population 1995	$_5r_a$	r_{a+}	$_5N_x$	C_{a+}	r_k	e(x+10)/e(x)	e_x^0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	994511	1028887	0.00680	0.03249	1011602	60.77			
55	760993	923625	0.03874	0.04377	839686	64.40	0.04389	0.6344	18.58
60	494845	673719	0.06171	0.04665	579690	68.34	0.04660	0.6444	15.82
65	375752	420873	0.02268	0.03686	397886	72.14	0.03692	0.6534	11.79
70	233308	293696	0.04604	0.04835	262345	75.89	0.04839	0.5731	10.20
75	127905	160498	0.04540	0.05099	143585	79.76	0.05095		7.70
80	54861	71267	0.05233	0.06033	62707	83.54			5.84
85+	18830	28370	0.08198	0.08198	23275				

Table 4

Estimation of $\frac{e(x+10)}{e(x)}$ for Korea Females during 1990-1995 at age fifty-five and above

Age	Population 1990	Population 1995	$_5r_a$	r_{a+}	$_5N_x$	C_{a+}	r_k	e(x+10)/e(x)	e_x^0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	1015507	1034881	0.00378	0.02959	1025163	62.86			
55	861860	989836	0.02769	0.03796	924372	66.21	0.03810	0.6731	24.26
60	662214	821363	0.04308	0.04220	738934	69.80	0.04220	0.6514	20.64
65	524562	623106	0.03443	0.04177	572421	73.39	0.04136	0.6033	16.33
70	361808	468848	0.05183	0.04629	413019	77.02	0.04628	0.5582	13.44
75	249266	295175	0.03381	0.04187	271574	80.63	0.04196		9.85
80	140451	174924	0.04390	0.05079	157057	84.09			7.51
85+	75496	103448	0.06300	0.06300	88739				

Table 5

Estimation of $\frac{e(x+10)}{e(x)}$ for China Males during 1982-1990 at age fifty-five and above

Age	Population 1982	Population 1990	$_5r_a$	r_{a+}	$_5N_x$	C_{a+}	r_k	e(x+10)/e(x)	e_x^0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	21528986	24117110	0.01419	0.02650	22798569	61.44			
55	17493925	21865620	0.02788	0.03116	19598577	64.83	0.03119	0.6238	19.07
60	13709397	17514640	0.03062	0.03274	15534420	68.36	0.03276	0.6095	15.08
65	10171973	12937720	0.03006	0.03405	11499467	71.98	0.03413	0.6001	11.89
70	6434731	8367690	0.03283	0.03742	7358949	75.77	0.03746	0.5815	9.19
75	3496703	4699180	0.03695	0.04282	4068367	79.60	0.04287		7.14
80	1350766	1996750	0.04886	0.05369	1652771	83.47			5.35
85+	474973	716430	0.06824	0.06824	552096				

Table 6

Estimation of $\frac{e(x+10)}{e(x)}$ for China Females during 1982-1990 at age fifty-five and above

Age	Population 1982	Population 1990	$_5r_a$	r_{a+}	$_5N_x$	C_{a+}	r_k	e(x+10)/e(x)	e_x^0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	19286515	21546710	0.01385	0.02327	20395744	62.81			
55	16400402	19887010	0.02410	0.02628	18087734	66.11	0.02629	0.6463	21.73
60	13652807	16540770	0.02399	0.02714	15050638	69.50	0.02715	0.6214	17.40
65	11088397	13457180	0.02420	0.02869	12234593	72.92	0.02875	0.5989	14.05
70	7913314	9751450	0.02611	0.03165	8800411	76.50	0.03167	0.5897	10.81
75	5120340	6271900	0.02536	0.03665	5676666	80.10	0.03669		8.41
80	2353829	3374530	0.04503	0.05244	2833606	83.73			6.38
85+	930439	1621540	0.06943	0.06943	1244162				

Table 7

Estimation of $\frac{e(x+10)}{e(x)}$ for Indian Males during 1981-1991 at age fifty-five and above

Age	Population 1981	Population 1991	$_5r_a$	r_{a+}	$_5N_x$	C_{a+}	r_k	e(x+10)/e(x)	e_x^0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	13794218	16904890	0.02034	0.02554	15296876	61.74			
55	8498073	10941747	0.02527	0.02781	9668496	65.75	0.02756	0.6620	18.54
60	9385925	11907237	0.02379	0.02877	10596636	68.88	0.02932	0.7003	15.85
65	4793728	6493630	0.03035	0.03231	5600750	73.42	0.03193	0.7939	12.28
70	4190266	5535950	0.02785	0.03349	4831917	76.97	0.03429	0.8591	11.10
75	1599673	2102284	0.02732	0.03958	1839549	81.80	0.03843		9.75
80	1360436	2094937	0.04317	0.04819	1701343	84.81			9.53
85+	692839	1229687	0.05737	0.05737	935736				

Table 8

Estimation of $\frac{e(x+10)}{e(x)}$ for Indian Females during 1981-1991 at age fifty- five and above

Age	Population 1981	Population 1991	$_5r_a$	r_{a+}	$_5N_x$	C_{a+}	r_k	e(x+10)/e(x)	e_x^0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	11602684	14208702	0.02026	0.02473	12861721	62.10			
55	7918507	10530755	0.02851	0.02646	9162653	65.81	0.02633	0.6141	19.77
60	8781635	10841739	0.02107	0.02568	9775535	68.96	0.02613	0.6783	15.78
65	4720693	6364869	0.02988	0.02883	5501897	73.38	0.02856	0.7873	12.14
70	4005876	5018131	0.02253	0.02817	4493015	77.04	0.02883	0.8499	10.70
75	1562823	2043289	0.02681	0.03405	1792336	81.75	0.03298		9.56
80	1348284	1893925	0.03398	0.03918	1605683	84.75			9.10
85+	725206	1173962	0.04817	0.04817	931640				

Table 9

Estimation of $\frac{e(x+10)}{e(x)}$ for Indian Males during 1991-2001 at age fifty-five and above

Age	Population 1991	Population 2001	$_5r_a$	r_{a+}	$_5N_x$	C_{a+}	r_k	e(x+10)/e(x)	e_x^0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	16904890	19851608	0.01607	0.02188	18338809	62.18			
55	10941747	13583022	0.02162	0.02422	12214827	66.07	0.02399	0.6924	19.28
60	11907237	13586347	0.01319	0.02517	12728338	69.21	0.02607	0.6551	17.45
65	6493630	9472103	0.03775	0.03259	7889383	73.35	0.03213	0.6822	13.35
70	5535950	7527688	0.03073	0.02940	6480890	76.98	0.02925	0.7134	11.43
75	2102284	3263209	0.04397	0.02802	2640346	81.63	0.02826		9.11
80	2094937	2257951	0.00749	0.01645	2175426	84.65			8.15
85+	1229687	1661029	0.03007	0.03007	1434566				

Table 10

Estimation of $\frac{e(x+10)}{e(x)}$ for Indian Females during 1991-2001 at age fifty- five and above

Age	Population 1991	Population 2001	$_5r_a$	r_{a+}	$_5N_x$	C_{a+}	r_k	e(x+10)/e(x)	e_x^0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	14208702	16735951	0.01637	0.02909	15437865	62.57			
55	10530755	14070325	0.02898	0.03348	12215188	66.03	0.03330	0.6192	22.70
60	10841739	13930432	0.02507	0.03516	12321632	69.22	0.03582	0.6185	18.86
65	6364869	10334852	0.04847	0.04128	8190121	73.28	0.04114	0.7034	14.05
70	5018131	7180956	0.03584	0.03648	6035089	77.15	0.03653	0.7563	11.67
75	2043289	3288016	0.04757	0.03711	2616492	81.69	0.03701		9.89
80	1893925	2307983	0.01977	0.02948	2094136	84.76			8.82
85+	1173962	1811755	0.04339	0.04339	1469868				